ITRF Combination: Theoretical and Practical Considerations and Lessons from ITRF2008

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Abstract

The current ITRF construction is based on a two-step approach, combining input data provided by space geodesy techniques (VLBI, SLR, GPS, DORIS) in the form of time series of station positions and Earth Orientation Parameters. In the first step, the individual technique time series are rigorously stacked (accumulated) yielding long-term secular solutions, while the second step forms the ITRF final combination of the four technique long-term solutions together with local ties at co-location sites. The combination model involves a 7- or 14-parameter similarity transformation formula, for time series stacking and multi-technique combination, respectively. Not all these parameters are necessarily estimated in the combination process, some or all of them could be eliminated from the constructed normal equation, depending on the combination purpose. The paper discusses the relevance of the combination model and its appropriateness for the ITRF combination activities, both from the theoretical and practical point of views, and in particular for the reference frame specifications (origin, scale, orientation and their time evolutions). Selected analysis tests of ITRF2008 input data and results are used to illustrate the discussion as well as to address lessons learned from ITRF2008 experience.

Keywords

Reference systems • Reference frames • Time evolution • ITRF

1 Introduction

With the advent of space geodesy in the early 1980s, the importance of reference frames has became more and more important, as a function of technological and data analysis advances. Appropriate definition of a Terrestrial Reference System (TRS) and its precise materialization through a Terrestrial Reference Frame (TRF) are fundamental to many applications in geosciences. The main TRS and TRF specifications are the origin, scale, orientation and their time evolutions. Any defect on these parameters would have an impact on the results and interpretation of geodetic and geophysical applications that require the usage of a reference frame, such as:

- Precise Orbit Determination, not only for Global Navigation Satellite Systems (GNSS), but also for other satellite missions dedicated to Altimetry, Oceanography, Gravity;
- Earth sciences applications, such as tectonic motion and crustal deformation, sea level variations and Earth rotation (Collilieux and Altamimi 2012).

Given the currently available reference frame products provided by space geodetic techniques, representations of terrestrial reference frames are divided in two categories:

- “Quasi-instantaneous” reference frame which gives access to mean station positions at “short” interval, using space geodesy observations over, e.g. 1 or several hours, 1 day, and up to 1 week. Note that over 1 month, stations may be subject to displacements of the order of

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1–10 mm due to plate tectonic motion. Time series of such frames encompass not only station linear motion due to plate tectonic, but also non-linear motion and discontinuities due to geophysical events such as Earthquakes.

- Long-term secular frame which gives access to mean station positions (X) at a reference epoch, t_0, and station linear velocities ($\dot{X}$). The propagation of station positions (as well as their variances) at any epoch t is operated using $X(t) = X(t_0) + \dot{X} \times (t - t_0)$.

The International Terrestrial Reference Frame (ITRF) is by definition a secular frame, and therefore precise definition of its defining parameters are of interest to many Earth science applications. For more details regarding the International Terrestrial Reference System and Frame (ITRS, ITRF) description and definition, the reader may refer to Chap. 4 of the IERS Conventions (Petit and Luzum 2010). As the input data of the current ITRF construction are in the form of time series of station positions (and Earth Orientation Parameters: EOPs), it is fundamental to evaluate the temporal behavior of not only the individual station positions, but also, and equally important, the frame physical parameters, namely the scale and the origin components. As it will be quantified in this paper, any temporal discontinuity or unexpected drift of these parameters will directly impact the estimated constant station velocities. We recall here that the current scientific requirement in terms of accuracy and stability over time of the origin and scale of the ITRF are believed to be at the level of 1 mm and 0.1 mm/year (Plag and Pearlman 2009; Blewitt et al. 2010; NRC 2010). This requirement is at least 10 times higher than what is achievable today, due mainly to the degradation of the network of space geodesy techniques and their intrinsic systematic errors.

In the following, we recall in Sect. 2 the CATREF combination model used for the ITRF computation and discuss the usefulness of including the transformation parameters in that model. Section 3 is devoted to numerical applications intended to verify that the estimation of weekly transformation parameters in the time series stacking has no impact on the ITRF secular frame. In Sect. 4 we summarize the main ITRF2008 lessons regarding the usage and consistency of local ties in the ITRF combination.

