

## Chapter 2

# Introduction to Option Management

*Εισαγωγή στη Διαχείριση Επιλογής*

*χρή δ' ἐπ' ἀξίοις πονεῖν ψυχὴν προβάλλοντ' ἐν κύβοισι δαίμονος*  
The prize must be worth the toil when one stakes one's life on  
fortune's dice.

Dolon to Hector, Euripides (Rhesus, 182)

In this chapter we discuss basic concepts of option management. We will consider both European and American call and put options and practice concepts of pricing, look at arbitrage opportunities and the valuation of forward contracts. Finally, we will investigate the put-call parity relation for several cases.

**Exercise 2.1 (Call and Put Options).** *A company's stock price is  $S_0 = 110$  USD today. It will either rise or fall by 20 % after one period. The risk-free interest rate for one period is  $r = 10$  %.*

- (a) *Find the risk-neutral probability that makes the expected return of the asset equal to the risk-free rate.*
- (b) *Find the prices of call and put options with the exercise price  $K = 100$  USD.*
- (c) *How can the put option be duplicated?*
- (d) *How can the call option be duplicated?*
- (e) *Check put-call parity.*

- (a) The risk-neutral probability in this one period binomial model satisfies

$$(1 + r)S_0 = E_Q S_t,$$

where  $Q$  denotes the risk neutral (Bernoulli) measure with probability  $q$ . Plugging in the given data  $S_0 = 110$ ,  $S_{11} = 110 \cdot 1.2$ ,  $S_{12} = 110 \cdot 0.8$  and  $r = 0.1$  leads to:

$$(1 + 0.1) \cdot 110 = q \cdot 1.2 \cdot 110 + (1 - q) \cdot 0.8 \cdot 110$$

$$1.1 = 1.2q + 0.8(1 - q)$$

$$0.3 = 0.4q$$

$$q = 0.75$$

Hence the risk neutral probability measure  $Q$  is

$$P_Q(S_{11} = 1.2 \cdot 110) = 0.75$$

$$P_Q(S_{12} = 0.8 \cdot 110) = 0.25$$

- (b) The call option price is  $C = (1 + r)^{-1} E_Q \Psi(S_1, K)$ , with  $K = 100$  and  $\Psi(S, K) = 1(S - K > 0)(S - K)$ . Denote  $c^u = \Psi(S_{11}, K)$  and  $c^d = \Psi(S_{12}, K)$ . Then  $C = \{qc^u + (1 - q)c^d\} / (1 + r)$  is the expected payoff discounted by the risk-free interest rate. Using the prior obtained values we know that the stock can either increase to  $S_{11} = 110 \cdot (1 + 0.2) = 132$  or decrease to  $S_{12} = 110 \cdot (1 - 0.2) = 88$ , whereas the risk-neutral probability is  $q = 0.75$ . Given the exercise price of  $K = 100$ , the payoff in case of a stock price increase is  $c^u = \max(132 - 100, 0) = 32$ , in case of a decrease is  $c^d = \max(88 - 100, 0) = 0$ . Thus, the call price is  $C = (0.75 \cdot 32 + 0.25 \cdot 0) / (1 + 0.1) = 21.82$  USD.

Then the put option price is calculated using  $P = \{qp^u + (1 - q)p^d\} / (1 + r)$ . Given the exercise price of  $K = 100$  the payoff for a stock price increase is  $p^u = \max(100 - 132, 0) = 0$  and for a decrease is  $p^d = \max(100 - 88, 0) = 12$ . Thus, the put price is  $P = (0.75 \cdot 0 + 0.25 \cdot 12) / (1 + 0.1) = 2.73$  USD.

- (c) Given an increase in the stock price, the value of the derivative is  $p^u = \Delta S_{11} + \beta(1 + r)$ , where  $\Delta$  is the the number of shares of the underlying asset,  $S_{11}$  is the value of the underlying asset at the top,  $\beta$  is the amount of money in the risk-free security and  $1 + r$  is the risk-free interest rate.

