Lempel–Ziv Model of Dynamical-Chaotic and Fibonacci-Quasiperiodic Systems

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Abstract Here we show that how the LZ-complexity concept connects to the concepts such as Lyapunov exponent and K-entropy and has an application in the theory of dynamical systems regardless of its main origin in the information theory. Furthermore, selecting the Fibonacci sequence as a sample of evolutionary arrays, it is proved that these systems' LZ complexity represents its long-range order.

1 Introduction

Chaos is complex and disordered [1]. However, the outstanding attribute of such behavior is that the state of the system cannot be predicted for a long period of time. This limitation occurs when one withdraws infinite accuracy from the common attitude toward the Newton dynamical system. Chaotic systems are very sensitive to initial conditions. It means that every slight change in initial conditions has a major impact on the final output [2–4].

2 LZ Complexity as a Dynamic Index

Here we consider the LZ complexity of the phase-space snapper of dynamic systems, which is coded to specific messages. Based on this formalism, the LZ complexity has been proposed as an index in the analysis of dynamic systems' behavior. Following this idea, by coding the dynamic observables, the Henon system is as follows:

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$$H\colon X_{n+1} = 1 - aX_n^2 + Y_n; \qquad Y_{n+1} = bX_n, \tag{1}$$

and the driven dissipative oscillating continuous system with the motion equation:

$$P\colon \ddot{X} + K\dot{X} + \sin X = g\cos(\omega_d t). \tag{2}$$

The LZ complexity has been calculated for the specific values of control parameters of these two systems. In the method of array-code generation for Henon mapping, the system's phase space is divided into four regions:$(X, -Y), (X, Y), (-X, Y), (-X, -Y)$. Next, when the asymptotic response of the dynamic system (X_n, Y_n) is placed on the first and third quarters for a specific value of a, it be-comes the symbol 1 in the array, otherwise it becomes 0. The resulted array is input to the LZ-complexity-calculation program. This process is iterated and calculated for each a in the interval [1,4] at step size $\Delta a = 0.001$ and $b = 0.3$. The calculation of LZ complexity for a driven dissipative oscillating system has been carried out as follows. Here the control parameters are the motive angular velocity ($\omega_d = 2/3$), friction factor ($K = 0.5$), and the motive force amplitude (g), which changes in the range $0.9 \leq g \leq 1.5$ by the step size $\Delta g = 0.001$. The array length is considered $n = 20000$. The generation of array codes for a specific motive force amplitude in the above interval has been performed based on a simple procedure such that whenever the oscillator angular velocity ($\dot{x}$) is larger than zero, it is considered the number corresponding to the single array, and otherwise zero. The complexity analysis of each array has been done by the computer program. The normalized LZ complexity, $\{C(n)/b(n)\}$, is near zero for the periodic trajectories and bifurcation points of both systems. It means that despite the fact that new bifurcations form and the periodic models become more complex, there is not a clear and easy-to-observe effect of these changes on the LZ complexity. This matter may result from the fact that data existing in the phase-space snapper is mapped to two-alphabet array symbols. At this level of calculation, it seems that we can just distinguish the determinable dynamic system's chaotic behavior ($0 < C(n)/b(n) < 1$) from ordered behavior ($C(n)/b(n) = 0$). Despite this difference in the periodic behavior region, the correspondence of these two indexes is obvious in the consistent explanation of the nature evolution of Henon and oscillator systems. This fact suggests the existence of a profound link between these two concepts. The correlation between Lyapunov exponent and LZ complexity was also proposed by Kasper and Shuster in 1987. This correlation can be explicitly understood. The quantities such as algorithmic complexity, LZ complexity, statistical complexity, and Rissanen complexity tend to the value of K-entropy at n $\rightarrow \infty$. On the other hand, the ith positive Lyapunov exponent (λ_i^+) determines the data generation rate under the system evolution along the ith coordinate of phase space. K-entropy is the total data generation rate in that dynamic process:

$$h_\mu(X) = \Sigma_i D_i \lambda_i^+, \tag{3}$$

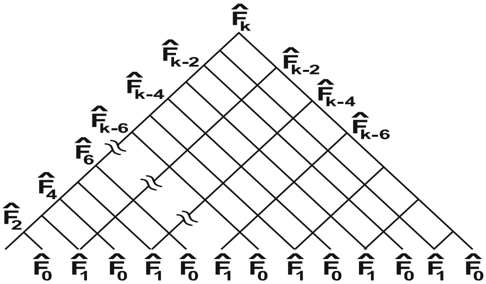

Fig. 1 The correlation between Fibonacci blocks and subblocks

where D_i is the data density for each bit of the ith coordinate of phase space. For directions that are dynamically unstable, although they are determinable, we cannot predict every single bit of system-evolution data. Therefore, D_i equals one for these directions. Consequently:

$$h_\mu(X) = \Sigma_i \lambda_i^+. \tag{4}$$

Accordingly, in chaotic modes, the LZ complexity directly relates to the sum of positive Lyapunov exponent. For the systems such as Henon mapping and dissipative driven oscillator in which there is only a positive Lyapunov exponent the LZ complexity corresponds with that only positive Lyapunov exponent.

