Chapter 2
Transport Processes in Stellar Interiors

Stéphane Mathis

Abstract Stars are rotating and magnetic bodies. Moreover, more and more constraints are obtained on such dynamical processes using, for example, seismology and spectropolarimetry. Therefore, it is now necessary to get a complete and coherent picture of dynamical processes in stellar interiors. However, to simulate such processes in a star in full details would require treating length scales and time scales spanning many orders of magnitude. This is clearly not feasible, even with the most powerful computers available today. This is the reason why it is nowadays necessary to use and couple 1-D, 2-D, and 3-D modelings to get a global picture of macroscopic MHD transport processes in stellar interiors. In this review, we report the state of the art of the modeling of transport processes in stellar interiors (both in radiation and in convection zones) aimed to study the stars angular momentum history, the related profile of differential rotation, and their magnetism.

2.1 Introduction

Stars are rotating and magnetic bodies. Moreover, more and more constraints are obtained on such dynamical processes, for example, using helioseismology (see, for example, [47]), asteroseismology [10, 34, 43, 82], and spectropolarimetry [38]. Therefore, it is now necessary to get a complete and coherent picture of dynamical processes in stellar interiors. However, to simulate such processes in a star in full details would require treating length scales and time scales spanning many orders of magnitude. This is clearly not feasible, even with the most powerful computers available today. Therefore, either one chooses to describe what occurs on a dynamical time scale (such as a convective turnover time), or one focuses on the long-time

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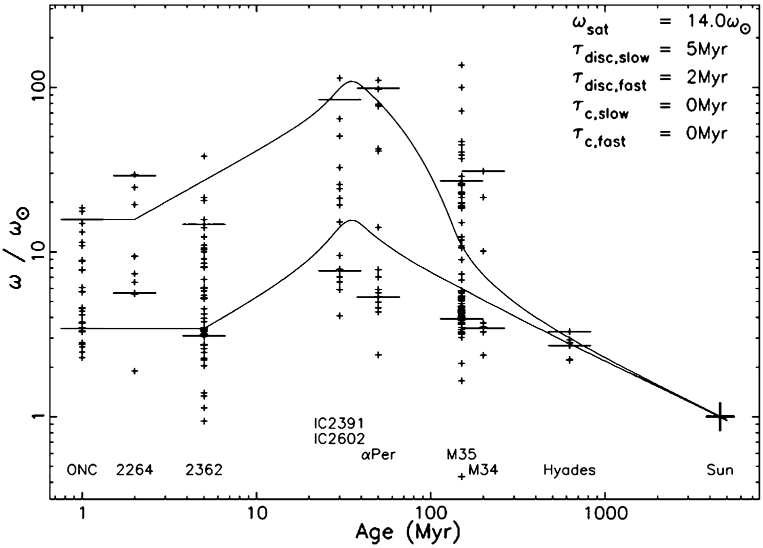


Fig. 2.1 Rotational angular velocity ω plotted as a function of time for stars with masses $0.9 < M/M_\odot < 1.1$. *Crosses* show the open cluster rotation period data such that each cluster collapses into a vertical stripe on the diagram, and *short horizontal lines* show the 25th and 90th percentiles of ω, used to characterize the slow and fast rotators, respectively. The *lines* show rotational evolution models for $1.0M_\odot$ stars, fitted to the percentiles for each cluster using a simple unweighted least squares method. For this plot, we have assumed that the stars rotate as solid bodies (i.e., constant ω as a function of radius inside the star). Plotted are the ONC, NGC 2264, NGC 2362, IC 2391, IC 2602, α Per, M35, M34, the Hyades, and the Sun. (Taken from [50], courtesy Cambridge University Press)

evolution where the typical time is either the Kelvin–Helmholtz time or that characterizing the dominant nuclear reactions. The same is true for the length scales. First, in the vertical direction, we have to choose the resolution that adequately represents the steepest gradients that develop during the evolution. Furthermore, in the horizontal direction, the resolution that allows one to describe convective structures, possible instabilities and turbulence has to be chosen. This is the reason why it is nowadays necessary to use and couple 1-D, 2-D, and 3-D modelings to get a global picture of macroscopic MHD transport processes in stellar interiors.

In this review, we report the state of the art of the modeling of transport processes in stellar interiors (both in radiation and in convection zones) aimed to study the star's angular momentum history (see [13, 50], Fig. 2.1), the related profile of differential rotation ([47], Fig. 2.2), and their magnetism. First, we recall the common methods used both on dynamical and secular time scales, namely the spectral expansion of MHD equations. Next, we focus on the behavior of convective regions and describe couplings between differential rotation, meridional circulation, turbulence, and magnetic field and the related dynamo processes. Then, we give a global picture of transport and mixing processes operating in stellar radiation zones. Finally, we conclude on the necessity of obtaining integrated models of stars from

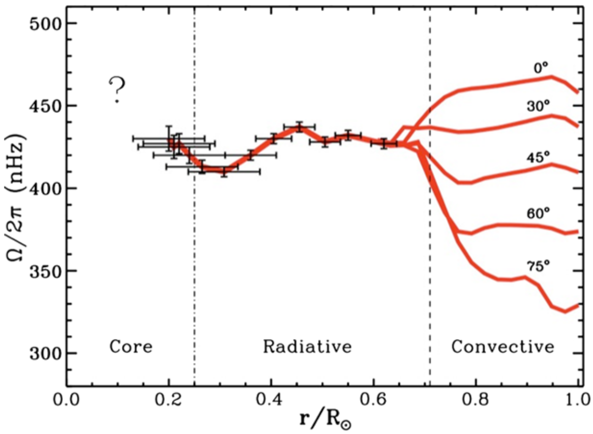


Fig. 2.2 Internal angular velocity in the Sun revealed by helioseismology ($R_\odot$ is the solar radius). It is conical in the convective envelope ($0.02M_\odot$, $M_\odot$ being the solar mass), uniform in the radiative core ($0.98M_\odot$), the transition layer, the tachocline being very thin (less than $0.04R_\odot$). (Taken from [47], courtesy Science)

their cores to their surfaces and to take into account in such models the action of the stellar environment.

2.2 Modeling

2.2.1 Preliminary Definitions

Here, we describe the method which is used to treat MHD equations in stellar interiors both in convective and in radiative regions.

First, the macroscopic velocity field is expanded

$$\mathbf{V}(\mathbf{r},t) = r\sin\theta\,\Omega(r,\theta,t)\hat{\mathbf{e}}_\varphi + \dot{r}\hat{\mathbf{e}}_r + \mathcal{U}_M(r,\theta,t) + \mathbf{u}(r,\theta,\varphi,t), \qquad (2.1)$$

where $\mathbf{r} = \{r,\theta,\varphi\}$ are the classical spherical coordinates with their associated orthonormal basis $\{\hat{\mathbf{e}}_j\}_{j=\{r,\theta,\varphi\}}$, with t being the time. The first term, where Ω is the internal angular velocity, is the azimuthal velocity field associated with the differential rotation. The second is the radial Lagrangian velocity due to the contractions and the dilatations of the star during its evolution. The third term is the large-scale meridional circulation velocity field ($\mathcal{U}_M$). Finally, $\mathbf{u}$ is the fluctuating velocity associated with turbulence or waves.

Next, the anelastic approximation is adopted to treat macroscopic large-scale transport processes. The continuity equation, $\partial_t\rho + \nabla\cdot(\rho\mathbf{V}) = 0$, ρ being the density, thus becomes $\nabla\cdot(\rho\mathbf{V}) = 0$, and acoustic waves are filtered.

Then, using the divergence-free property of the magnetic field, we expand it

$$\mathbf{B}(\mathbf{r},t) = \nabla\wedge\nabla\wedge\left[\xi_P(\mathbf{r},t)\hat{\mathbf{e}}_r\right] + \nabla\wedge\left[\xi_T(\mathbf{r},t)\hat{\mathbf{e}}_r\right], \qquad (2.2)$$

where ξ_P and ξ_T are the poloidal and the toroidal magnetic stream functions.

For an axisymmetric magnetic field, the poloidal field $\mathbf{B}_P(r,\theta,t)$ is in the meridional plane $(\hat{\mathbf{e}}_r,\hat{\mathbf{e}}_\theta)$, while the toroidal one $\mathbf{B}_T(r,\theta,t)$ is purely azimuthal along $\hat{\mathbf{e}}_\varphi$.

The different fields being now defined, we have to consider the transport equations that have to be solved in stellar interiors.

2.2.2 Transport Equation System

The first transport equation we consider is the one related to the transport of magnetic field, namely the induction equation

$$\partial_t \mathbf{B} = \mathbf{\nabla} \wedge (\mathbf{V} \wedge \mathbf{B}) - \mathbf{\nabla} \wedge (\eta \mathbf{\nabla} \wedge \mathbf{B}), \tag{2.3}$$

η being the magnetic diffusivity. Two equations, for ξ_P and ξ_T respectively, are then obtained when the expansion given in Eq. (2.2) is introduced. First, they describe the advection of both poloidal and toroidal components of the field by the large-scale meridional currents, both in radiation and convection zones. Then, we get the creation of toroidal magnetic field by shearing the poloidal through differential rotation (the so-called Ω effect). Next, they give the Ohmic diffusion of each component. Finally, they describe the action of the turbulent magnetic field fluctuations, which will be possible in the case where the dynamo threshold is reached, to generate and sustain magnetic energy; this is the so-called α-effect in the mean-field formalism. Furthermore, in the case where secular time-scales are studied we take into account the Lagrangian variation of magnetic field due to the contractions and dilatations of the star during its evolution. This is the reason why we have introduced the time-Lagrangian derivative $\mathrm{d}/\mathrm{d}t = \partial_t + \dot{r}\partial_r$.