The value of  $p^d$  is calculated respectively as  $p^d = \Delta S_{12} + \beta(1 + r)$ . Using  $p^u = 0$ ,  $p^d = 12$ ,  $S_{11} = 132$  and  $S_{12} = 88$  we can solve the two equations:  $\Delta 132 + \beta(1 + 0.1) = 0$  and  $\Delta 88 + \beta(1 + 0.1) = 12$  and obtain  $\Delta = -0.27$ ,  $\beta = 32.73$ . This means that one should sell 0.27 shares of stock and invest 32.73 USD at the risk-free rate.

- (d) For the call option, we can analogously solve the following two equations:  $\Delta 132 + \beta(1 + 0.1) = 32$ ,  $\Delta 88 + \beta(1 + 0.1) = 0$ . Finally, we get  $\Delta = 0.73$ ,  $\beta = -58.18$ . This means that one should buy 0.73 shares of stock and borrow 58.18 USD at a risk-free rate.
- (e) The principle of put-call parity refers to the equivalence of the value of a European call and put option which have the same maturity date  $T$ , the same delivery price  $K$  and the same underlying. Hence, there are combinations of options which can create positions that are the same as holding the stock itself.

These option and stock positions must all have the same return or an arbitrage opportunity would be available to traders.

Formally, the relationship reads  $C + K/(1+r) = P + S_0$ . Refer to Franke et al. (2011) for the derivation. Plugging in the above calculated values yields  $21.82 + 100/1.1 = 2.73 + 110$ . Obviously, the equivalence holds, so the put-call parity is satisfied.

**Exercise 2.2 (American Call Option).** *Consider an American call option with a 40 USD strike price on a specific stock. Assume that the stock sells for 45 USD a share without dividends. The option sells for 5 USD 1 year before expiration. Describe an arbitrage opportunity, assuming the annual interest rate is 10 %.*

Short a share of the stock and use the 45 USD you receive to buy the option for 5 USD and place the remaining 40 USD in a savings account. The initial cash flow from this strategy is zero. If the stock is selling for more than 40 USD at expiration, exercise the option and use your savings account balance to pay the strike price. Although the stock acquisition is used to close out your short position, the  $40 \cdot 0.1 = 4$  USD interest in the savings account is yours to keep. If the stock price is less than 40 USD at expiration, buy the stock with funds from the savings account to cancel the short position. The 4 USD interest in the savings account and the difference between the 40 USD (initial principal in the savings account) and the stock price is yours to keep (Table 2.1).

**Exercise 2.3 (European Call Option).** *Consider a European call option on a stock with current spot price  $S_0 = 20$ , dividend  $D = 2$  USD, exercise price  $K = 18$  and time to maturity 6 months. The annual risk-free rate is  $r = 10$  %. What is the upper and lower bound (limit) of the price of the call and put options?*

The upper bound for a European call option is always the current market price of the stock  $S_0$ . If this is not the case, arbitrageurs could make a riskless profit by buying the stock and selling the call option. The upper limit for the call is therefore 20.

Based on  $P + S_0 - K \exp(-r\tau) - D = C$  and  $P \geq 0$ , the lower bound for the price of a European call option is given by:

$$C \geq S_0 - K \exp(-r\tau) - D$$

$$C \geq 20 - 18 \exp(-0.10 \cdot 6/12) - 2$$

$$C \geq 20 - 17.12 - 2$$

$$C \geq 0.88$$

Consider for example, a situation where the European call price is 0.5 USD. An arbitrageur could buy the call for 0.5 USD and short the stock for 20 USD. This provides a cash flow of  $20 - 0.5 = 19.5$  USD which grows to  $19.5 \exp(0.1 \cdot 0.5) = 20.50$  in 6 months. If the stock price is greater than the exercise price at maturity,

**Table 2.1** Cash flow table for this strategy

Action	$CF_t$	$CF_T$	
		$S_T < 40$	$S_T \geq 40$
Short a share of the stock	45	$-S_T$	$-S_T$
Buy a call	-5	0	$S_T - 40$
Rest to savings account	-40	44	44
Total	0	$(40 - S_T) + 4$	4

the arbitrageur will exercise the option, close out the short position and make a profit of  $20.50 - 18 = 2.50$  USD.