2 CATREF Combination Model

Although the CATREF combination model is extensively described in several publications (see for instance Altamimi et al. 2002, 2007, 2011), we review here its main equations for the purpose of this paper, discussing the benefit of including the transformation parameters in this model. The main two equations are written as:

$$\begin{cases} X_s^i = X_c^i + (t_s^i - t_0)\,\dot{X}_c^i \\ \qquad + T_k + D_k X_c^i + R_k X_c^i \\ \qquad + (t_s^i - t_k)\left[\dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i\right] \\ \dot{X}_s^i = \dot{X}_c^i + \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i \end{cases} \tag{2.1}$$

$$\begin{cases} x_s^p = x_c^p + R2_k \\ y_s^p = y_c^p + R1_k \\ UT_s = UT_c - \frac{1}{f} R3_k \\ \dot{x}_s^p = \dot{x}_c^p \\ \dot{y}_s^p = \dot{y}_c^p \\ LOD_s = LOD_c \end{cases} \tag{2.2}$$

where for each point i, X_s^i (at epoch t_s^i) and $\dot{X}_s^i$ are positions and velocities of technique solution s and X_c^i (at epoch t_0) and $\dot{X}_c^i$ are those of the combined solution c. For each individual frame k, as implicitly defined by solution s, D_k is the scale factor, T_k the translation vector and R_k the rotation matrix. The dotted parameters designate their derivatives with respect to time. The translation vector T_k is composed of three origin components, namely T_x, T_y, T_z, and the rotation matrix of three small rotation parameters: R_x, R_y, R_z, following the three axes, respectively X, Y, Z. t_k is a conventionally selected epoch of the seven transformation parameters. In addition to Eq. 2.1 involving station positions (and velocities), the EOPs are added by Eq. 2.2 making use of pole coordinates x_s^p, y_s^p and universal time UT_s as well as their daily rates $\dot{x}_s^p$, $\dot{y}_s^p$ and LOD_s. The link between the combined frame and the EOPs is ensured via the three rotation parameters appearing in the first three lines of Eq. 2.2. Detailed derivation of the above equations could also be found in Altamimi and Dermanis (2011), and more discussion regarding the polar motion rate equations is available in Altamimi et al. (2011).

CATREF combination model was designed to be as general as possible in order to be able dealing with reference frame solutions of different natures. In time series stacking, the first line of Eq. 2.1, nullifying its last terms involving the rates of the transformation parameters, and the entire Eq. 2.2 are used. The entire two equations are used in the combination of multi-technique long-term solutions. Note that in both combination cases, the EOPs could be included or discarded. Note also that polar motion rate equations (fourth and fifth lines of Eq. 2.2) are included in the combination model in order to be able to take into account solutions where the polar motion representation is in the form of offset and drift. For solutions where the polar motion representation is in the form of a continuous piece-wise linear function, the polar motion rate equations are of course discarded.

It should be noted here that including the transformation parameters in the combination model does not imply systematically estimating them. Depending on the application, some or all of them could/should be eliminated from the constructed normal equation system. However, from the ITRF combination perspective, we list below the main advantages of estimating the transformation parameters in the two-step procedure: time series stacking and multi-technique combination of long-term solutions. In the step of time series stacking, the main advantages are to allow:

- Evaluating the temporal behavior of the reference frame parameters and in particular the physical ones, namely the scale and origin components. In the same way as a station presenting discontinuities in its position time series (due for example to earthquakes) should not be part of the list of reference frame stations, a particular parameter having such discontinuities should be excluded from the reference frame definition. Otherwise adopting such a parameter in the ITRF definition would introduce biases in its time evolution and consequently in the estimated station velocities. In a simulated study, we found that a discontinuity of 1.5 ppb in the middle of a scale time series covering 4 years induces a bias of 3 mm/year in the vertical velocities of all the stations, and 1 mm/year over a time-span of 16 years. In the same study, a simulated scale drift of 0.15 ppb/year, induces a vertical velocity bias of 1 mm/year. Therefore estimating the transformation parameters in the time series analysis is a tool to evaluate the level of stability over time of the frame defining parameters;
- Assessing robustly the repeatability (internal precision) of the analyzed solutions by computing the Weighted Root Mean Scatter (WRMS) of each epoch (daily, weekly) solution with respect to the combined long-term solution;
- Applying the inner minimal constraints as described in Altamimi et al. (2007), having the advantage of preserving mean origin and scale of satellite technique and the mean scale of VLBI solutions;

