Preface

This volume is an attempt to write a textbook providing a modern introduction to stochastic geometry, spatial statistics, the theory of random fields and related topics. It has been made out of selected contributions to the Summer Academy on Stochastic Geometry, Spatial Statistics and Random Fields

http://www.uni-ulm.de/summeracademy09

which took place during 13–26 Sep 2009 at Söllerhaus, an Alpine conference centre of the University of Stuttgart and RWTH Aachen, in the village Hirschegg (Austria). It was organized by the Institute of Stochastics of Ulm University in cooperation with the Chair of Probability Theory of Lomonosov Moscow State University. In contrast with previous schools on this subject (Sandbjerg 2000, Martina Franca 2004, Sandbjerg 2007), this summer academy concentrated on the asymptotic theory of random sets, fields and geometric graphs. At the same time, it provided an introduction to more classical subjects of stochastic geometry and spatial statistics, giving (post)graduate students an opportunity to start their own research within a couple of weeks. The summer academy hosted 38 young participants from 13 countries (Australia, Austria, Denmark, Germany, France, Mongolia, Russia, Romania, Sweden, Switzerland, UK, USA and Vietnam). Twelve experts gave lectures on various domains of geometry, probability theory and mathematical statistics. Moreover, students and young researchers had the possibility to give their own short talks.

As it was pointed out above, this volume is focused on the asymptotic methods in the theory of random geometric objects (point patterns, sets, graphs, trees, tessellations and functions). It reflects advances in this domain made within the last two decades. This especially concerns the limit theorems for random tessellations, random polytopes, finite point processes and random fields.

The book is organized as follows. The first chapter provides an introduction to the theory of random closed sets (RACS). It starts with the foundations of geometric probability (Buffon needle problem, Bertrand's paradox) and continues with the classical theory of random sets by Matheron. Then it gives laws of large numbers and limit theorems for Minkowski sums and unions of independent identically distributed (i.i.d.) RACS. Chapter 2 provides basics of the classical integral geometry and its applications to stereology, a part of spatial stochastics which deals with the reconstruction of the higher-dimensional properties of geometric objects from lower-dimensional sections. In Chap. 3, principal classes of spatial point processes (Poisson-driven point processes, finite point processes) are introduced. Their simulation and statistical inference techniques (partially using the Markov Chain Monte Carlo (MCMC) methods) are discussed as well. Chapter 4 provides an account of the theory of marked point processes and the asymptotic statistics for them in growing domains. Ergodicity, mixing and m-dependence properties of marked point processes are studied in detail. Random tessellation models are the matter of Chap. 5. There, Poisson-driven tessellations as well as Cox processes on them and hierarchical networks constructed on their basis are considered. Scaling limits for some characteristics of these networks are found. Applications to telecommunication networks are also discussed. Distribution tail asymptotics and limit theorems for the characteristics of the (large) typical cell of Poisson hyperplane and Poisson–Voronoi tessellations are given in Chap. 6. The shape of large cells of hyperplane tessellations as well as limit theorems for some geometric functionals of convex hulls of a large number of i.i.d. points within a convex body and of random polyhedra are dealt with in Chap. 7. Weak laws of large numbers and central limit theorems for functionals of finite point patterns are discussed in Chap. 8. Additionally, their applications to various topics ranging from optimization to sequential packings of convex bodies are touched upon. Chapter 9 surveys the elementary theory of random functions with the focus on random fields. Basic classes of random field models as well as an account of the correlation theory, statistical inference and simulation techniques are provided. Special attention is paid to infinitely divisible random functions. Dependence concepts for random fields (such as mixing, association, (BL, θ)-dependence) as well as central limit theorems for weakly dependent random fields are the subject of Chap. 10. They are applied to establish the limiting behaviour of the volume of excursion sets of weakly dependent stationary random fields. Chapter 11 focuses on almost sure limit theorems for partial sums (or increments) of random fields on $\mathbb{N}^d$ such as laws of large numbers, laws of single or iterated logarithm and others. In the final chapter, the geometry of large rooted plane random trees with nearest neighbour interaction is studied. A law of large numbers, a large deviation principle for the branching type statistics and scaling limits of the tree are considered. Connections of these results with the solutions of some partial differential equations are discussed as well. Some of the chapters are written in a more formal and rigorous way than others which reflects the personal taste and style of the authors.

The topics of this volume are (almost) self-contained. Thus, we recommend Chaps. 1, 2 and 7 for the first acquaintance with the theory of random sets. A reader interested in the (asymptotic theory of) point processes might start reading with Chap. 3 and continue with Chaps. 4, 5 and 8 following the references to Chap. 1 if needed. Readers with an interest in tessellations and random polytopes might additionally read Chaps. 2, 6 and 7. To get a concise introduction to random fields and limit theorems for them, one could read Chaps. 9–11 occasionally following the references to earlier chapters. For random graphs and trees, Chaps. 8 and 12 are a good starting point.

All in all, the authors hope that the present volume will be helpful to graduates and PhD students in mathematics to get a first glance of the geometry of random objects and its asymptotical methods. Written in the spirit of a textbook (with a significant number of proofs and exercises for active reading), it might be also instrumental to lecturers in preparing their own lecture courses on this subject.

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