If the price is less than 18 USD, the stock is bought in the market and the short position is closed out. For instance, if the price is 15 USD, the arbitrageur makes a profit of  $20.50 - 15 = 5.50$  USD.

Thus, the price of the call option lies between 0.88 and 20 USD.

The upper bound for the put option is always the strike price  $K = 18$  USD, while the lower bound is given by:

$$P \geq K \exp(-r\tau) - S_0 + D$$

$$P \geq 18 \exp(-0.10 \cdot 0.5) - 20 + 2$$

$$P \geq 17.12 - 20 + 2$$

$$P \geq -0.88$$

However, the put option price cannot be negative and therefore it can be further refined as:

$$P \geq \max\{K \exp(-r\tau) - S_0 + D, 0\}.$$

Thus, the price of this put option lies between 0 and 18 USD.

**Exercise 2.4 (Spread between American Call and Put Option).** Assume that the above stock and option market data does not refer to European put and call options but rather to American put and call options. What conclusions can we draw about the relationship between the upper and lower bounds of the spread between the American call and put for a non-dividend paying stock?

The relationship between the upper and lower bounds of the spread between American call and put options can be described by the following relationship:  $S_0 - K \leq C - P \leq S_0 - K \exp(-r\tau)$ . In this specific example, the spread between the prices of the American put and call options can be described as follows:

$$20 - 18 \leq C - P \leq 20 - 18 \exp(-0.10 \cdot 6/12)$$

$$2 \leq C - P \leq 2.88$$

**Table 2.2** Portfolio value for some future time  $t'$ 

	$S_{t'} \leq K_1$	$K_1 \leq S_{t'} \leq K_\lambda$	$K_\lambda \leq S_{t'} \leq K_0$	$K_0 \leq S_{t'}$
1.	$\lambda(K_1 - S_{t'})$	0	0	0
2.	$(1 - \lambda)(K_0 - S_{t'})$	$(1 - \lambda)(K_0 - S_{t'})$	$(1 - \lambda)(K_0 - S_{t'})$	0
3.	$-(K_\lambda - S_{t'})$	$-(K_\lambda - S_{t'})$	0	0
Sum	0	$\lambda(S_{t'} - K_1)$	$(1 - \lambda)(K_0 - S_{t'})$	0

**Exercise 2.5 (Price of American and European Put Option).** *Prove that the price of an American or European put option is a convex function of its exercise price.*

*Additionally, consider two put options on the same underlying asset with the same maturity. The exercise prices and the prices of these two options are  $K_1 = 80$  and 38.2 EUR and  $K_2 = 50$  and 22.6 EUR.*

*There is a third put option on the same underlying asset with the same maturity. The exercise price of this option is 60 EUR. What can be said about the price of this option?*

Let  $\lambda \in [0, 1]$  and  $K_1 < K_0$ . Consider a portfolio with the following assets:

1. A long position in  $\lambda$  puts with exercise price  $K_1$
2. A long position in  $(1 - \lambda)$  puts with exercise price  $K_0$
3. A short position in 1 put with exercise price  $K_\lambda \stackrel{\text{def}}{=} \lambda K_1 + (1 - \lambda)K_0$

The value of this portfolio for some future time  $t'$  can be seen in Table 2.2:

The value of the portfolio is always bigger than or equal to 0. For no arbitrage to happen, the current value of the portfolio should also be non-negative, so:

$$\lambda P_{K_1, T}(S_t, \tau) + (1 - \lambda)P_{K_0, T}(S_t, \tau) - P_{K_\lambda, T}(S_t, \tau) \geq 0$$

The above inequality proves the convexity of the put option price with respect to its exercise price.