The second transport equation which has to be treated is the momentum equation, i.e., the Navier–Stokes equation

$$\rho\left[\partial_t \mathbf{V} + (\mathbf{V}\cdot\mathbf{\nabla})\mathbf{V}\right] = -\mathbf{\nabla} P - \rho\mathbf{g} + \mathbf{\nabla}\cdot\mathsf{T} + \left[\frac{1}{\mu_0}(\mathbf{\nabla}\wedge\mathbf{B})\right]\wedge\mathbf{B}. \tag{2.4}$$

Here P and $\mathbf{g}$ are respectively the pressure and the gravity, T is the viscous stress tensor, and the last term is the Lorentz force, where μ_0 is the magnetic permeability of vacuum. Taking its azimuthal component and averaging over φ, the 2-D equation (in r and θ) for the transport of angular momentum is obtained (see [27, 69]). In the case of both radiation and convection zones, it describes the advection of angular momentum by meridional flows and the actions of the viscous diffusion, of Reynolds stresses, of large-scale magnetic torque, and of Maxwell stresses. Furthermore, taking the curl of this equation and focusing on the azimuthal component gives the so-called thermal wind equation, which describes the baroclinicity of flows inside the star and which links the differential rotation to entropy fluctuations along isobars. In the convective regions the study of this equation allows one to study the behavior of the differential rotation, while in the radiation zones it allows one to close the transport loop.

The last transport equation that has to be solved is the heat transport equation for the macroscopic entropy (S):

$$\rho T\left[\partial_t S + (\mathbf{V}\cdot\nabla)S\right] = \nabla\cdot(\chi\nabla T) + \rho\varepsilon + \mathcal{Q}. \tag{2.5}$$

It describes the transport of entropy by advection, the thermal diffusion (χ is the thermal conductivity), the production of energy associated to nuclear reactions (ε is the nuclear energy production rate per unit mass), and the heating due to turbulence and Ohmic effects.

One should note that assuming hydrostatic and thermal equilibrium hypotheses in Eqs. (2.4) and (2.5) returns us to the standard stellar evolution equations.

Finally, the equation for the transport of chemicals

$$\rho\left[\partial_t c_i + (\mathbf{V}\cdot\nabla)c_i\right] = \nabla\cdot\left[\rho(D_{\rm mic} + \mathsf{D})\otimes\nabla c_i\right] + \rho\dot{c}_i \tag{2.6}$$

has to be solved to study elements mixing. Here c_i is the concentration of the considered ith element, $\dot{c}_i$ its creation (destruction) rate, $D_{\rm mic}$ the microscopic diffusivity, and D the diffusivity tensor, which can be anisotropic, for example, in radiative regions that are highly stable stratified zones. Since turbulent movements in the convection zone are vigorous, chemicals are assumed to be instantaneously mixed there. In the radiation zone, the situation is different, and the secular action of transport processes in this region induces extra-mixing, which is necessary to explain observed surface chemical element abundances.

2.2.3 *The Spectral Method*

Once the system of MHD dynamical equations has been established, we solve it applying spectral methods in the horizontal direction. Then, because of the spherical geometry of the star, we describe both scalar quantities and vectorial fields using spherical harmonics.

Indeed, scalar quantities (X) are expanded as

$$X(\mathbf{r},t) = \overline{X}(r) + X'(r,\theta,\varphi,t) = \overline{X}(r) + \sum_{l=0}^{N}\sum_{m=-l}^{l}\left\{X_m^l(r,t)Y_l^m(\theta,\varphi)\right\}, \tag{2.7}$$

$\overline{X}$ being the hydrostatic value, and X' the fluctuation induced by transport processes.

Likewise, we expand the poloidal and toroidal magnetic stream functions:

$$\begin{aligned}\xi_P(\mathbf{r},t) &= \sum_{l=1}^{N_{\rm mag}}\sum_{m=-l}^{l}\left\{\xi_{l,m}^P(r,t)Y_l^m(\theta,\varphi)\right\},\\ \xi_T(\mathbf{r},t) &= \sum_{l=1}^{N_{\rm mag}}\sum_{m=-l}^{l}\left\{\xi_{l,m}^T(r,t)Y_l^m(\theta,\varphi)\right\}.\end{aligned} \tag{2.8}$$

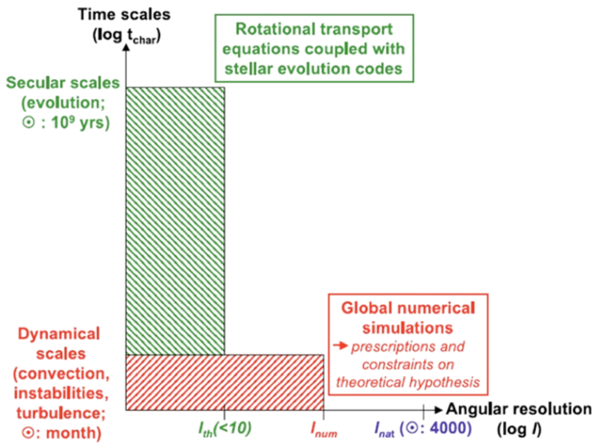


Fig. 2.3 Modeling strategy to study dynamical stellar evolution. The diagram presents time scales of the typical physical processes as functions of the angular resolution needed to properly describe these processes. The angular resolution is expressed in terms of the l index of the spherical harmonics $Y_l^m(\theta,\varphi)$. $l_{\rm num} \approx 600$ indicates the maximum angular resolution (in terms of spherical harmonics modes) presently achieved in global numerical simulations. (Adapted from [33], courtesy Astronomy & Astrophysics)

The expansion degree N (and $N_{\rm mag}$) on spherical harmonics depends on the problem which is treated (cf. Fig. 2.3). On the one hand, in a convection zone or if we study highly nonaxisymmetric and nonlinear processes in a radiative region such as shear-induced instabilities or fossil field MHD ones, we will use an expansion involving high-degree spherical harmonics. On the other hand, if transport processes in a radiation zone are studied on secular time scales, we will use only low-degree spherical harmonics but with an high radial accuracy to capture the strongest gradients which develop during the star evolution. This approach has allowed us to study, across the various scales of time and space, the problems of dynamical stellar evolution which cannot yet be modeled with direct numerical simulations.

Then, since transport equations such as the induction equation for the magnetic field and the momentum equation are three-dimensional vector equations, we expand vector fields (magnetic field and macroscopic velocity) in vectorial spherical harmonics as in stellar oscillations theory

$$\mathbf{u}(r,\theta,\varphi,t) = \sum_{l=0}^{N}\sum_{m=-l}^{l}\left\{u_m^l(r,t)\mathbf{R}_l^m(\theta,\varphi) + v_m^l(r,t)\mathbf{S}_l^m(\theta,\varphi) + w_m^l(r,t)\mathbf{T}_l^m(\theta,\varphi)\right\}, \tag{2.9}$$

where we have defined $\mathbf{R}_l^m(\theta,\varphi) = Y_l^m(\theta,\varphi)\hat{\mathbf{e}}_r$, $\mathbf{S}_l^m(\theta,\varphi) = \nabla_{\rm H} Y_l^m(\theta,\varphi)$, and $\mathbf{T}_l^m(\theta,\varphi) = \nabla_{\rm H} \wedge [Y_l^m(\theta,\varphi)\hat{\mathbf{e}}_r]$, while $\nabla_{\rm H} = \hat{\mathbf{e}}_\theta\partial_\theta + \hat{\mathbf{e}}_\varphi\frac{1}{\sin\theta}\partial_\varphi$. These expansions (Eqs. (2.7) and (2.9)) allow us to separate variables in transport equations. Thus, modal equations in r and t are solved in MHD spectral codes and in dynamical stellar evolution ones.

2.3 Dynamical Processes in Stellar Convection Zones

2.3.1 Convection, Differential Rotation, and Meridional Flows

First, in the case of the Sun, the most recent studies of global rotating convection in the external envelope have been achieved by the ASH group [25, 78, 80]. For the first time, the solar convection is now examined using global models with a strong density contrast of 150 from top to bottom. This leads to significant results on the turbulent convection spectra from large-scale (giant cell-like) down to supergranular-like convection patterns and their correlation with temperature fluctuations. Then, large-scale flows such as differential rotation and meridional circulation are associated with the turbulent convection action. Indeed, [24] discussed the respective role of Reynolds stresses, baroclinic effects, and latitudinal heat transport in establishing the conical differential rotation profile observed by Helioseismology (see, for example, Fig. 2.2). In fact, the so-called Taylor–Proudman constraint for rotating fluids implies that the differential rotation should be invariant along the rotation axis, thus leading to a cylindrical rotation profile. Since this is not observed, it is necessary to understand the source of the breaking of these constraint. First, [79] have shown that baroclinic effects are, in part, associated with temperature latitudinal variations because of the poleward heat transport. Moreover, using helioseismic inversions of the temperature, of the entropy fluctuations, and angular velocity profiles, [29] show that the Sun is not in strict thermal-wind balance and that departure from baroclinic forcing by Reynolds stresses are likely. Thus, assessing the meridional circulation pattern in the convective envelope is very important. In fact, if this flow contributes little to the heat transport and to the kinetic energy budget, it plays a pivotal role in the angular momentum transport by opposing and balancing the equatorial transport by Reynolds stresses [25]. In most simulations, meridional flows are found to be multicellular and fluctuate significantly over a solar rotation. Moreover, if temporal average are performed, it is possible to find solution exhibiting a large-single shell per hemisphere. Finally, the first complete 3-D hydrodynamical models of the Sun from the top of the convective envelope to the radiative core have been computed [30] showing the consistent establishment of the tachocline and the profile of the large-scale flows, i.e., the differential rotation that matches with helioseismic inversions and the meridional circulation.

Next, such studies have been extended to other stars. Indeed, the convection properties and the associated differential rotation have been examined in young solar-type stars as a function of their rotation rate [6, 18], in T-Tauri [12], in G & K stars [73], in F stars [3], in red giant stars [23], and in the convective cores of more massive stars [4, 21]. These works have allowed us to study scaling laws for the differential rotation, its properties (pro- or retrograde at the equator) and the associated thermal imbalance, the meridional flow properties, etc., in the whole Hertzsprung–Russell diagram. Finally, the properties of convective penetration, which is important for chemicals mixing, have been examined both at the base of a convective envelope in the case of the Sun [30] and at the top of a convective core [21].