In the second step of the ITRF combination involving multi-technique long-term solutions and local ties, estimating the transformation parameters has the following advantages:

- The ITRF defining parameters are eliminated from the normal equation system, offering in this way various possible options to define the combined reference frame among the incorporated solutions, e.g. adopting SLR origin; SLR, VLBI or their average to define the scale.
- Possible biases between the technique solutions are rigourously quantified. This is the case of the scale bias of 1.05 ppb at epoch 2005.0 and 0.049 ppb/year between VLBI and SLR solutions determined from the ITRF2008 combination (Altamimi et al. 2011);
- Assessing the uncertainties of the transfer of the SLR origin and VLBI and SLR mean scale to GPS and DORIS frames as it will be shown in this paper.

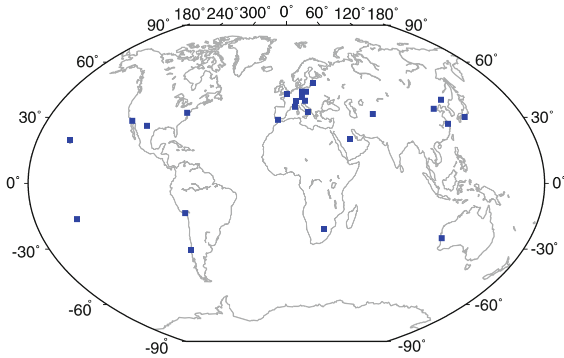

Fig. 2.1 Distribution of the 26 SLR stations used in the test combinations

In the following section devoted to numerical applications using some ITRF2008 input data, we discuss the difference in the results between estimating and not estimating the scale and origin parameters in the time series stacking and in a test combination of multi-technique long term solutions. We show in particular that including or not these parameters in the observation equations model produces the same mean origin and scale of the obtained two long-term solutions.

However, it should be noted that an obvious alternative to the two-step procedure is the one-step approach where all the technique time series are stacked together with local ties. Although this one-step procedure is computationally prohibitive, it should mathematically be equivalent to the two-step approach.

3 Numerical Applications

In order to evaluate the difference in the results between estimating and not the origin components and the scale when stacking time series provided by satellite techniques, we use hereafter, as an example, the ILRS SLR time series that contributed to the ITRF2008 (Pavlis et al. 2010). We extracted from these weekly solutions 26 stations (illustrated by Fig. 2.1) with long observation histories (from 7 to 16 years) and conducted two types of stacking: with and without including the weekly origin and scale parameters in the observation equations model. We first analyzed the results on three aspects: the weekly WRMS with respect to the combined/stacked frame, the precision of the estimated station positions and velocities and the 14

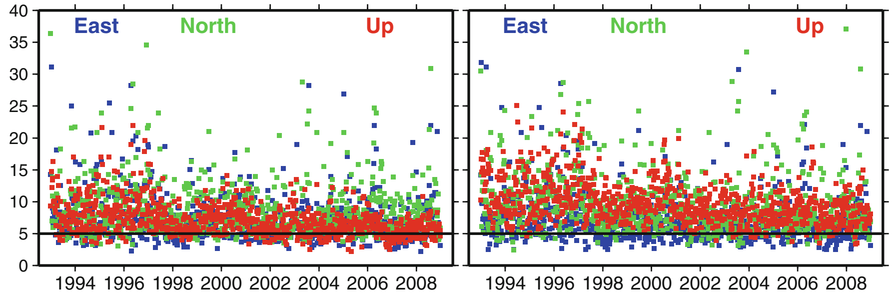

Fig. 2.2 Weekly WRMS of SLR time series as results of their stacking with (*left*) and without (*right*) estimating the weekly translations and scales