The price of a put option increases as the exercise price increases. So in this specific example:

$$P_{50, T} \leq P_{60, T} \leq P_{80, T}$$

and hence:

$$22.6 \leq P_{60, T} \leq 38.2$$

Moreover, we also know that the prices of call and put options are convex, so

$$\lambda K_1 + (1 - \lambda)K_2 = 60$$

$$\lambda = 1/3$$

**Table 2.3** Cash flow table for this strategy

Action	$CF_t$	$CF_T$	
		$S_T < 235$	$S_T \geq 235$
Buy a put	- 5.25	$235 - S_T$	0
Short a call	21.88	0	$235 - S_T$
Buy a forward with $K = 235$	-16.17	$S_T - 235$	$S_T - 235$
Total	0.46	0	0

$$1/3 \cdot 38.2 + 2/3 \cdot 22.6 \geq P_{60,T}$$

$$27.8 \geq P_{60,T}$$

The price of the put option in this example should be between 22.6 and 27.8 EUR

**Exercise 2.6 (Put-Call Parity).** *The present price of a stock without dividends is 250 EUR. The market value of a European call with strike price 235 EUR and time to maturity 180 days is 21.88 EUR. The annual risk-free rate is 1 %.*

- Assume that the market price for a European put with same strike price and time to maturity is 5.25 EUR. Show that this is inconsistent with put-call parity.*
- Describe how you can take advantage of this situation by finding a combination of purchases and sales which provides an instant profit with no liability 180 days from now.*

- Put-call parity gives:

$$\begin{aligned}
 P_{K,T}(S_t, \tau) &= C_{K,T}(S_t, \tau) - \{S_t - K \exp(-r\tau)\} \\
 &= 21.88 - \{250 - 235 \exp(-0.01 \cdot 0.5)\} \\
 &= 21.88 - 16.17 \\
 &= 5.71
 \end{aligned}$$

Thus, the market value of the put is too low and it offers opportunities for arbitrage.

- Puts are underpriced, so we can make profit by buying them. We use  $CF_t$  to denote the cash flow at time  $t$ . The cash flow table for this strategy can be seen in Table 2.3.

**Exercise 2.7 (Hedging Strategy).** *A stock currently selling at  $S_0$  with fixed dividend  $D_0$  is close to its dividend payout date. Show that the parity value for the futures price on the stock can be written as  $F_0 = S_0(1 + r)(1 - d)$ , where  $d = D_0/S_0$  and  $r$  is the risk-free interest rate for a period corresponding to the term of the futures contract. Construct an arbitrage table demonstrating the riskless strategy assuming that the dividend is reinvested in the stock. Is your result*

**Table 2.4** Cash flow table for this strategy

Action	$CF_0$	$CF_T$
Buy one share immediately	$-S_0$	$S_T$
Reinvest the dividend	0	$S_T d / (1 - d)$
Sell $1/(1 - d)$ forwards	0	$(F_0 - S_T)/(1 - d)$
Borrow $S_0$ euros	$S_0$	$-S_0(1 + r)$
Total	0	$F_0/(1 - d) - S_0(1 + r)$

consistent with the parity value  $F_0 = S_0(1 + r) - FV(D_0)$  where the forward value  $FV(x) = (1 + r)x$ ? (Hint: How many shares will you hold after reinvesting the dividend? How will this affect your hedging strategy? )

The price of the stock will be  $S_0(1 - d)$  after the dividend has been paid, and the dividend amount will be  $dS_0$ . So the reinvested dividend could purchase  $d/(1 - d)$  shares of stock, and you end up with  $1 + d/(1 - d) = 1/(1 - d)$  shares in total. You will need to sell that many forward contracts to hedge your position. Here is the strategy (Table 2.4):

To remove arbitrage, the final payoff should be zero, which implies:

$$\begin{aligned}
 F_0 &= S_0(1 + r)(1 - d) \\
 &= S_0(1 + r) - S_0(1 + r)(D_0/S_0) \\
 &= S_0(1 + r) - D_0(1 + r) \\
 &= S_0(1 + r) - FV(D_0)
 \end{aligned}$$

**Exercise 2.8 (No-Arbitrage Theory).** Prove that the following relationship holds, using no-arbitrage theory.

$$F(T_2) = F(T_1)(1 + r)^{T_2 - T_1} - FV(D)$$

where  $F_0(T)$  is today's futures price for delivery time  $T$ ,  $T_2 > T_1$ , and  $FV(D)$  is the future value to which any dividends paid between  $T_1$  and  $T_2$  will grow if invested risklessly until time  $T_2$  (Table 2.5).

