Optimal Pollution, Optimal Population, and Sustainability

Ulla Lehmijoki

1 Introduction

Is it possible that current utility maximization takes place at the cost of human lives? This possibility was already implied in the long-run consumer optimization models of Dasgupta and Heal (1974), Solow (1974), Stiglitz (1974), Krautkraemer (1985), and Pezzey and Withagen (1998) who argued that *the scarcity of natural resources* may lead to ever-decreasing per capita consumption. Per capita consumption may also decrease if *excessive pollution* impairs production and compromises life-supporting systems as was argued by Keeler et al. (1971), Plourde (1972), Foster (1973) and Smulders and Gradus (1996). However, in both types of models the demographic aspect is deficient as population either keeps constant or grows at a constant rate in spite of decreasing consumption numbers.

In this paper, I explicitly assume that population is endogenous to the environment, i.e., there is feedback from the environment to mortality which rises if population is not environmentally supported, this feedback being defined as a "positive check" by Robert Malthus (1914). Positive check may occur either because of the increasing scarcity of resources or because of the continuing concentration of pollutants. In this paper, I focus on pollutants as emerging evidence on the lethal effects of the pollutants maintains that the positive check is at work. This evidence consists of medical and econometric studies, showing that there already is a statistically significant increase in mortality due to urban air pollution, and that climate change may induce further increases in the future. Other global concerns, such as the pollution of ground waters and oceans, are also possible, but less evidence on their mortality effects has been received thus far.[1]

[1] In spite of my emphasis on pollutants, the model can be generalized to natural resources since resource depletion can be seen as pollution in the extended sense (Keeler et al. 1971).

U. Lehmijoki (✉)
University of Helsinki and HECER, "Economicum" Building, Arkadiankatu 7, P.O. Box 17, 00014 Helsinki, Finland
e-mail: Ulla.Lehmijoki@helsinki.fi

J. Crespo Cuaresma et al. (eds.), *Green Growth and Sustainable Development*, Dynamic Modeling and Econometrics in Economics and Finance 14,
DOI 10.1007/978-3-642-34354-4_2,

Section 2 of this paper reviews the empirical evidence on the positive check and Sect. 3 introduces a model of optimal pollution with endogenous population. Section 4 discusses the sustainability implications and a new definition for sustainability is supplied. The role of technical progress is shown to be less positive than what is usually suggested. Section 5 gives a parametric example and Sect. 6 closes the paper. To concentrate on population, the simplest model of optimal pollution is provided. Even so, endogenous population tends to make the model "murky" (Solow 1974) but excessive complexity can be avoided by modeling in virtual time.

2 The Positive Check—Recent Evidence

This Section reviews the global evidence on environmental mortality with focus on air pollution and climate change (CO_2 emissions). Mortality induced by air pollution has been debated since the smog in the Meuse Valley in 1930 and London in 1952 took the lives of 60 and 4000 people (Nemery et al. 2001 and Logan 1953). Recently, the Clean Air for Europe program (*CAFE*) and *WHO* have summarized the European research by collecting 629 peer-reviewed time-series studies and 160 individual or panel studies up to February 2003 (WHO 2004). In the original studies, daily adult mortality in several European cities was regressed against daily changes in air pollution as indicated by particular matter (PM) and ozone.[2] The summary estimates show that there is a statistically significant 0.6% and 0.3% increase in mortality for each 10 μg/m^3 increase in *PM* and ozone respectively.

The study for the effects of long-term *PM* exposure got its onset in the United States as Pope et al. (2002) analyzed questionnaires from 1982 which provided data on sex, race, smoking, alcohol consumption, etc., so that controlling for alternative risk sources was possible. The mortality data which were collected until 1998 implied that there was 4%, 6%, and 8% increases in all-cause, respiratory, and lung cancer mortality respectively for each 10 μg/m^3 increase in *PM*. Evans and Smith have estimated similar increases (Evans and Smith 2005). For a recent review of long-term study literature, see Raaschou-Nielsen et al. (2011). The estimates of Pope et al. (2002) were applied to the European data by *CAFE* and *WHO* to calculate that the short-term and long-term exposures were together responsible for 370 000 premature deaths in 2000 in Europe (WHO 2004). The infant mortality risk