3 LZ Complexity of Fibonacci Sequence

The subarray $A_i A_{i+1} \ldots A_{i+m}$ with length $l \geq m$ from the sequence $A_1 A_2 \ldots A_l \ldots A_{l+m} \ldots A_L$ is called a block.

Fibonacci block: The Fibonacci sequence $\{f_n\}_n^N = 0$ in which $f_n = f_{n-1} + f_{n-2},\ \ n \leq 2$ is considered. The Fibonacci block is a finite array whose formation results from a command based on the formation of Fibonacci sequence, which is due to the sort order operation ($\oplus$) of two previous blocks. The simplest block is called zero or one block. Figure 1 shows each block's relation with its constitutive subblocks. Furthermore, the correlation between each block and basic blocks F_0 and F_1 is clear. For example, the block F_6 is composed of F_4 and F_5. On the other hand, F_6 consists of a subblock F_5, two subblocks F_4, three subblocks F_3, and five subblocks F_2. The calculation of these blocks' LZ complexity is of importance. The following simple example shows a procedure for calculating the LZ complexity.

Trick 1: The LZ complexity of the third Fibonacci block (F_2) is equal to two.

It is proved by using the description of LZ complexity:

$$S' = F_2 = 10;\ 1)S^2 = 1/;\ 2)Q = 0; SQ = 1/0; V(S\hat{Q}) = 1; \because\ 1 \varepsilon V(S\hat{Q}); \therefore$$
$$S' = S = 1/0 \Rightarrow C(S') = C(F_2) = 2. \tag{5}$$

Trick 2: If we add each block of S with length $n - r$ to the sequence S with length r ($\oplus$), the LZ complexity of the resulted sequence (S') will be:

$$C(S') = C(S) + 1, \tag{6}$$

$$S' = S \oplus S'', \quad S'' \subset_B S; \quad L(S') = L(S \oplus S'') = n, \tag{7}$$

$$S'' = s_1 s_{i+1} \ldots s_j = s_{r+1} s_{r+2} \ldots s_n, 1 \le i \le j \le r;$$

$$S' = s_1 s_2 \ldots s_r s_{r+1} \ldots s_n. \tag{8}$$

Using the description, we calculate the LZ complexity of S'.

$$\begin{aligned}
&j)\ S' = S\backslash = s_1 s_2 \ldots s_r \backslash; Q = s_{r+1}; SQ = S\backslash s_{r+1}; S\hat{Q} = S\backslash;\\
&S'' \subset_B S \Rightarrow (Q = s_{r+1}) \in S; \therefore\ Q \in V(S\hat{Q})\\
&j+1)\ Q = s_{r+1} s_{r+2}; SQ = S\backslash s_{r+1}\ s_{r+2}; S\hat{Q} = S\backslash s_{r+1};\\
&S'' \subset_B S \Rightarrow (Q = s_{r+1} s_{r+2}) \in S; \therefore\ Q \in V(S\hat{Q}).
\end{aligned} \tag{9}$$

Accordingly, this process is iterated up to the last term ($j = r$). For the last step, we write:

$$\begin{aligned}
&j = r)\ Q = s_{r+1} s_{r+2} \ldots s_n; SQ = S\backslash s_{r+1} s_{r+2} \ldots s_n; S\hat{Q} = S\backslash s_{r+1} s_{r+2} \ldots s_{n-1}\\
&S'' \subset_B S \Rightarrow Q \subset_B S; \therefore\ Q \in V(S\hat{Q}); \text{COPY } Q = S''; \therefore\ S' = S/Q = S/S''; \therefore\\
&C(S') = C(S) + 1.
\end{aligned} \tag{10}$$

4 Conclusion

The calculation of Lyapunov exponent for large-scale systems faces some problems such as lack of convergence in numerical solutions. Herein it is shown that the LZ complexity can be also utilized for systems more complex than one-dimensional mappings such as logistic mappings. Therefore, by selecting the appropriate coding method or more alphabets (more than two alphabets), the LZ complexity, which is simpler and less time-consuming in terms of calculation, can be used as a dynamical index equivalent to Lyapunov exponent.

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