2.3.2 Dynamo Action

Using such powerful 3-D global numerical simulations, it is now possible to study in detail the nonlinear interactions between turbulence, rotation, and the magnetic field in the Sun. Then, if an initial weak magnetic field is introduced in global solar turbulent convection simulations, the magnetic energy grows by many orders of magnitude through dynamo action if a critical value of the magnetic Reynolds number ($R_m = V_f L_f / \eta$, where V_f and L_f are characteristic velocity and length of the flow) is reached [27]. After a linear phase of exponential growth, magnetic energy saturates, because of the nonlinear feed-back of the Lorentz force, to a fraction of its kinetic energy retaining that level over many Ohmic decay times. Then, when saturation is reached, the kinetic energy is reduced when compared to the initial hydrodynamical value because of the reduction of the energy contained in the differential rotation. Moreover, the kinetic energy contained in the convective motions is less modified, and the nonaxisymmetric contribution to the kinetic energy is increased. The radial magnetic field is concentrated in the cold downflows. There, the Lorentz force modifies the flow, influencing the evolution of the strong downflows through magnetic tension that reduces the vorticity creation and inhibits the shear. The magnetic field and radial velocity are highly intermittent both in space and in time. The poloidal field is found to reverse in simulations, but it is not yet possible to get the observed 11-year solar cycle. To solve this problem, [22] have introduced in such simulations a shallow stably stratified tachocline at the bottom of the computed domain (Fig. 2.4). They show in this work that this stable layer plays a crucial role in organizing the irregular field produced in the convective zone into axisymmetric toroidal structures. The presence of such a large-scale mean field in the tachocline seems to influence the behavior of the model with much less frequent magnetic reversals. This is the reason why it is now clear that the presence of the tachocline is one of the key factors in generating large-scale magnetic structures and associated magnetic cycles. From now on, butterfly diagrams are only reproduced using 2-D mean field kinematic dynamo models with prescribed differential rotation, meridional circulation, magnetic diffusivity, and α-effect (see, for example, [51]).

This work has now been extended to solar-like stars. In the series of papers [19, 20] magnetic wreaths (coherent toroidal magnetic structures) have been identified and shown to survive within turbulent convective zones. This is a major result as it was thought until then that this was not possible because of magnetic flux expulsion. These results are also in accord with current spectropolarimetric observations of fast-rotating suns, which exhibit a dominant toroidal field topology. Activity cycles have also been obtained with regular reversals of field polarity found to exist in

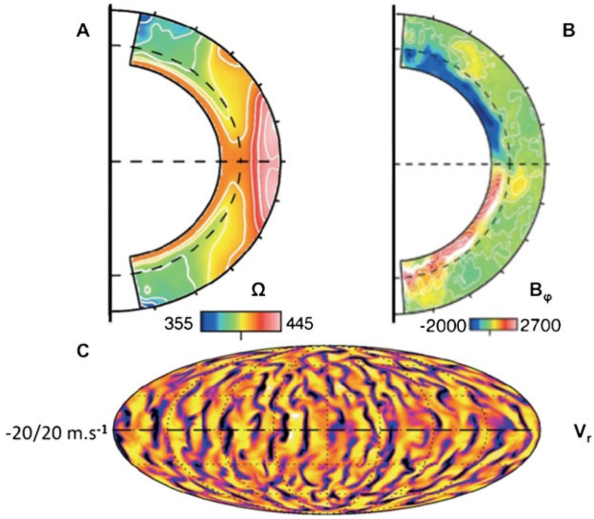


Fig. 2.4 *Top-left* (**A**): Angular velocity profile, *top-right* (**B**): azimuthal mean toroidal magnetic field, and *bottom* (**C**): convective radial velocities obtained using integrated ASH 3-D numerical simulations of the solar convective envelope and the tachocline. (Adapted from [22], courtesy The Astrophysical Journal)

turbulent conditions. In the same time, as in the Solar case, 2-D mean field dynamo models have been developed in order to study the impact of various physical processes on stellar magnetic cycles guided by 3-D numerical simulations. It has been shown that multicelled meridional flows and turbulent pumping have a significant impact on the cycle period and the butterfly diagram that are used to interpret the observed scaling law linking the star's cycle period to its rotation rate [37, 52].

Finally, the dynamo action due to turbulent convection has been undertaken in convective cores of more massive stars [4, 28]. In [44], the interaction of a convective core dynamo with various fossil fields in its surrounding radiative envelope (see Sect. 2.4.3) has been studied. It has demonstrated that such interaction (if the external field possesses a poloidal component) can lead to a strong dynamo branch in the core with magnetic field strength reaching the mega-Gauss level.

2.4 Dynamical Processes in Stellar Radiation Zones

In the classical approach, stellar radiation zones are supposed to be motionless. However, they are the seat of dynamical processes that act on secular time scales to transport angular momentum and chemicals. Four main processes are then identified (see, for example, [106, 121]): the meridional circulation, turbulence, fossil magnetic fields, and internal waves. These mechanisms form what is called the rotational transport (cf. Fig. 2.5). We will now describe each of them and the state of the art of their modeling.

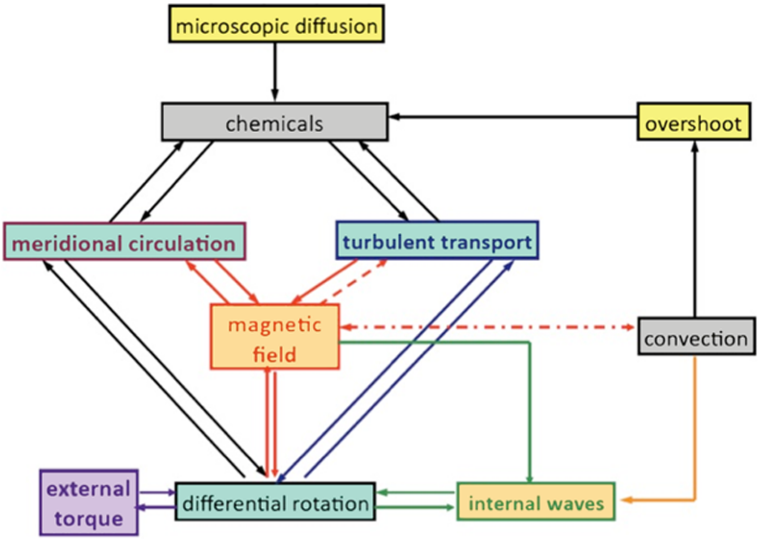


Fig. 2.5 The highly nonlinear rotational transport in the solar radiative core. Meridional circulation, shear-induced turbulence, fossil magnetic field, and internal waves act together to transport angular momentum and chemicals. In the solar case, the two latter processes are necessary to explain the observed angular velocity profile

2.4.1 Meridional Circulation

In a first step, the large-scale meridional circulation, which occurs in stellar radiation zones, was ascribed to the deformation of the isothermal surfaces by the centrifugal acceleration (and by the Lorentz force if there is a magnetic field and the tidal force if there is a close companion). Then, the radiative flux is no longer divergence-free [113] and must be balanced by heat advection. Therefore, in the original treatment [42, 105, 112] the meridional circulation velocity (and the induced mixing) was linked to the ratio of the centrifugal acceleration (and other perturbing forces) to the gravity. The characteristic time was derived by Sweet and named the Eddington–Sweet time $t_{\mathrm{ES}} = t_{\mathrm{KH}} \frac{GM}{\Omega^2 R^3}$, $t_{\mathrm{KH}} = \frac{GM^2}{RL}$ being the Kelvin–Helmholtz time (R, M, and L are respectively the stellar radius, mass, and luminosity). These results indicated that rapid rotators should be well mixed due to this circulation that should modify their evolution to the giant branch. This was then corrected by [75] who invoked the effects of μ-gradients (i.e., the chemical composition) to cancel such circulation effects.

However, the fact that meridional circulation advects angular momentum was ignored. Then, after a transient phase which lasts about an Eddington–Sweet time, the star settles into an asymptotic regime where the circulation is mainly driven by the torques applied to the star and internal stresses as those related to the shear-induced turbulence (if we do not take into account fossil magnetic fields and internal waves); their cases will be discussed respectively in Sects. 2.4.3 and 2.4.4. On one hand, if

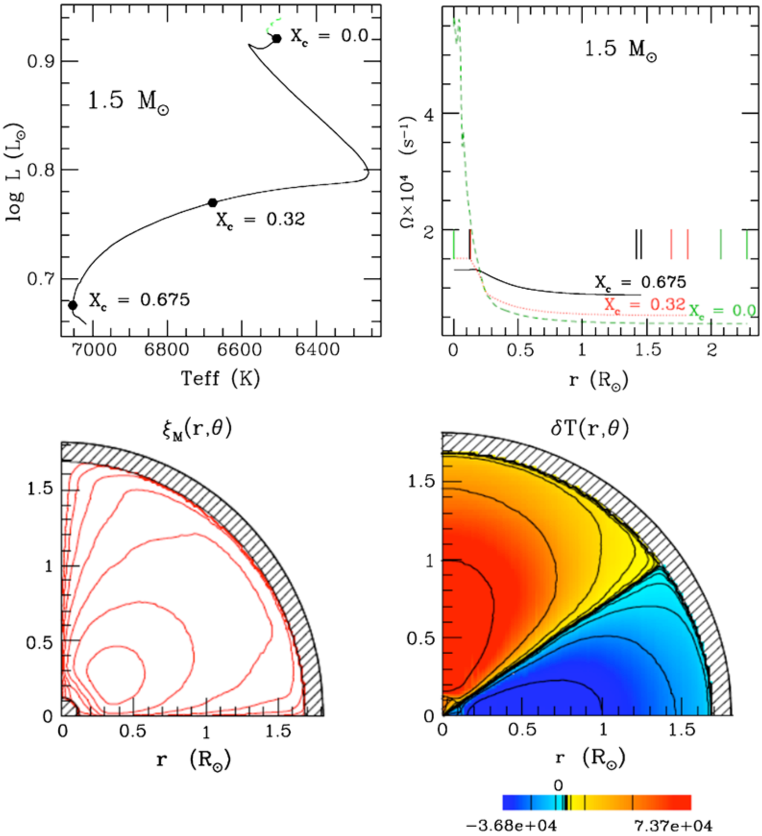


Fig. 2.6 Highly nonlinear interactions between the differential rotation (*upper-right panel*), the meridional circulation (*bottom-left panel*, where we represent the iso-contours of the stream meridional function (ξ); the meridional circulation is tangent to these lines), and the related thermal imbalance (δT) (*bottom-right panel*) during the evolution on the main sequence of a $1.5 M_\odot$ star with a solar metallicity and an initial equatorial velocity $V_{\rm ini} = 100$ km s^{-1}. The rotation profile is represented for three positions in the Hertzsprung–Russell diagram (*upper-left corner*): $X_c = \{0.675, 0.32, 0.0\}$, where X_c is the central fraction in mass in hydrogen. The meridional currents, which are driven here by the torque of the wind that slows down the surface, and the related temperature fluctuation are represented for $X_c = 0.32$. (Adapted from [33], courtesy Astronomy & Astrophysics)