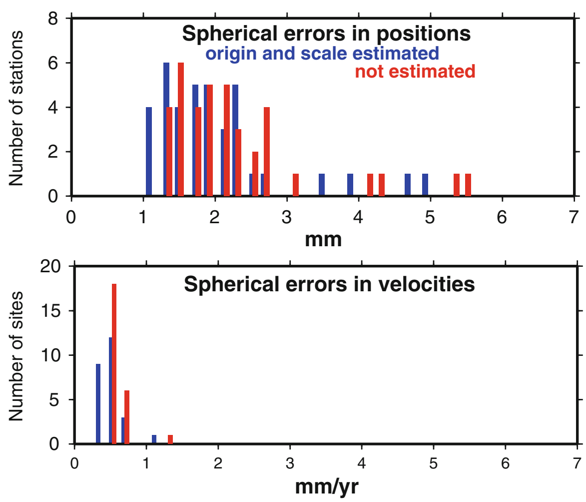

Fig. 2.3 Spherical errors in station positions and velocities of the long-term SLR solutions obtained by stacking the time series with and without including the weekly origin and scale parameters in the observation equations model

transformation parameters between the two long-term solutions.

When accumulating (rigorously stacking) time series of station positions (including or not EOPs), the constructed normal equation that includes the transformation parameters has a rank deficiency of 14, corresponding to the combined frame parameters that have to be defined. In this case we used the inner minimal constraints as described in Altamimi et al. (2007) that make use of the time series of the 7 estimated parameters. In the second stacking where the origin and scale parameters were eliminated, the normal equation has a rank deficiency of six corresponding to the three rotations and their rates. In this case we also used the inner minimal constraints over the time series of the rotation parameters.

Figure 2.2 illustrates the weekly WRMS with respect to the combined frame, with and without estimating the weekly origins and scales. This figure shows expected increase of the residuals when weekly origins and scales are not estimated, given the fact that the transformation parameters absorb part of the time series biases. These biases include technique systematic errors, as well as geocenter motion and aliased loading effects (Collilieux et al. 2009, 2010). However, as shown by Collilieux et al. (2010), applying a loading model to the station position time series before the stacking decreases the annual signal present in the residuals, but would not absorb the systematic errors in these tested SLR time series. The mean of these WRMS shown at Fig. 2.2 are 7.3, 8.7, 7.2 mm, versus 7.2, 9.5, 9.8 mm in east, north, and vertical components, with and without estimating the transformation parameters, respectively.

Another interesting feature to examine is the level of precision of the estimated station positions and velocities of the two accumulated frames, with and without including the weekly transformation parameters in the observation equations model. Figure 2.3 displays the spherical formal errors of both estimations, computed by the square sum of the formal errors resulting from the least squares adjustment, following Altamimi et al. (2002). This figure indicates that the precision of station positions and velocities is higher when the transformation parameters are estimated. This is also an expected result because the formal errors shown at Fig. 2.3 are function of the variance factor of unit weight computed with the residuals illustrated by Fig. 2.2.

We also estimated 14 transformation parameters between the two estimated long-term frames and found that all are insignificantly different from zero. The WRMS values of the 14-parameter fit are of the order of 1 mm in positions and 0.2 mm/year in velocities. These results indicate that the two estimated frames are equivalent at the level of their intrinsic uncertainties. They also confirm that the non-linear variations related to loading effects which are partly absorbed by the estimated weekly translations and scales does not affect the ITRF frame parameters as demonstrated by Collilieux et al. (2010). These results are also expected since the loading effects induce seasonal (annual or semi-annual) variations, with no impact on the estimated linear velocities for stations with long time-span, whether these variations leak to the station residuals or to the epoch transformation parameters. Blewitt and Lavallée (2002) showed for instance that in case of GPS, 2.5 years is the minimum time-span necessary to minimize the effect of annual signals on the estimated linear station velocity. This also means that despite the poor SLR network geometry (see Fig. 2.1 where only 5 stations out of 26 are located in the southern hemisphere), estimating the weekly translations and scales does not affect the estimated linear velocities. This fact was demonstrated theoretically by Collilieux (2008), using a simpler case of translation parameters only, and uniform weighting.