[2] Air pollution consists of several components, of which particulate matter (*PM*) and ozone are the most dangerous (WHO 2004). The term particulate matter (*PM*) refers to solid airborne particles of varying size, chemical composition and origin. For example, the particles in PM_{10} have a diameter of less than 10 μm and are mainly combustion-derived, either from traffic or from energy production, often from long-distance sources. Existing evidence suggests that the smaller the particles are, the more deeply into the lung they penetrate (WHO 2004). Air pollution increases mortality mainly through an increase in respiratory and cardiovascular diseases and lung cancer (Samet et al. 2000), but an increase in skin cancer is also reported (Brunekreef and Holgate 2002). All age groups are affected, but unborn and young children as well as the elderly are the most vulnerable (Pope and Dockery 2006).

has been studied by for example by Currie and Neidell (2004), Chay and Greenstone (2003) and Scheers et al. (2011). WHO has summarized that, taken all types of deaths together, urban outdoor air pollution causes 1.3 million deaths worldwide per year (WHO 2011).

In climate-change studies, the mortality estimates are based on simulations (Pitcher et al. 2008). Tanser et al. (2003), for example, have applied the Hadley Centre's climate model to estimate that the increase in malaria distribution and the prolonged malaria season would lead to a 25% increase in the risk of death from malaria by 2100, mainly in Africa. The abundant literature on climate change has been collected and analyzed by the UN's Intergovernmental Panel on Climate Change (*IPCC*). Its Third Assessment Report suggests that mortality will increase because of weather extremes, because of environmental changes which lead to diseases or to water and food shortages, or because of conflicts in displaced populations (IPCC 2003, updated 2007). Relying on the *IPCC*, *WHO* has published a summary report on human health and climate change (WHO 2003). This report projects a maximum increase in the risk of 83%, 17%, and 32% for the great killers; malaria, diarrhoea, and malnutrition, respectively. There is also a great projected risk increase in coastal floods, but the number of deaths may be low (Gosling et al. 2009). The mortality effects of climate change are unequally distributed and are particularly severe in countries with already high disease burdens, such as sub-Saharan Africa and Asia (IPCC 2003). Nevertheless, Deschênes and Greenstone (2011) suggest that, under a business-as-usual scenario, climate change will also lead to an increase in the overall U.S. annual mortality rate ranging from 0.5% to 1.7% by the end of the 21st century. WHO has also summarized that, currently, climate change contributes to 150 000 deaths each year (WHO 2012).

3 The Model

3.1 *Modeling the Positive Check*

To model the positive check, note that the population growth rate $\dot{L}/L = n$ is the difference between fertility and mortality. In what follows, I assume that only mortality depends on pollution while fertility is constant.[3]

Pollution may increase mortality (decrease population growth) both as emissions E and as stocks S, but it seems appropriate to model in terms of stocks because their mortality effects are more longstanding. Hence, let:

$$n = n(S), \qquad n(0) > 0, \qquad n'(S) < 0, \qquad n(\hat{S}) = 0, \tag{1}$$

[3]Some studies suggest, however, that fertility may respond to environmental degradation both because it is causing poverty and because toxins etc. cause miscarriage (Lutz et al. 2005). Because the emphasis of this paper is on the positive check, the fertility effects are excluded, for simplicity.

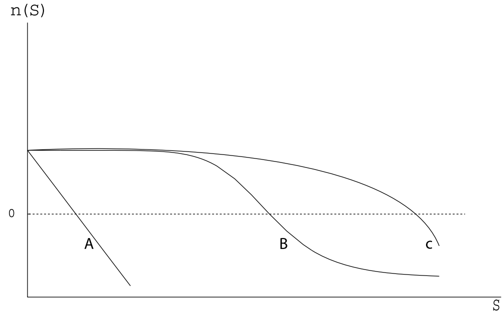

Fig. 1 Possible functional formulas for the positive check. Meadows et al. (1972)

where $\hat{S}$ is the critical stock beyond which population starts to decrease. Normalizing the initial level of population to unity it holds

$$L(t) = \exp \int_0^t n\big[S(\tau)\big] d\tau. \tag{2}$$

Several functional formulas satisfy the assumption of the population growth function (1). Some alternatives, repeated in Fig. 1, have been suggested already in the Report of Rome (Meadows et al. 1972). In A population growth decreases linearly, in C the negative effect is exponential, and in B mortality increases as pollution stock bypasses the threshold level after which the positive check cuts in and mortality starts to increase (population growth starts to decrease). Section 5 gives a closer look at these alternative cases.

