Preface

Random fields appeared for the very first time in the 1920s in classical research papers about turbulence written by A.A. Friedmann, J. Kampé de Fériet, L.P. Keller, A.N. Kolmogorov, T. von Kármán, A.M. Obukhov, among others. Later on, the mathematical theory of random fields was developed and successfully applied in computer graphics, earthquake engineering, medicine, quantum physics, statistical mechanics, etc. The above theory uses such mathematical tools as Abstract Harmonic Analysis, Special Functions, Abelian and Tauberian theorems, etc.

At the end of the last millennium, new applications appeared. At that time, cosmology transformed into a predictive science whose predictions were tested against precise observations. In particular, tiny fluctuations of the cosmic microwave background were discovered. In order to build the rigourous mathematical model of the above fluctuations, one has to construct an isotropic random section of a special tensor bundle over the two-dimensional sphere. Therefore, the set of actively used mathematical tools for a specialist in random fields has to be extended with Differential Geometry, Lie Groups and Lie Algebras, just to mention a few.

New applications generated new challenges. In particular, currently there exists no single book written for specialists in probability that describes the current state of the spectral theory of invariant random fields and includes all necessary material from the above-mentioned non-probabilistic parts of mathematics. Our book originated from an attempt to fill this gap in the literature. The contents of the book are described in detail in Chap. 1. Here, we just present the book's scope.

Most random fields that appear in applied areas are invariant under an action of some group G. The group G acts either on the parametric space T of a random field or in some space of sections of a vector or tensor bundle over T. In Chap. 2 we describe a unified approach to spectral expansions of such fields based on the theory of induced group representations.

We divide the remaining part of the theory of random fields into two areas. In the first area, the so-called L^2 theory, a random field is considered as a function on T with values in the Hilbert space of square integrable random vectors. Some aspects of this theory are presented in Chap. 3. In the second area, some restrictions on the finite-dimensional distributions of the random field under consideration are imposed. In Chap. 4 we consider some questions of the theory of Gaussian random fields.

In Chap. 5 we consider applications of the above-described material to approximation theory, cosmology, and earthquake engineering. Finally, in the Appendix (Chap. 6) we consider mathematical tools outside the scope of probability and statistics, that are necessary for a specialist in random fields. Bibliographical remarks conclude each chapter and the Appendix (Chap. 6). Throughout the book, we use Halmos' abbreviation iff for "if and only if".

The book is intended for specialists in the theory of random fields, for mathematicians who would like to study the above theory, as well as for specialists in applied areas. It may be useful for graduate and postgraduate students in probability, statistics, functional analysis, cosmology, earthquake engineering, etc.

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Springer

http://www.springer.com/978-3-642-33405-4

Invariant Random Fields on Spaces with a Group Action
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2013, XVIII, 262 p., Hardcover
ISBN: 978-3-642-33405-4