

Preface

For us, the authors of the monograph *Asymptotic Solutions of Strongly Nonlinear Systems of Differential Equations*—the first Russian edition (1996) by Moscow University Press and the second edition (2009) by the publisher R&C Dynamics (<http://shop.rcd.ru>)—the decision by Springer-Verlag to publish an English translation of the book is an important event. It is not merely an offspring born from the pangs of creative struggle and thus a favorite, for there are many books, but we present to the Western reader a monograph that is a very special book.

In what way special? First, even at the moment the first Russian edition appeared in 1996, it was the fruit of a decade and a half of research. The first publication of V.V. Kozlov on the subject goes back to 1982 [103, 104, 117] and is dedicated to a very important fundamental problem: the inversion of Lagrange’s theorem on the stability of equilibrium—not a trivial task, over which researchers had struggled for more than half a century. The idea behind that 1982 paper belongs to N.G. Chetaev: “To prove the instability of equilibrium is to find just one trajectory of the system that tends to the equilibrium position as time decreases indefinitely”. But the word “find” is easily said! The 1982 paper showed that the solution of the equations of motion of a natural mechanical system, with some restrictions, can be constructed in the form of generalized power series whose convergence is proved by quite sophisticated methods of functional analysis.

For a long time our efforts were devoted to this issue and matters related to it. Years passed, and at last it dawned upon us that the method—developed to address a very famous, interesting, and yet particular problem—was in fact universal, giving scientists the opportunity to “view from above” the problem of the asymptotic behavior of differential equations in the neighborhood of a nonelementary singular point. In addition, it was discovered that the technique used in the 1982 paper, which seemed so successful and even elegant, has a deep connection with the fundamental works of such great classics as Lyapunov and Poincaré. This “view from above” which we have perhaps only by the grace of God, gave access to an immense number of applications. Today we scarcely can find an example from any of the previously studied critical cases of the stability of equilibrium of autonomous systems of ordinary differential equations for which sufficient conditions for instability cannot

be obtained by the method developed. Moreover, we were able to obtain conditions for instability in some previously unexplored cases and it was found that the method works well for periodic and quasi-periodic time-dependence of systems of differential equations. We also managed to make significant progress on the problem of inverting Lagrange's theorem on the stability of equilibrium and its "little sister": Routh's theorem, for which—as we know by the existence of the phenomenon of gyroscopic stabilization—the converse is simply not true without the imposition of additional conditions. Finally, the method provides enhanced results on the instability of equilibrium for systems with time lags. Work on the method led (as so often happens) to unexpected secondary effects: it turned out that the behavior of solutions in the neighborhood of a trajectory, whose existence is guaranteed by the method, can indicate when systems are nonintegrable or chaotic in one sense or another.

We are absolutely certain that our published work ends not with a period but rather with a comma and that the possibilities that the method offers are far from exhausted. We therefore think of the publication of our monograph by Springer-Verlag as an invitation to Western scholars to continue research in this direction. To the great Russian poet Sergei Yesenin belong the words: "big things can only be seen at a distance". We thus hope that our readers can bring a fresh perspective to the methodology that we have developed and extend the range of problems that can be solved with it.

We are deeply grateful to Springer-Verlag and in particular to Ruth Allewelt, without whose perseverance the preparation time for the book's publication might have become asymptotically infinite, and to the translator, Lester Senechal, whose interest and assistance have helped preserve our hopes for the book's realization.

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