Since the cashflow at  $T_2$  is riskless and no net investment is made, any profits would represent an arbitrage opportunity. Therefore, the zero-profit no-arbitrage restriction implies that

$$F(T_2) = F(T_1)(1 + r)^{T_2 - T_1} - FV(D)$$

**Exercise 2.9 (Arbitrage Opportunity).** Suppose that the current DAX index is 3,200, and the DAX index futures which matures exactly in 6 months are priced at 3,220.

**Table 2.5** Cash flow table for this strategy

Action	$CF_0$	$CF_{T_1}$	$CF_{T_2}$
Long futures with $T_1$ maturity	0	$S_1 - F(T_1)$	0
Short futures with $T_2$ maturity	0	0	$F(T_2) - S_2$
Buy the asset at $T_1$ , sell at $T_2$ .	0	$-S_1$	$S_2 + FV(D)$
Invest dividends paid until $T_2$			
At $T_1$ , borrow $F(T_1)$	0	$F(T_1)$	$-F(T_1) \times (1 + r)^{T_2 - T_1}$
Total	0	0	$F(T_2) - F(T_1) \times (1 + r)^{T_2 - T_1} + FV(D)$

(a) If the bi-annual current interest rate is 2.5 %, and the bi-annual dividend rate of the index is 1.5 %, is there an arbitrage opportunity available? If there is, calculate the profits available on the strategy.

(b) Is there an arbitrage opportunity if the interest rate that can be earned on the proceeds of a short sale is only 2 % bi-annually?

(a) The bi-annual net cost of carry is  $1 + r - d = 1 + 0.025 - 0.015 = 1.01 = 1\%$ . The detailed cash flow can be seen in Table 2.6.

Thus, the arbitrage profit is 12.

(b) Now consider a lower bi-annual interest rate of 2 %. From Table 2.7 which displays the detailed cash flow, we could see the arbitrage opportunity has gone.

**Exercise 2.10 (Hedging Strategy).** A portfolio manager holds a portfolio that mimics the S&P 500 index. The S&P 500 index started at the beginning of this year at 800 and is currently at 923.33. The December S&P 500 futures price is currently 933.33 USD. The manager's fund was valued at ten million USD at the beginning of this year. Since the fund has already generated a handsome return last year, the manager wishes to lock in its current value. That is, the manager is willing to give up potential increases in order to ensure that the value of the fund does not decrease. How can you lock in the value of the fund implied by the December futures contract? Show that the hedge does work by considering the value of your net hedged position when the S&P 500 index finishes the year at 833.33 and 1,000 USD.

First note that at the December futures price of 933.33 USD, the return on the index, since the beginning of the year, is  $933.33/800 - 1 = 16.7\%$ . If the manager is able to lock in this return on the fund, the value of the fund will be  $1.1667 \cdot 10 = 11.67$  million USD. Since the notional amount underlying the S&P 500 futures contract is  $500 \cdot 933.33 = 466,665$  USD, the manager can lock in the 16.67 % return by selling  $11,666,625/466,665 = 25$  contracts.

Suppose the value of the S&P 500 index is 833.33 at the end of December. The value of the fund will be  $833.33/800 \cdot 10 = 10.42$  million USD. The gain on the futures position will be  $-25 \cdot 500(833.33 - 933.33) = 1.25$  million USD. Hence,



**Table 2.6** Cash flow table for this strategy

Action	$CF_0$	$CF_T$
Buy futures contract	0	$S_T - 3,220$
Sell stock	3,200	$-S_T - 0.015 \cdot 3,200$
Lend proceeds of sale	$-3,200$	$3,200 \cdot 1.025$
	0	12

**Table 2.7** Cash flow table with a lower interest rate

Action	$CF_0$	$CF_T$
Buy futures contract	0	$S_T - 3,220$
Sell stock	3,200	$S_T - 0.015 \cdot 3,200$
Lend proceeds of sale	$-3,200$	$3,200 \cdot 1.02$
	0	$-4$

the total value of the hedged position is  $10.42 + 1.25 = 11.67$  million USD, locking in the 16.67 % return for the year.

