# Preface

Electromagnetic metamaterials are artificially structured composite materials that exhibit a frequency band where the effective index of refraction becomes negative. Since the successful construction of such metamaterials in 2000, the study of metamaterials has attracted great attention of researchers across many disciplines. There is currently an enormous effort in the electrical engineering, material science, physics, and optics communities to come up with various ways of constructing efficient metamaterials and using them for potentially revolutionary applications in antenna and radar design, subwavelength imaging, and invisibility cloak design. Hence, simulation of electromagnetic phenomena in metamaterials becomes a very important issue, which is the subject of this book. In the mathematics community, there is an increasing interest in the study of metamaterials as evidenced by the Hot Topics Workshops on Negative Index Materials held at IMA (Institute for Mathematics and its Applications) of the University of Minnesota during October 2–4, 2006, which was the first public exposure of this subject to the mathematics community. During January 25–29, 2010, the leading author (Jichun Li) cochaired a workshop on “Metamaterials: Applications, Analysis and Modeling” at IPAM (Institute for Pure and Applied Mathematics) of the University of California at Los Angeles to expose this subject once more to the general mathematics community.

The purpose of this book is to provide a detailed introduction to the basic mathematical analysis of those model equations resulting from metamaterial simulations. We focus on developing and analyzing time-domain finite element methods for solving those metamaterial model equations. The book is intended to be self-contained in terms of finite element methods. Though there are many other types of numerical methods developed for metamaterial simulations, we restrict the contents to finite element methods because of our own research interests and experiences. The book starts with a brief introduction to metamaterials in Chap. 1. Here we discuss the origins of metamaterials, their basic electromagnetic and optical properties, some metamaterial structures and potential applications in subwavelength imaging, antenna design, invisibility cloak, and biosensing. At the end of this chapter, we introduce the governing equations for modeling wave propagation in metamaterials.

In Chap. 2, we provide a self-contained introduction to finite element methods. We start with the basic Lagrange finite elements and the corresponding interpolation error estimates. Then we present the basic finite element error analysis techniques for the second-order elliptic problems and teach readers how to code a simple Q1 element for solving elliptic problems.

After the preparatory work of Chap. 2, we move on to introduce the divergence-conforming and curl-conforming finite elements in Chap. 3. Since these elements play very important roles in metamaterial simulations, detailed constructions of these elements and their interpolation error estimates are discussed. After these, we present both explicit and implicit schemes for solving the Drude metamaterial model. The stability and error estimate analysis are carried out for those schemes. Finally, we extend similar schemes and analysis developed for the Drude model to the Lorentz model, and the Drude-Lorentz model, which are popular metamaterial models used by physicists and engineers.

In Chap. 4, we introduce the discontinuous Galerkin method and present its application to metamaterial simulations. Here, three types of discontinuous Galerkin methods are presented: one for integro-differential vector wave equations; and the other two for metamaterial Maxwell's equations written in conservation laws. MATLAB codes are provided for the practical implementation.

From our computational experiments with the lowest-order rectangular and cubic edge elements, we found that at element centers, these edge elements achieve one order higher convergence rate than the theoretical analysis suggested. This is a new superconvergence phenomenon; hence we devote Chap. 5 to the analysis of this phenomenon. The results and proofs are original, since no other books cover such superconvergence results in the infinity norm.

To develop an efficient adaptive finite element method, a posterior error estimator plays a very important role. There are several books covering this topic, but they mainly focus on classic elliptic and parabolic equations. To fill the gap, in Chap. 6 we venture to introduce some basic techniques recently developed for a posterior error analysis of Maxwell's equations. Here we first present detailed derivations of a posterior error estimator for the standard time-harmonic Maxwell's equations, then extend the analysis to the time-dependent integro-differential Maxwell's equations in cold plasma.

In Chap. 7, we present a detailed discussion on how to code the two-dimensional edge element for solving metamaterial Maxwell's equations. Considering that programming edge element is difficult and no other book has a detailed discussion on this task, we cover the whole programming process including mesh generation, calculation of the element matrices, assembly process, and postprocessing of numerical solutions. The complete MATLAB source codes are provided in the hope that the readers can easily modify our codes to solve other similar models interesting to them.

In order to model practical wave propagation problems in unbounded domains, we feel that readers have to understand how to construct the Perfectly Matched Layers (PMLs). In Chap. 8, we provide a succinct discussion of PMLs developed for free space, lossy media, dispersive media, and metamaterials.

In the last chapter (Chap. 9) of this book, we present several interesting simulations of wave propagation in metamaterials. Here we demonstrate the negative refraction index phenomenon (i.e., backward wave propagation inside metamaterials), invisibility cloak in both frequency domain and time domain, and solar cell designs with metamaterials. Finally, we mention some open issues which need more attention or have not been well studied.

Overall, this book is intended to bring readers to the front field of metamaterial simulations by finite element methods. Inevitably, there are some interesting topics left out of this book, since there is a tremendous effort going on in this area and it is hard for us to keep abreast of the vast amount of literature across many disciplines. The contents are a reflection of our own interests and related subjects. Part of the material has been given as a series of lectures by Jichun Li at Xiangtan University of China in December 2010, in the 2011 Winter Enrichment Program at King Abdullah University of Science and Technology (KAUST) of Saudi Arabia in January 2011, and at Peking University of China in August 2012. Hence, the book can also be used as a one-semester course for graduate students in physics, engineering, material sciences, optics, and mathematics interested in wave propagation simulations. We assume that all potential readers should have some basic knowledge about electromagnetic theory, partial differential equations, functional analysis, and have some training in numerical methods for solving differential equations.

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