the star losses angular momentum through a wind, the circulation adjusts to transport angular momentum to the surface [33, 62, 68, 120]. The induced rotation, resulting from the angular momentum advection, is then nonuniform and a baroclinic state sets in where the temperature varies with latitude along the isobar. On the other hand, if the star does not exchange angular momentum, the advection by the circulation balances the turbulent transport induced by the shear of the differential rotation; in the case of a uniform rotation without any turbulent transport the circulation thus dies [31]. In the case of solar-type stars (cf. Fig. 2.6) it is thus the loss or the gain of angular momentum which drives the circulation and not the amplitude of the angular velocity. Note also that the circulation can be driven by star's structural adjustments during their evolution. Therefore, the loop of the transport in stellar radiation zones in the case where we treat the highly nonlinear interactions between the differential rotation, the related shear-induced turbulence, and the meridional circulation is identified (see [33, 88]): first, meridional currents are sustained by the torques applied at the stellar surface, by internal stresses such as the viscous ones related to turbulence and by structural adjustments; next, the temperature relaxes to balance the advection of entropy by the meridional circulation; finally, because of the baroclinic torque induced by the latitudinal distribution of temperature fluctuations on the isobar, a new differential rotation profile is established because of the so-called thermal-wind balance that may again generate turbulence, and the loop is closed.

2.4.2 Shear-Induced Turbulence

Stellar radiation zones are stably stratified. In such regions, hydrodynamical turbulence is thus strongly influenced by the buoyancy force, which inhibits vertical displacements. The vertical and horizontal shear instabilities thus give birth to a supposed strongly anisotropic turbulent transport much efficient in the horizontal direction, as in oceans [99, 120]. Vertical turbulent diffusion of angular velocity and chemicals is thus weaker than the horizontal and horizontal gradients of all quantities are thus strongly damped allowing reduced expansion in few spherical harmonics in Eq. (2.7). A review of prescriptions used for describing shear-induced turbulent transport in stellar radiation zones is given in [70]. Other instabilities such as baroclinic and double diffusive instabilities can also develop (see, for example, [118] and [49]).

Secular modeling has shown that combined action of such turbulent transport and meridional circulation advection are successful, for example, in explaining properties of massive stars [77]. However, since such mechanisms are unable to explain the observed solar radiation zone angular velocity profile (see, for example, [84, 111]) and the rotation of the core of subgiants and red giant stars [10, 34], fossil magnetic fields that are detected, for example, at the surface of the radiative envelope of some of the massive stars [114] and internal waves that are excited by convective regions must be studied.

2.4.3 The Fossil Magnetic Field Dynamics

The core of solar-type stars and the envelopes of massive ones are supposed to be the seat of a fossil magnetic field, which has been trapped during the star's birth and is a remnant of the stellar formation and of the PMS dynamo. The possible configurations of this field are now understood as resulting from an MHD turbulent

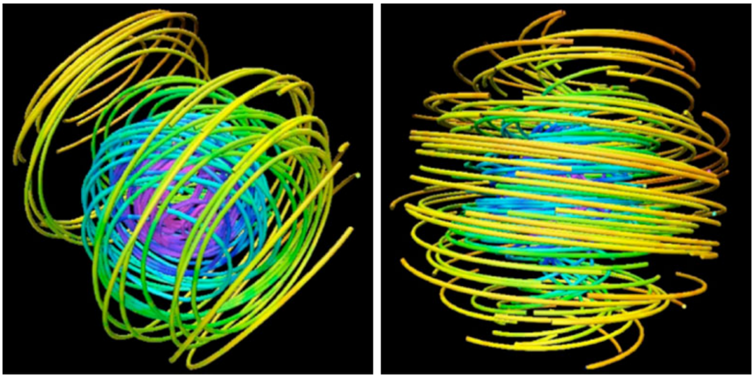


Fig. 2.7 *Left*: Unstable purely toroidal fossil magnetic field, once the Tayler instability has reached its saturation (the colorscale is a function of the density). *Right*: Mixed stable fossil field configuration resulting from an initial MHD turbulent relaxation. (Taken from [41], courtesy The Astrophysical Journal)

relaxation process, in which the magnetic energy has decreased while conserving the global magnetic helicity of the initial field. Numerical simulations [15–17, 39] and semi-analytical work [40, 41] have been devoted to study such initial conditions for fossil fields. They conclude that it must be, to a first approximation, in a non-force-free stable relaxed equilibrium with a mixed configuration, both poloidal and toroidal (cf. the right panel of Fig. 2.7) because purely toroidal and poloidal fields are unstable in stellar radiation zones (see respectively [110] and [63] and the left panel of Fig. 2.7).

Once the initial non-force-free magnetic configuration (axisymmetric or nonaxisymmetric) has been established by the initial MHD turbulent relaxation processes, it interacts with differential rotation. Then, two cases are possible as described by [100]. In the first case, if the field is strong, the rotation becomes uniform on magnetic surfaces due to Alfvén wave phase mixing, which damps the differential rotation; in the axisymmetric case this leads to the Ferraro's state where the rotation is frozen along the poloidal field lines (see, for example, [98]) and to a uniform rotation in the nonaxisymmetric case (the oblique rotators case for example; see, for example, [81]). Then, the field can only be modified by structural adjustments, and the mixing is frozen. In the second case, if the field is weak, it could first become axisymmetric if it is nonaxisymmetric because of rotational smoothing, and then, because of phase mixing, this leads to the Ferraro's state. However, this picture could be strongly modified by MHD magnetic instabilities if during the first step of the phase mixing, the residual differential rotation on each magnetic surface is able to generate a strong toroidal component of the field that becomes unstable and if this instability becomes able to trigger a dynamo action through an α-effect; this question still remains open [2, 14, 93, 101, 123]. The critical value of the field that gives the limit between the weak and the strong field regimes has been given in [5]. A summary is given in Fig. 2.8.

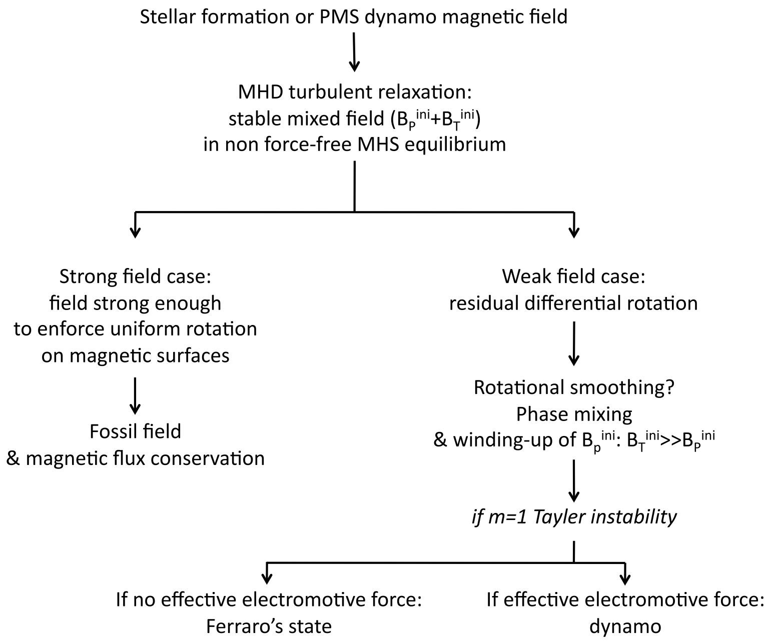


Fig. 2.8 Interaction of fossil magnetic field and differential rotation (see also [65, 102])

Let us now take into account the meridional circulation. To understand its interaction with the other dynamical processes (the differential rotation and the shear induced turbulence) in presence of a fossil magnetic field, we shall adopt the picture of rotational transport as described in Sect. 2.4.1 in the purely hydrodynamical case and generalize it to the magnetized case. As described previously, meridional circulation in radiation zones are driven by applied torques (internal, like the Lorentz torque or external, like those induced by stellar winds), structural adjustments during stellar evolution and turbulent transport. In the case where all these sources vanish, the meridional circulation dies after an Eddington–Sweet time, and the star settles in a baroclinic state described by the MHD thermal wind equation. If we apply this picture to the case of radiation zones with a fossil magnetic field, we thus understand that the meridional circulation (if we consider a star without structural adjustments and external torques) will be mainly driven by the residual magnetic torque until the phase mixing leads the star to a torque-free state. Then, the meridional circulation advection of angular momentum balances the residual Lorentz torque (see [76] and [69] and Fig. 2.9).

