

# Preface

This book gives an introduction to the finite element method as a general computational method for solving partial differential equations approximately. Our approach is mathematical in nature with a strong focus on the underlying mathematical principles, such as approximation properties of piecewise polynomial spaces, and variational formulations of partial differential equations, but with a minimum level of advanced mathematical machinery from functional analysis and partial differential equations.

In principle, the material should be accessible to students with only knowledge of calculus of several variables, basic partial differential equations, and linear algebra, as the necessary concepts from more advanced analysis are introduced when needed.

Throughout the text we emphasize implementation of the involved algorithms, and have therefore mixed mathematical theory with concrete computer code using the numerical software MATLAB<sup>1</sup> and its PDE-Toolbox. A basic knowledge of the MATLAB language is therefore necessary. The PDE-Toolbox is used for pre and post processing (i.e., meshing and plotting).

We have also had the ambition to cover some of the most important applications of finite elements and the basic finite element methods developed for those applications, including diffusion and transport phenomena, solid and fluid mechanics, and also electromagnetics.

The book is loosely divided into two parts Chaps. 1–6 which provides basic material and Chaps. 7–14 which covers more advanced material and applications. In the first part Chaps. 1–4 gives an introduction to the finite element method for stationary second order elliptic problems. Here we consider the one dimensional case in Chaps. 1 and 2 and then extend to two dimensions in Chaps. 3 and 4. In Chap. 5 we consider time dependent problems and in Chap. 6 we give an introduction to numerical linear algebra for sparse linear systems of equations. In the second more advanced part we present the abstract theory in Chap. 7, various finite elements in Chap. 8, and a short introduction to nonlinear problems

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<sup>1</sup>MATLAB is a registered trademark of The MathWorks Inc. ([www.mathworks.com](http://www.mathworks.com))

in Chap. 9. In Chaps. 10–13 we consider applications to transport problems, solid mechanics, fluid mechanics, and electromagnetics. Finally, in Chap. 14 we give a short introduction to discontinuous Galerkin methods.

The book is based on lecture notes used in various courses given by the authors and their coworkers during the last eight years at Chalmers University of Technology, Umeå University, Uppsala University, and at the University of Oslo. Several different courses in engineering and mathematics programs can be based on the material in the book. The following are some examples of possible courses that we have had in mind while developing the material:

- Short introduction to finite elements as part of a calculus or numerical analysis course. Chapters 1 and 2.
- Introduction to finite elements as part of a calculus or numerical analysis course, with focus on one dimensional stationary and time dependent problems. Chapters 1 and 2 and parts of Chapter 5.
- Introduction to finite elements as part of a calculus or numerical analysis course, with focus on stationary problems in one and two dimensions. Chapters 1–4.
- First course on finite elements. Chapters 1–6.
- Second course on finite elements and its applications. Chapters 7–14.

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The Finite Element Method: Theory, Implementation,  
and Applications

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2013, XVII, 395 p., Hardcover

ISBN: 978-3-642-33286-9