Now suppose that the value of the S&P 500 index is 1,000 at the end of December. The value of the fund will be  $1,000/800 \cdot 10 = 12.5$  million USD. The gain on the futures position will be  $-25 \cdot 500(1,000 - 933.33) = -0.83$  million USD. Hence, the total value of the hedged position is  $12.5 - 0.83 = 11.67$  million USD, again locking in the 16.67 % return for the year.

**Exercise 2.11 (Forward Exchange Rate).** *The present exchange rate between the USD and the EUR is 1.22 USD/EUR. The price of a domestic 180-day Treasury bill is 99.48 USD per 100 USD face value. The price of the analogous EUR instrument is 99.46 EUR per 100 EUR face value.*

- (a) *What is the theoretical 180-day forward exchange rate?*  
 (b) *Suppose the 180-day forward exchange rate available in the marketplace is 1.21 USD/EUR. This is less than the theoretical forward exchange rate, so an arbitrage is possible. Describe a risk-free strategy for making money in this market. How much does it gain, for a contract size of 100 EUR?*

- (a) The theoretical forward exchange rate is

$$1.22 \cdot 0.9946 / 0.9948 = 1.2198 \text{ USD/EUR.}$$

- (b) The price of the forward is too low, so the arbitrage involves buying forwards. Firstly, go long on a forward contract for 100 EUR with delivery price 1.21 USD/EUR. Secondly, borrow  $\exp(-qT)$  EUR now, convert to dollars at 1.22 USD/EUR and invest at the dollar rate.

At maturity, fulfill the contract, pay  $1.21 \cdot 100$  USD for 100 EUR, and clear your cash positions. You have  $(1.2198 - 1.21) \cdot 100 = 0.0098 \cdot 100$  USD. That is, you make 0.98 USD at maturity risk-free.

**Table 2.8** Cash flow table for zero-net-investment arbitrage portfolio

Action	$CF_0$	$CF_T$
Short shares	2,500	$-(S_T + 40)$
Long futures	0	$S_T - 2,530$
Long zero-bonds	-2,500	2,576.14
Total	0	6.14

**Exercise 2.12 (Valuation of a Forward Contract).** *What is the value of a forward contract with  $K = 100$ ,  $S_t = 95$ ,  $r = 10\%$ ,  $d = 5\%$  and  $\tau = 0.5$ ?*

The payoff of the forward contract can be duplicated with buying  $\exp(-d\tau)$  stocks and short selling zero bonds with nominal value  $K \exp(-r\tau)$ . So

$$\begin{aligned} V_{K,T}(S_t, \tau) &= \exp(-0.05 \cdot 0.5) \cdot 95 - 100 \cdot \exp(-0.10 \cdot 0.5) \\ &= -2.4685 \end{aligned}$$

Thus, the buyer of the forward contract should be paid 2.4685 for this deal.

**Exercise 2.13 (Put-Call Parity).** *Suppose there is a 1-year future on a stock-index portfolio with the future price 2,530 USD. The current stock index is 2,500, and a 2,500 USD investment in the index portfolio will pay a year-end dividend of 40 USD. Assume that the 1-year risk-free interest rate is 3%.*

- Is this future contract mispriced?*
  - If there is an arbitrage opportunity, how can an investor exploit it using a zero-net investment arbitrage portfolio?*
  - If the proceeds from the short sale of the shares are kept by the broker (you do not receive interest income from the fund), does this arbitrage opportunity still exist?*
  - Given the short sale rules, how high and how low can the futures price be without arbitrage opportunities?*
- (a) The price of a future can be found as follows:

$$\begin{aligned} F_0 &= S_0 \exp(r\tau) - D \\ &= 2500 \cdot \exp(0.03) - 40 \\ &= 2576.14 - 40 \\ &= 2536.14 > 2530 \end{aligned}$$

This shows that the future is priced 6.14 EUR lower.