However, this global picture is only respected if the poloidal field lines are entirely confined in a radiation zone. If the poloidal field connects with the differentially convective envelope in the case of solar-type stars, the differential rotation is then transmitted along the field lines to their radiative core (see, for example, [32]). To resolve this problem, [48] (see also [92]) proposed a scenario in which the Ohmic diffusion of the fossil field into the convective envelope is prevented by its advection

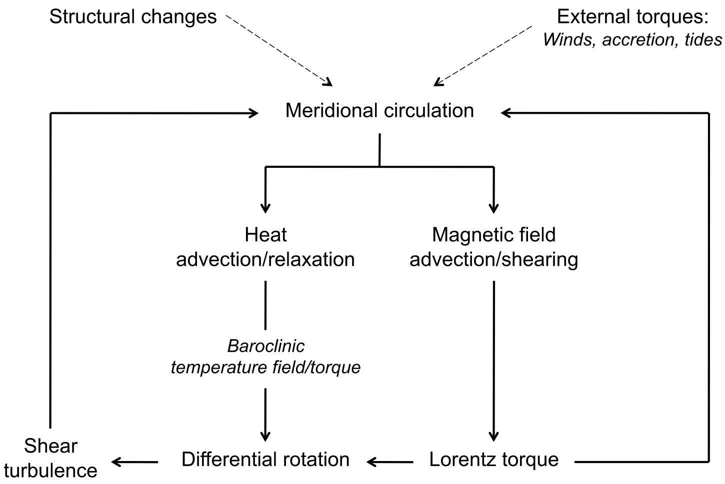


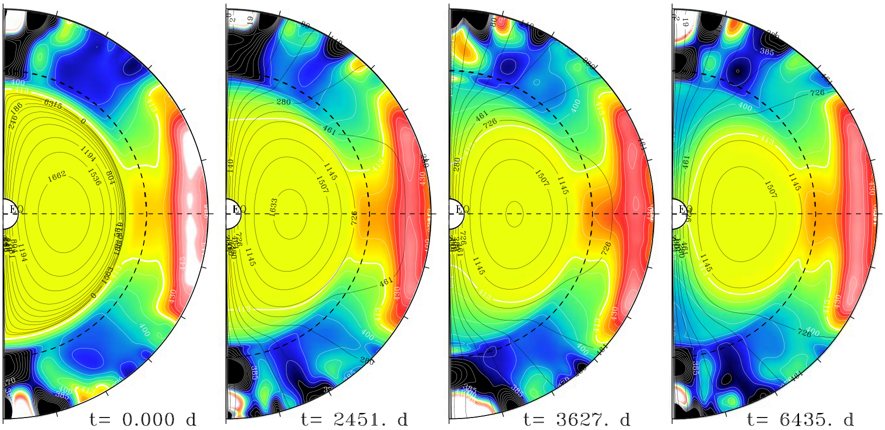
Fig. 2.9 The transport loop in a differentially rotating magnetized stellar radiation zone (see also [65])

by the down-flow part of the thermally driven circulation in the tachocline. However, this circulation has also an up-flow component, which would advect the field into the convection zone that would then imprint a small differential rotation in the radiation zone. The first detailed calculations to settle the question were achieved in [45, 46] (see also [56, 115]), who considered 2-D stationary solutions for various parameters. This work showed that the field lines would penetrate into the convection zone over a broad region at high latitudes, thus imprinting differential rotation to the radiation zone. This behavior is now confirmed by 3-D consistent calculations ([26, 103, 104], Fig. 2.10) where authors showed that even a deeply buried magnetic field will eventually connect with the convection zone, inducing differential rotation in the radiative core both in the case of axi- and nonaxisymmetric configurations. Therefore, another transport process seems to be responsible for the uniform rotation in the radiative core of solar-type stars.

2.4.4 Internal Waves

Since the fossil magnetic field seems unable to enforce the observed uniform rotation in the radiative core of the Sun, we must examine the last possible mechanism: the transport of angular momentum by the internal waves excited by the turbulent motions at convection/radiation transitions in stellar interiors, namely the bases of convective envelopes in low-mass stars and the tops of convective cores in intermediate-mass and massive ones. The importance of these processes was first anticipated by [86, 96, 122] (see also [97]).

Let us first discuss the case where modifications of gravity waves by rotation and magnetic field are not taken into account. In this simplest case, the restoring



(a) (b) (c) (d)

Fig. 2.10 Interaction of a deeply buried axisymmetric fossil magnetic field in the Solar radiative core with the surrounding convective envelope. During its evolution, the fossil field connects with the convection zone transmitting its differential rotation to the radiation zone. (Taken from [104], courtesy Astronomy & Astrophysics)

force operating on waves is the buoyancy force. They thus propagate only in stably stratified regions, i.e., in the radiative core of low-mass stars. After an excitation of a whole spectrum of such waves at the base of the convective envelope, they penetrate into the radiation zone, transporting angular momentum which is deposited where they are dissipated mainly through thermal diffusion as first described by [117].

Waves that are of short wavelength are first dissipated close to the convection zone. Prograde waves carry positive angular momentum, and retrograde waves negative angular momentum. If they propagate in a radiative region, which is rotating faster than the excitation region, their frequency is Doppler-shifted, leading to higher dissipation for the prograde waves than for the retrograde waves. This is the reason why the angular velocity first increases where it was already high, and its slope with depth steepens until the shear becomes unstable. The induced turbulent layer then merges with the convective envelope. In the same time, retrograde waves extract angular momentum at a deeper layer, thus building there a “negative” shear layer, which takes the place of the former and the cycle repeats. This phenomenon, which depends on the waves turbulent excitation, is called the Shear Layer Oscillation [107] and is similar to the quasi-biennial oscillation observed in the Earth’s atmosphere. The question, which must then be answered, is how this thin shear layer modifies the propagation of waves of lowest frequencies, which are less damped. If there is no slope in angular velocity, the shear layer is symmetric, and the effect is the same on the prograde as on the retograde waves. However, if the angular velocity increases with depth, the prograde waves will be more dissipated, which allows the retrograde waves to extract angular momentum from the deep interior. This picture was first studied by [109] using a first approach where only gravity waves and shear-induced turbulence are taken into account. This was confirmed using complete

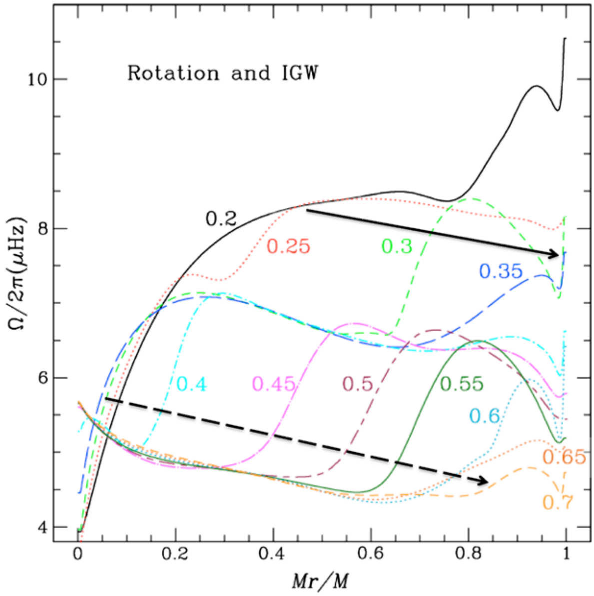


Fig. 2.11 Evolution of the (differential) rotation profile (averaged on latitudes) in a $1.2M_{\odot}$ star (its metallicity is $Z = 0.02$) with an initial rotation velocity of 50 $\mathrm{km\,s^{-1}}$. This is obtained making simulations using the STAREVOL code, where the transport of angular momentum by the meridional circulation, the shear vertical turbulence, and the internal gravity waves are taken into account. The *black arrows* show the successive angular momentum extraction fronts (the first is represented by the *continuous line*, and the second one by the *dashed line*), which are mainly driven by the angular momentum extraction at the surface by the wind. The *curves* are labeled according to the corresponding ages in Gyrs. (Adapted from [107], courtesy Astronomy & Astrophysics)

calculation including also meridional circulation [107]. In this modeling, where horizontal differential rotation is damped by the strong horizontal turbulence, internal gravity waves succeed in achieving nearly uniform rotation in solar-type stars at the solar age with successive angular momentum extraction fronts (cf. Fig. 2.11).

These fronts are mainly driven by wave-damping and the meridional circulation, while the shear-induced turbulence acts only in regions of strong differential rotation. Furthermore, efficiency of this transport is strongly correlated with the external extraction of angular momentum by magnetic wind, which creates a differential rotation between the convective envelope and the radiative core, which feeds the transport by waves. Moreover, as in Sects. 2.4.1 and 2.4.3, the transport loop can be identified: first, external torques and internal stresses (i.e., viscous ones related to the shear-induced turbulence and waves Reynolds stresses) sustain the large-scale meridional circulation that advects entropy leading to a new temperature distribution and thus to a new differential rotation profile (see Fig. 2.12). Furthermore, models including waves predict the observed Li abundance, which is a strong constraint for stellar modeling.

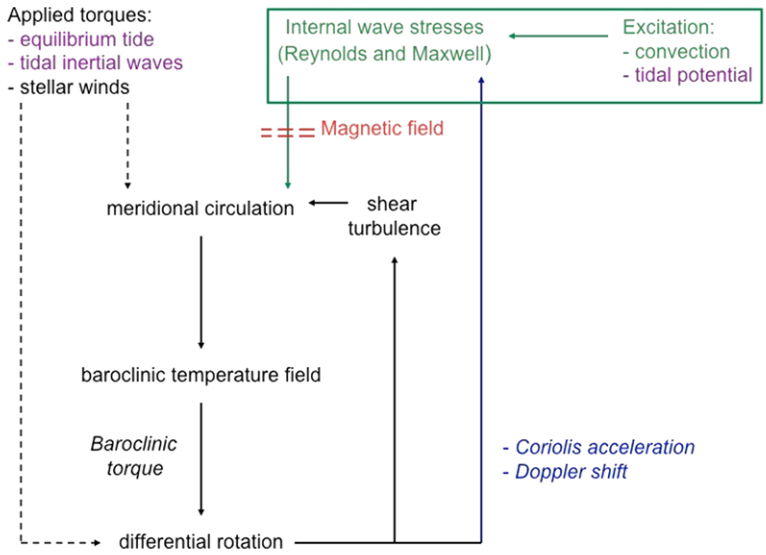


Fig. 2.12 Hydrodynamical transport loop in stellar radiation zones when taking into account internal waves. The different applied torques (winds and tides) are shown. (Adapted from [67], courtesy Astronomy & Astrophysics)

Now, work has been undertaken to obtain a more complete picture of this promising mechanism in the whole Hertzsprung–Russell diagram [108].