In order to evaluate the behavior of the two estimated long-term solutions in a multi-technique combination, two combinations were tested, involving the long-term GPS solution used in the ITRF2008 elaboration. Local ties at 21 GPS-SLR co-location sites were used, adopting the same weighting as the one used in the ITRF2008 combination. The results of these two combinations show that the estimated 14 transformation parameters between GPS and SLR solutions are consistent in both cases, their differences being within the uncertainties of the estimated parameters. The resulting two combined frames are also statistically equivalent: the differences are in average at the level of 1 mm in station positions and 0.2 mm/year in velocities. The level of their agreement with local ties is also within the noise and consistent with the results obtained in the ITRF2008 combination.

4 Lessons from ITRF2008

Detailed ITRF2008 results and discussion are published in Altamimi et al. (2011), and in particular regarding the impact of local ties on the ITRF2008 combination. Consistency between local ties and space geodesy estimates are critically analyzed. We showed in particular that 50 % of the available SLR and VLBI tie vectors to GPS exhibit residuals larger than 6 mm, and about 30 % have residuals larger than 10 mm. These discrepancies are to be understood as differences between local ties and space geodesy estimates. The reasons for these discrepancies are difficult to pinpoint, since they could be due to errors in local ties, in space geodesy estimates or in both. There are indications discussed in Altamimi et al. (2011) that these discrepancies are most likely to be due to systematic errors in space geodesy estimates, rather than in local surveys. In addition to the critical analysis discussed in Altamimi et al. (2011), we report here the impact of the usage of local ties on the level of uncertainty of the transfer of SLR origin and SLR and VLBI mean scale to GPS frame. Table 2.1 lists the levels of that uncertainty for three different cases of used local tie sets: (1) ties where the discrepancies with space geodesy estimates do not exceed 6 mm, (2) ties where the discrepancies are less than 10 mm, and (3) all available ties used in the ITRF2008 combination provided in SINEX files where 63 % of them are with full variance-covariance information. The results listed in this table indicate that the more ties used the better is the precision of the translations and scale of the ITRF2008 frame and their transfer to GPS frame. This kind of assessment is possible thanks to the inclusion of the transformation in the CATREF combination model.

Table 2.1 Uncertainties (in mm) of the transfer of SLR origin and SLR and VLBI mean scale to GPS frame, as a function of local ties used

Ties used	TX	TY	TZ	Scale
Ties discrepancies <6 mm	2.5	2.5	2.5	1.4
Ties discrepancies up to 10 mm	1.4	1.1	1.2	1.2
All tie SINEX files (ITRF2008 results)	0.6	0.5	0.6	0.6

5 Conclusion

CATREF combination model was developed and designed for the purpose of the ITRF combinations and is well adapted for time series stacking and multi-technique combinations. It is based on the 7- or 14-parameter similarity transformation, a classical mathematical formula allowing transformation between distinct reference frames. Although these parameters are not all systematically estimated in all combination processes, we identified the advantages of including them in the ITRF combination procedure.

Using a sub-set of SLR time series incorporated in the ITRF2008 combination, we found that with or without including the weekly scales and origin translation components in the observation equations model, we obtain equivalent long-term solutions of station positions and velocities, at the level of their intrinsic uncertainties. Combining separately these two long-term solutions with GPS solution included in the ITRF2008, involving 21 local ties gave also equivalent results within the uncertainties of the estimated parameters.

One of the main lessons of the ITRF2008 results reported by Altamimi et al. (2011) is the large inconsistency between local ties and space geodesy estimates: 50 % of the available SLR and VLBI tie vectors to GPS exhibit residuals larger than 6 mm, and about 30 % have residuals larger than 10 mm. Thanks to the inclusion of the transformation parameters in the combination model, we were able to assess the level of uncertainty of the transfer of SLR origin and mean SLR and VLBI scale to GPS frame. We showed here that the more ties used (properly weighted) the better is the precision of the origin and scale determination.

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