- (b) Zero-net-investment arbitrage portfolio Cash flow for this portfolio is described in Table 2.8.

**Table 2.9** Cash flow table for the no interest income case

Action	$CF_0$	$CF_T$
Short shares	2,500	$-(S_T + 40)$
Long futures	0	$S_T - 2,530$
Long zero-bonds	-2,500	2,500
Total	0	-70

**Table 2.10** Cash flow table for this strategy

Action	$CF_0$	$CF_T$
Short shares	2,500	$-(S_T + 40)$
Long futures	0	$S_T - F_0$
Long zero-bonds	-2,500	2,500
Total	0	$2,460 - F_0$

## (c) No interest income case

According to Table 2.9, the arbitrage opportunity does not exist.

- (d) To avoid arbitrage,  $2,460 - F_0$  must be non-positive, so  $F_0 \geq 2,460$ . On the other hand, if  $F_0$  is higher than 2,536.14, an opposite arbitrage opportunity (buy stocks, sell futures) opens up. Finally we get the no-arbitrage band  $2,460 \leq F_0 \leq 2,536.14$  (Table 2.10)

**Exercise 2.14 (Hedging Strategy).** *The price of a stock is 50 USD at time  $t = 0$ . It is estimated that the price will be either 25 or 100 USD at  $t = 1$  with no dividends paid. A European call with an exercise price of 50 USD is worth  $C$  at time  $t = 0$ . This call will expire at time  $t = 1$ . The market interest rate is 25 %.*

- (a) *What return can the owner of the following hedge portfolio expect at  $t = 1$  for the following actions: sell 3 calls for  $C$  each, buy 2 stocks for 50 USD each and borrow 40 USD at the market interest rate*
- (b) *Calculate the price  $C$  of a call.*
- (a) By setting up a portfolio where 3 calls are sold  $3C$ , 2 stocks are bought  $-2 \cdot 50$  and 40 USD are borrowed at the market interest rate at the current time  $t$ , the realised immediate profit is  $3C - 60$ . The price of the call option can be interpreted as the premium to insure the stocks against falling below 50 USD. At time  $t = 1$ , if the price of the stock is less than the exercise price ( $S_t < K$ ) the holder does not exercise the call options, otherwise he does. When the price of the stock at time  $t = 1$  is equal to 25 USD, the holder does not exercise the call option, but he does when the price of the stock at time  $t = 1$  is 100 USD. Also at time  $t = 1$ , the holder gets the value  $2S_1$  by purchasing two stocks at  $t = 0$  and pays back the borrowed money at the interest rate of 25 %. The difference of the value of the portfolio with the corresponding stock price 25 USD or 100 USD at  $t = 1$  is shown in Table 2.11. At time  $t = 1$ , the cash flow is independent of

**Table 2.11** Portfolio value at time  $t = 1$  of Exercise 2.14

Action	$CF_0$	$CF_1(S_1 = 25)$	$CF_1(S_1 = 100)$
Sell 3 calls	$3C$	0	$-3(100 - 50) = -150$
Buy 2 stocks	$-2 \cdot 50 = -100$	$2 \cdot 25 = 50$	$2 \cdot 100 = 200$
Borrow	$40n$	$-40(1 + 0.25) = -50$	$-50$
Total	$3C - 60$	0	0

the stock, which denotes this strategy as risk-free. That is, the owner does not expect any return from the described hedge portfolio.

- (b) The price of the call of this hedge portfolio is equal to the present value of the cash flows at  $t = 1$  minus the cash flow at  $t = 0$ . In this case we have that the present value of cash flows at  $t = 1$  is equal to zero and the cash flow at time  $t = 0$  is  $3C - 60$ . Therefore, the value of the call option is equal to  $C = 20$ . Here the martingale property is verified, since the conditional expected value of the stock price at time  $t = 1$ , given the stock prices up to time  $t = 0$ , is equal to the value at the earlier time  $t = 0$ .

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Borak, S.; Härdle, W.K.; López-Cabrera, B.

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