Firstly, theoretical studies [11, 94] and numerical simulations aim to determine the wave spectrum excited by convective envelopes and cores. For numerical simulations, results in Cartesian simulations [36, 54, 55] show a broad spectrum with an overevaluated energy flux in 2-D. More recently, integrated simulation of the Sun, first in 2-D in the solar equatorial plane [89] and then in 3-D [30], show a complex excited spectrum, which we should aim to understand.

Secondly, in the case of waves that have frequencies close to the inertial (2Ω) or to the Alfvén (ω_A) frequencies, the Coriolis acceleration and the Lorentz force, which modifies their propagation, their damping, and thus the related transport, must be taken into account. First, in the purely hydrodynamical case, waves become gravito-inertial waves [7, 8, 35, 60]. Two main wave families are then identified: the super-inertial gravito-inertial waves ($\sigma > 2\Omega$), which propagate in the whole sphere, and the sub-inertial ($\sigma \leq 2\Omega$), which are trapped in an equatorial belt (see Fig. 2.13). In this context, transport by gravito-inertial waves has been studied both in the case of a weak [71] and of a general differential rotation [58, 59, 64]. Then, the Coriolis acceleration's main action is to reduce the efficiency of the transport. Next, the effect of a toroidal magnetic field at the bottom of the tachocline and in the radiative core has been studied [57, 61, 66, 67, 90, 91, 95], and latitudinal trapping phenomena such as in the gravito-inertial wave case have been isolated. Furthermore, the azimuthal field acts, depending on the considered frequency, as a vertical filter. However, when taking simultaneously the action of the Coriolis acceleration

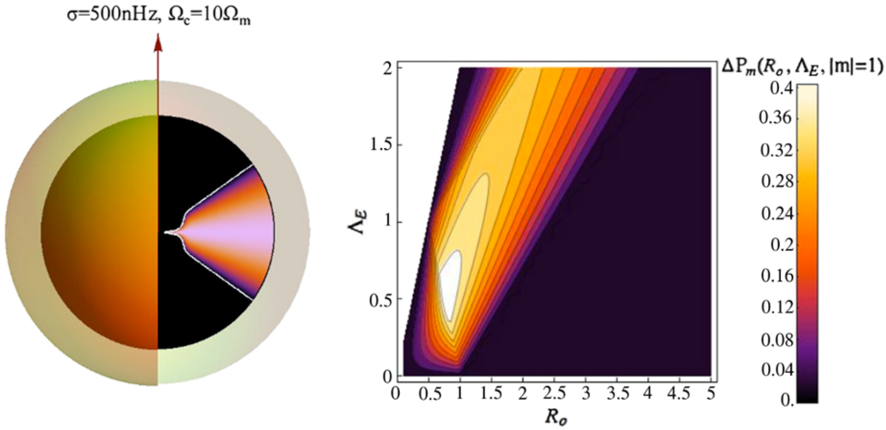


Fig. 2.13 Impact of (differential) rotation and magnetic fields on internal waves propagation and the related transport of angular momentum. *Left*: Equatorial trapping of a sub-inertial gravito-inertial wave (with a frequency $\sigma = 500$ nHz) in the differentially rotating radiation zone of a $1 M_\odot$ star, which has a given uniform rotation $\Omega_\mathrm{m} = 430$ nHz from 0.2 to $0.7 R_\odot$ and a core that rotates 10 times faster [64]. The colored region is the one where waves are propagative, while the black one is the "dead-region" for such propagation. *Right*: Net bias in the convection's energy transmission to prograde and retrograde waves (here for an azimuthal number $m = 1$) as a function of the wave Rossby ($R_\mathrm{o} = \sigma/2\Omega$) and Elsasser ($\Lambda_\mathrm{E} = \omega_\mathrm{A}^2/\Omega\sigma$) numbers; because of the stronger equatorial trapping of prograde waves, the retrograde are favored as well as the related net extraction of angular momentum. (Taken from [67], courtesy Astronomy & Astrophysics)

and of the Lorentz force related to a toroidal field into account, even if the efficiency of the transport can be strongly affected by trappings both in the horizontal and in the vertical directions, the results obtained by [107] of a net extraction of angular momentum because of a net bias in favor of retrograde waves is conserved in low-mass stars (see Fig. 2.13).

Finally, nonlinear effects such as wave braking at the center of the star are now studied [1]; some signs appear of a possible acceleration of the central rotation by this mechanism [9].

2.5 Conclusion

Work to go beyond the classical static picture of stars to get a global understanding of their dynamics is thus in a golden age. From dynamical to secular time scales 1-D, 2-D, and 3-D stellar models, including macroscopic transport processes, now allow us to enter into the details of the highly nonlinear couplings between differential rotation, meridional circulations, turbulence, dynamo and fossil magnetic fields, and waves. Moreover, we feel now how the impact of the star's environment is a key actor for internal transport processes. Therefore, as physical modeling becomes more and more complex and refined, it is clear that integrated models that take into account all the known internal processes and external constraints such as winds or accretion [53, 72, 74, 85] and tides [83, 87, 116, 119] must be built. In order to guide such construction, constraints obtained by helioseismology, asteroseismology, and spectropolarimetry will be crucial.

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References

1. Alvan, L., Mathis, S.: Critical layers for internal waves in stellar radiation zones. In: Alecian, G., Belkacem, K., Samadi, R., Valls-Gabaud, D. (eds.) SF2A-2011: Proceedings of the Annual Meeting of the French Society of Astronomy and Astrophysics, pp. 443–447 (2011)
2. Arlt, R., Rüdiger, G.: Amplification and stability of magnetic fields and dynamo effect in young A stars. Mon. Not. R. Astron. Soc. **412**, 107–119 (2011)
3. Augustson, K., Brown, B.P., Brun, A.S., Toomre, J.: Dynamo action, magnetic activity, and rotation in F stars. Bull. Am. Astron. Soc. **39**, 117 (2007)
4. Augustson, K.C., Brun, A.S., Toomre, J.: Convection and dynamo action in B stars. In: Brummell, N.H., Brun, A.S., Miesch, M.S., Ponty, Y. (eds.) Astrophysical Dynamics: From Stars to Galaxies. IAU Symposium, vol. 271, pp. 361–362 (2011)
5. Aurière, M., Wade, G.A., Silvester, J., Lignières, F., Bagnulo, S., Bale, K., Dintrans, B., Donati, J.F., Folsom, C.P., Gruberbauer, M., Hui Bon Hoa, A., Jeffers, S., Johnson, N., Landstreet, J.D., Lèbre, A., Lueftinger, T., Marsden, S., Mouillet, D., Naseri, S., Paletou, F., Petit, P., Power, J., Rincon, F., Strasser, S., Toqué, N.: Weak magnetic fields in Ap/Bp stars. Evidence for a dipole field lower limit and a tentative interpretation of the magnetic dichotomy. Astron. Astrophys. **475**, 1053–1065 (2007)
6. Ballot, J., Brun, A.S., Turck-Chièze, S.: Simulations of turbulent convection in rotating young solarlike stars: differential rotation and meridional circulation. Astrophys. J. **669**, 1190–1208 (2007)
7. Ballot, J., Lignières, F., Reese, D.R., Rieutord, M.: Gravity modes in rapidly rotating stars. Limits of perturbative methods. Astron. Astrophys. **518**, A30 (2010)
8. Ballot, J., Lignières, F., Prat, V., Reese, D.R., Rieutord, M.: 2D computations of g modes. arXiv:1109.6856 [astro-ph]
9. Barker, A.J., Ogilvie, G.I.: On internal wave breaking and tidal dissipation near the centre of a solar-type star. Mon. Not. R. Astron. Soc. **404**, 1849–1868 (2010)
10. Beck, P.G., Montalban, J., Kallinger, T., De Ridder, J., Aerts, C., García, R.A., Hekker, S., Dupret, M.A., Mosser, B., Eggenberger, P., Stello, D., Elsworth, Y., Frandsen, S., Carrier, F., Hillen, M., Gruberbauer, M., Christensen-Dalsgaard, J., Miglio, A., Valentini, M., Bedding, T.R., Kjeldsen, H., Girouard, F.R., Hall, J.R., Ibrahim, K.A.: Fast core rotation in red-giant stars as revealed by gravity-dominated mixed modes. Nature **481**, 55–57 (2012)
11. Belkacem, K., Samadi, R., Goupil, M.J., Dupret, M.A., Brun, A.S., Baudin, F.: Stochastic excitation of nonradial modes. II. Are solar asymptotic gravity modes detectable? Astron. Astrophys. **494**, 191–204 (2009)
12. Bessolaz, N., Brun, A.S.: Hunting for giant cells in deep stellar convective zones using wavelet analysis. Astrophys. J. **728**, 115 (2011)
13. Bouvier, J.: Lithium depletion and the rotational history of exoplanet host stars. Astron. Astrophys. **489**, L53–L56 (2008)
14. Braithwaite, J.: A differential rotation driven dynamo in a stably stratified star. Astron. Astrophys. **449**, 451–460 (2006)
15. Braithwaite, J.: On non-axisymmetric magnetic equilibria in stars. Mon. Not. R. Astron. Soc. **386**, 1947–1958 (2008)

16. Braithwaite, J., Nordlund, Å.: Stable magnetic fields in stellar interiors. Astron. Astrophys. **450**, 1077–1095 (2006)
17. Braithwaite, J., Spruit, H.C.: A fossil origin for the magnetic field in A stars and white dwarfs. Nature **431**, 819–821 (2004)
18. Brown, B.P., Browning, M.K., Brun, A.S., Miesch, M.S., Toomre, J.: Rapidly rotating suns and active nests of convection. Astrophys. J. **689**, 1354–1372 (2008)
19. Brown, B.P., Browning, M.K., Brun, A.S., Miesch, M.S., Toomre, J.: Persistent magnetic wreaths in a rapidly rotating sun. Astrophys. J. **711**, 424–438 (2010)
20. Brown, B.P., Miesch, M.S., Browning, M.K., Brun, A.S., Toomre, J.: Magnetic cycles in a convective dynamo simulation of a young solar-type star. Astrophys. J. **731**, 69 (2011)
21. Browning, M.K., Brun, A.S., Toomre, J.: Simulations of core convection in rotating A-type stars: differential rotation and overshooting. Astrophys. J. **601**, 512–529 (2004)
22. Browning, M.K., Miesch, M.S., Brun, A.S., Toomre, J.: Dynamo action in the solar convection zone and tachocline: pumping and organization of toroidal fields. Astrophys. J. Lett. **648**, L157–L160 (2006)
23. Brun, A.S., Palacios, A.: Numerical simulations of a rotating red giant star. I. Three-dimensional models of turbulent convection and associated mean flows. Astrophys. J. **702**, 1078–1097 (2009)
24. Brun, A.S., Rempel, M.: Large scale flows in the solar convection zone. Space Sci. Rev. **144**, 151–173 (2009)
25. Brun, A.S., Toomre, J.: Turbulent convection under the influence of rotation: sustaining a strong differential rotation. Astrophys. J. **570**, 865–885 (2002)
26. Brun, A.S., Zahn, J.P.: Magnetic confinement of the solar tachocline. Astron. Astrophys. **457**, 665–674 (2006)
27. Brun, A.S., Miesch, M.S., Toomre, J.: Global-scale turbulent convection and magnetic dynamo action in the solar envelope. Astrophys. J. **614**, 1073–1098 (2004)
28. Brun, A.S., Browning, M.K., Toomre, J.: Simulations of core convection in rotating A-type stars: magnetic dynamo action. Astrophys. J. **629**, 461–481 (2005)
29. Brun, A.S., Antia, H.M., Chitre, S.M.: Is the solar convection zone in strict thermal wind balance? Astron. Astrophys. **510**, A33 (2010)
30. Brun, A.S., Miesch, M.S., Toomre, J.: Modeling the dynamical coupling of solar convection with the radiative interior. Astrophys. J. **742**, 79 (2011)
31. Busse, F.H.: On the problem of stellar rotation. Astrophys. J. **259**, 759–766 (1982)
32. Charbonneau, P., MacGregor, K.B.: Angular momentum transport in magnetized stellar radiative zones. II. The solar spin-down. Astrophys. J. **417**, 762 (1993)
33. Decressin, T., Mathis, S., Palacios, A., Siess, L., Talon, S., Charbonnel, C., Zahn, J.P.: Diagnoses to unravel secular hydrodynamical processes in rotating main sequence stars. Astron. Astrophys. **495**, 271–286 (2009)
34. Deheuvels, S., Garcia, R.A., Chaplin, W.J., Basu, S., Antia, H.M., Appourchaux, T., Benomar, O., Davies, G.R., Elsworth, Y., Gizon, L., Goupil, M.J., Reese, D.R., Regulo, C., Schou, J., Stahn, T., Casagrande, L., Christensen-Dalsgaard, J., Fischer, D., Hekker, S., Kjeldsen, H., Mathur, S., Mosser, B., Pinsonneault, M., Valenti, J., Christiansen, J.L., Kinemuchi, K., Mullally, F.: Seismic evidence for a rapidly rotating core in a lower-giant-branch star observed with Kepler. arXiv:1206.3312 [astro-ph]
35. Dintrans, B., Rieutord, M.: Oscillations of a rotating star: a non-perturbative theory. Astron. Astrophys. **354**, 86–98 (2000)
36. Dintrans, B., Brandenburg, A., Nordlund, Å., Stein, R.F.: Spectrum and amplitudes of internal gravity waves excited by penetrative convection in solar-type stars. Astron. Astrophys. **438**, 365–376 (2005)
37. Do Cao, O., Brun, A.S.: Effects of turbulent pumping on stellar activity cycles. Astron. Nachr. **332**, 907 (2011)
38. Donati, J.F., Landstreet, J.D.: Magnetic fields of nondegenerate stars. Astron. Astrophys. Rev. **47**, 333–370 (2009)

39. Duez, V.: Numerical simulations of magnetic relaxation in rotating stellar radiation zones. Astron. Nachr. **332**, 983 (2011)
40. Duez, V., Mathis, S.: Relaxed equilibrium configurations to model fossil fields. I. A first family. Astron. Astrophys. **517**, A58 (2010)
41. Duez, V., Braithwaite, J., Mathis, S.: On the stability of non-force-free magnetic equilibria in stars. Astrophys. J. Lett. **724**, L34–L38 (2010)
42. Eddington, A.S.: Circulating currents in rotating stars. Observatory **48**, 73–75 (1925)
43. Eggenberger, P., Meynet, G., Maeder, A., Miglio, A., Montalban, J., Carrier, F., Mathis, S., Charbonnel, C., Talon, S.: Effects of rotational mixing on the asteroseismic properties of solar-type stars. Astron. Astrophys. **519**, A116 (2010)
44. Featherstone, N.A., Browning, M.K., Brun, A.S., Toomre, J.: Effects of fossil magnetic fields on convective core dynamos in A-type stars. Astrophys. J. **705**, 1000–1018 (2009)
45. Garaud, P.: Dynamics of the solar tachocline—I. An incompressible study. Mon. Not. R. Astron. Soc. **329**, 1–17 (2002)
46. Garaud, P., Garaud, J.D.: Dynamics of the solar tachocline—II. The stratified case. Mon. Not. R. Astron. Soc. **391**, 1239–1258 (2008)
47. García, R.A., Turck-Chièze, S., Jiménez-Reyes, S.J., Ballot, J., Pallé, P.L., Eff-Darwich, A., Mathur, S., Provost, J.: Tracking solar gravity modes: the dynamics of the solar core. Science **316**, 1591 (2007)
48. Gough, D.O., McIntyre, M.E.: Inevitability of a magnetic field in the Sun's radiative interior. Nature **394**, 755–757 (1998)
49. Hirschi, R., Maeder, A.: The GSF instability and turbulence do not account for the relatively low rotation rate of pulsars. Astron. Astrophys. **519**, A16 (2010)
50. Irwin, J., Bouvier, J.: The rotational evolution of low-mass stars. In: Mamajek, E.E., Soderblom, D.R., Wyse, R.F.G. (eds.) The Ages of Stars. IAU Symposium, vol. 258, pp. 363–374 (2009)
51. Jouve, L., Brun, A.S.: On the role of meridional flows in flux transport dynamo models. Astron. Astrophys. **474**, 239–250 (2007)
52. Jouve, L., Brown, B.P., Brun, A.S.: Exploring the $P_{\rm cyc}$ vs. $P_{\rm rot}$ relation with flux transport dynamo models of solar-like stars. Astron. Astrophys. **509**, A32 (2010)
53. Kawaler, S.D.: Angular momentum loss in low-mass stars. Astrophys. J. **333**, 236–247 (1988)
54. Kiraga, M., Jahn, K., Stępień, K., Zahn, J.P.: Direct numerisimulations of penetrative convection and generation of internal gravity waves. Acta Astron. **53**, 321–339 (2003)
55. Kiraga, M., Stepien, K., Jahn, K.: Generation and propagation of internal gravity waves: Comparison between two- and three-dimensional models at low resolution. Acta Astron. **55**, 205–217 (2005)
56. Kitchatinov, L.L., Rüdiger, G.: Magnetic field confinement by meridional flow and the solar tachocline. Astron. Astrophys. **453**, 329–333 (2006)
57. Kumar, P., Talon, S., Zahn, J.P.: Angular momentum redistribution by waves in the sun. Astrophys. J. **520**, 859–870 (1999)
58. Lee, U.: R modes of slowly pulsating B stars. Mon. Not. R. Astron. Soc. **365**, 677–687 (2006)
59. Lee, U., Saio, H.: Angular momentum transfer by non-radial oscillations in massive main-sequence stars. Mon. Not. R. Astron. Soc. **261**, 415–424 (1993)
60. Lee, U., Saio, H.: Low-frequency nonradial oscillations in rotating stars. I. Angular dependence. Astrophys. J. **491**, 839 (1997)
61. MacGregor, K.B., Rogers, T.M.: Reflection and ducting of gravity waves inside the sun. Sol. Phys. **270**, 417–436 (2011)
62. Maeder, A., Zahn, J.P.: Stellar evolution with rotation. III. Meridional circulation with μ-gradients and non-stationarity. Astron. Astrophys. **334**, 1000–1006 (1998)
63. Markey, P., Tayler, R.J.: The adiabatic stability of stars containing magneticfields—II. Poloidal fields. Mon. Not. R. Astron. Soc. **163**, 77 (1973)
64. Mathis, S.: Transport by gravito-inertial waves in differentially rotating stellar radiation zones. I—Theoretical formulation. Astron. Astrophys. **506**, 811–828 (2009)

65. Mathis, S.: Dynamics of fossil magnetic fields in massive star interiors. In: Neiner, C., Wade, G., Meynet, G., Peters, G. (eds.) Active OB Stars: Structure, Evolution, Mass Loss, and Critical Limits. IAU Symposium, vol. 272, pp. 160–165 (2011)
66. Mathis, S., de Brye, N.: Low-frequency internal waves in magnetized rotating stellar radiation zones. I. Wave structure modification by a toroidal field. Astron. Astrophys. **526**, A65 (2011)
67. Mathis, S., de Brye, N: Low-frequency internal waves in magnetized rotating stellar radiation zones. II. Angular momentum transport with a toroidal field. Astron. Astrophys. **540**, A37 (2012)
68. Mathis, S., Zahn, J.P.: Transport and mixing in the radiation zones of rotating stars. I. Hydrodynamical processes. Astron. Astrophys. **425**, 229–242 (2004)
69. Mathis, S., Zahn, J.P.: Transport and mixing in the radiation zones of rotating stars. II. Axisymmetric magnetic field. Astron. Astrophys. **440**, 653–666 (2005)
70. Mathis, S., Palacios, A., Zahn, J.P.: On shear-induced turbulence in rotating stars. Astron. Astrophys. **425**, 243–247 (2004)
71. Mathis, S., Talon, S., Pantillon, F.P., Zahn, J.P.: Angular momentum transport in the sun's radiative zone by gravito-inertial waves. Sol. Phys. **251**, 101–118 (2008)
72. Matt, S., Pudritz, R.E.: The spin of accreting stars: dependence on magnetic coupling to the disc. Mon. Not. R. Astron. Soc. **356**, 167–182 (2005)
73. Matt, S.P., Do Cao, O., Brown, B.P., Brun, A.S.: Convection and differential rotation properties of G and K stars computed with the ASH code. Astron. Nachr. **332**, 897 (2011)
74. Matt, S.P., MacGregor, K.B., Pinsonneault, M.H., Greene, T.P.: Magnetic braking formulation for sun-like stars: dependence on dipole field strength and rotation rate. arXiv:1206.2354 [astro-ph]
75. Mestel, L.: Rotation and stellar evolution. Mon. Not. R. Astron. Soc. **113**, 716 (1953)
76. Mestel, L., Tayler, R.J., Moss, D.L.: The mutual interaction of magnetism, rotation and meridian circulation in stellar radiative zones. Mon. Not. R. Astron. Soc. **231**, 873–885 (1988)
77. Meynet, G., Maeder, A.: Stellar evolution with rotation. V. Changes in all the outputs of massive star models. Astron. Astrophys. **361**, 101–120 (2000)
78. Miesch, M.S., Elliott, J.R., Toomre, J., Clune, T.L., Glatzmaier, G.A., Gilman, P.A.: Three-dimensional spherical simulations of solar convection. I. Differential rotation and pattern evolution achieved with laminar and turbulent states. Astrophys. J. **532**, 593–615 (2000)
79. Miesch, M.S., Brun, A.S., Toomre, J.: Solar differential rotation influenced by latitudinal entropy variations in the tachocline. Astrophys. J. **641**, 618–625 (2006)
80. Miesch, M.S., Brun, A.S., De Rosa, M.L., Toomre, J.: Structure and evolution of giant cells in global models of solar convection. Astrophys. J. **673**, 557–575 (2008)
81. Moss, D.: Magnetic fields and differential rotation in stars. Mon. Not. R. Astron. Soc. **257**, 593–601 (1992)
82. Neiner, C., Mathis, S., Saio, H., Lovekin, C., Eggenberger, P., Lee, U.: Seismic modelling of the late Be stars HD 181231 and HD 175869 observed with CoRoT: a laboratory for mixing processes. Astron. Astrophys. **539**, A90 (2012)
83. Ogilvie, G.I., Lin, D.N.C.: Tidal dissipation in rotating solar-type stars. Astrophys. J. **661**, 1180–1191 (2007)
84. Pinsonneault, M.H., Kawaler, S.D., Sofia, S., Demarque, P.: Evolutionary models of the rotating sun. Astrophys. J. **338**, 424–452 (1989)
85. Pinto, R.F., Brun, A.S., Jouve, L., Grappin, R.: Coupling the solar dynamo and the corona: Wind properties, mass, and momentum losses during an activity cycle. Astrophys. J. **737**, 72 (2011)
86. Press, W.H.: Radiative and other effects from internal waves in solar and stellar interiors. Astrophys. J. **245**, 286–303 (1981)
87. Remus, F., Mathis, S., Zahn, J.P.: The equilibrium tide in stars and giant planets: I—the coplanar case. arXiv:1205.3536 [astro-ph]

88. Rieutord, M.: The dynamics of the radiative envelope of rapidly rotating stars. I. A spherical Boussinesq model. Astron. Astrophys. **451**, 1025–1036 (2006)
89. Rogers, T.M., Glatzmaier, G.A.: Gravity waves in the sun. Mon. Not. R. Astron. Soc. **364**, 1135–1146 (2005)
90. Rogers, T.M., MacGregor, K.B.: On the interaction of internal gravity waves with a magnetic field—I. Artificial wave forcing. Mon. Not. R. Astron. Soc. **401**, 191–196 (2010)
91. Rogers, T.M., MacGregor, K.B.: On the interaction of internal gravity waves with a magnetic field—II. Convective forcing. Mon. Not. R. Astron. Soc. **410**, 946–962 (2011)
92. Rudiger, G., Kitchatinov, L.L.: The slender solar tachocline: a magnetic model. Astron. Nachr. **318**, 273 (1997)
93. Ruediger, G., Kitchatinov, L.L., Elstner, D.: Helicity and dynamo action in magnetized stellar radiation zones. arXiv:1107.2548 [astro-ph]
94. Samadi, R., Belkacem, K., Goupil, M.J., Dupret, M.A., Brun, A.S., Noels, A.: Stochastic excitation of gravity modes in massive main-sequence stars. Astrophys. Space Sci. **328**, 253–258 (2010)
95. Schatzman, E.: Filtering of gravity waves. Astron. Astrophys. **271**, L29 (1993)
96. Schatzman, E.: Transport of angular momentum and diffusion by the action of internal waves. Astron. Astrophys. **279**, 431–446 (1993)
97. Schatzman, E.: Diffusion process produced by random internal waves. J. Fluid Mech. **322**, 355–382 (1996)
98. Spada, F., Lanzafame, A.C., Lanza, A.F.: A semi-analytic approach to angular momentum transport in stellar radiative interiors. Mon. Not. R. Astron. Soc. **404**, 641–660 (2010)
99. Spiegel, E.A., Zahn, J.P.: The solar tachocline. Astron. Astrophys. **265**, 106–114 (1992)
100. Spruit, H.C.: Differential rotation and magnetic fields in stellar interiors. Astron. Astrophys. **349**, 189–202 (1999)
101. Spruit, H.C.: Dynamo action by differential rotation in a stably stratified stellar interior. Astron. Astrophys. **381**, 923–932 (2002)
102. Spruit, H.C.: Angular momentum transport and mixing by magnetic fields (invited review). In: Maeder, A., Eenens, P. (eds.) Stellar Rotation. IAU Symposium, vol. 215, p. 356 (2004)
103. Strugarek, A., Brun, A.S., Zahn, J.P.: Magnetic confinement of the solar tachocline: the oblique dipole. Astron. Nachr. **332**, 891 (2011)
104. Strugarek, A., Brun, A.S., Zahn, J.P.: Magnetic confinement of the solar tachocline: II. Coupling to a convection zone. Astron. Astrophys. **532**, A34 (2011)
105. Sweet, P.A.: The importance of rotation in stellar evolution. Mon. Not. R. Astron. Soc. **110**, 548 (1950)
106. Talon, S.: Transport processes in stars: Diffusion, rotation, magnetic fields and internal waves. In: Charbonnel, C., Zahn, J.P. (eds.) Stellar Nucleosynthesis: 50 Years After B^2FH, Aussois, France. EAS Publications Series, vol. 32, pp. 81–130 (2008)
107. Talon, S., Charbonnel, C.: Hydrodynamical stellar models including rotation, internal gravity waves, and atomic diffusion. I. Formalism and tests on Pop I dwarfs. Astron. Astrophys. **440**, 981–994 (2005)
108. Talon, S., Charbonnel, C.: Angular momentum transport by internal gravity waves. IV. Wave generation by surface convection zone, from the pre-main sequence to the early-AGB in intermediate mass stars. Astron. Astrophys. **482**, 597–605 (2008)
109. Talon, S., Kumar, P., Zahn, J.P.: Angular momentum extraction by gravity waves in the sun. Astrophys. J. Lett. **574**, L175–L178 (2002)
110. Tayler, R.J.: The adiabatic stability of stars containing magnetic fields—I. Toroidal fields. Mon. Not. R. Astron. Soc. **161**, 365 (1973)
111. Turck-Chièze, S., Palacios, A., Marques, J.P., Nghiem, P.A.P.: Seismic and dynamical solar models. I. The impact of the solar rotation history on neutrinos and seismic indicators. Astrophys. J. **715**, 1539–1555 (2010)
112. Vogt, H.: Zum Strahlungsgleichgewicht der Sterne. Astron. Nachr. **223**, 229 (1925)
113. Von Zeipel, H.: The radiative equilibrium of a rotating system of gaseous masses. Mon. Not. R. Astron. Soc. **84**, 665–683 (1924)

114. Wade, G.A., Alecian, E., Bohlender, D.A., Bouret, J.C., Cohen, D.H., Duez, V., Gagné, M., Grunhut, J.H., Henrichs, H.F., Hill, N.R., Kochukhov, O., Mathis, S., Neiner, C., Oksala, M.E., Owocki, S., Petit, V., Shultz, M., Rivinius, T., Townsend, R.H.D., Vink, J.S.: The MiMeS project: overview and current status. In: Neiner, C., Wade, G., Meynet, G., Peters, G. (eds.) Active OB Stars: Structure, Evolution, Mass Loss, and Critical Limits. IAU Symposium, vol. 272, pp. 118–123 (2011)
115. Wood, T.S., McCaslin, J.O., Garaud, P.: The sun's meridional circulation and interior magnetic field. Astrophys. J. **738**, 47 (2011)
116. Zahn, J.P.: Les marées dans une étoile double serrée (suite). Ann. Astrophys. **29**, 489 (1966)
117. Zahn, J.P.: The dynamical tide in close binaries. Astron. Astrophys. **41**, 329–344 (1975)
118. Zahn, J.P.: Instability and mixing processes in upper main sequence stars. In: Cox, A.N., Vauclair, S., Zahn, J.P. (eds.) Astrophysical Processes in Upper Main Sequence Stars. Saas-Fee Advanced Courses, vol. 13, p. 253 (1983)
119. Zahn, J.P.: Tidal evolution of close binary stars. I—Revisiting the theory of the equilibrium tide. Astron. Astrophys. **220**, 112–116 (1989)
120. Zahn, J.P.: Circulation and turbulence in rotating stars. Astron. Astrophys. **265**, 115–132 (1992)
121. Zahn, J.P.: Modeling stellar interiors with rotational mixing. In: Straka, C.W., Lebreton, Y., Monteiro, M.J.P.F.G. (eds.) Proceedings of the Joint HELAS and CoRoT/ESTA Workshop. EAS Publications Series, vol. 26, pp. 49–64 (2007)
122. Zahn, J.P., Talon, S., Matias, J.: Angular momentum transport by internal waves in the solar interior. Astron. Astrophys. **322**, 320–328 (1997)
123. Zahn, J.P., Brun, A.S., Mathis, S.: On magnetic instabilities and dynamo action in stellar radiation zones. Astron. Astrophys. **474**, 145–154 (2007)

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