The accumulation of the pollution stock is dictated by emissions and abatements which are given by an abatement function $\delta(S)$,

$$\dot{S} = E - \delta(S). \tag{3}$$

The first component of (3) can be rewritten as $E = (E/L) \cdot L$ to see that the environmental burden of population comes from two sources, namely from per capita emission E/L and from the number of people L.

The role of the second component, the abatement function $\delta(S)$ has been broadly debated in the literature.[4] In this paper, I assume a simple hump-shaped abatement function which is strictly concave. Thus, let $\delta(0) = \delta(\tilde{S}) = 0$ and $\delta'(0) > 0$, $\delta'(\tilde{S}) < 0$, $\delta''(S) < 0$ where $\tilde{S} > 0$ is the carrying capacity of the environment. To allow the possibility of negative population growth in the area $0 < S < \tilde{S}$, I assume $\hat{S} < \tilde{S}$.

3.2 *The Household Optimization*

Consider an infinitely living representative household which wants to maximize its Benthamian total utility. At each instant of time, the total utility then becomes

[4] For a review, see Tahvonen and Salo (1996).

$u(C/L) \cdot L$, where u satisfies the standard concavity properties and Inada conditions.[5] In its intertemporal choice, the household faces the discount factor $\rho > 0$. To focus on population and pollution in the absence of production problems, I adopt the simplest formulation for the rest of the model in line with Foster (1973) who assumes that consumption C takes place directly at the cost of environment, i.e., $C = E$. The representative household then chooses emissions $E(t)$ to maximize

$$U = \int_0^\infty u\big[E(t)/L(t)\big]L(t)e^{-\rho t}dt = \int_0^\infty u\big[E(t)/L(t)\big]e^{-\int_0^t\{\rho - n[S(\tau)]\}d\tau}dt, \quad (4)$$

subject to (3). The mechanism of the model is the following: by choosing the optimal path for $E(t)$, the household determines $S(t)$, which in turn dictates the optimal population growth rate $n(t)$ and the optimal population $L(t)$. Finally, per capita emissions $E(t)/L(t)$ are determined.

Because the discount factor in (4) is not constant, I apply the virtual time technique suggested by Uzawa (1968). Let us denote

$$\Delta(t) \equiv \int_0^t \big\{\rho - n\big[S(\tau)\big]\big\}d\tau$$

to get $\frac{d\Delta(t)}{dt} = \rho - n[S(t)]$ and $dt = \frac{d\Delta(t)}{\rho - n[S(t)]}$. The problem can now be rewritten in virtual time as:

$$U = \int_0^\infty \frac{u(E/L)}{\rho - n(S)} \cdot e^{-\Delta} \cdot d\Delta,$$
$$\mathring{S} \equiv \frac{dS}{d\Delta} = \frac{dS}{dt}\frac{dt}{d\Delta} = \frac{E - \delta(S)}{\rho - n(S)},$$

where $E \equiv E[\Delta(t)]$, $S \equiv S[\Delta(t)]$, $L \equiv L[\Delta(t)]$. This concave problem with constant discount factor can be solved in virtual time by using standard methods (Benveniste and Scheinkman 1982). Given that both the population size L and its growth rate n depend on the pollution stock S through (1) and (2), the current value Hamiltonian and the necessary conditions become:

$$H(S, E, \lambda) = \frac{1}{\rho - n(S)}\big\{u(E/L) + \lambda(\Delta)\big[E - \delta(S)\big]\big\},$$

$$\frac{\partial H(S, E, \lambda)}{\partial E} = 0 \quad \iff \quad -u'(E/L) = \lambda(\Delta) \cdot L, \quad (5)$$

$$\mathring{\lambda} \equiv \frac{d\lambda(\Delta)}{d\Delta} = -\frac{\partial H}{\partial S} + \lambda(\Delta), \quad (6)$$

$$\lim_{\Delta\to\infty} \lambda(\Delta)e^{-\Delta}S = 0. \quad (7)$$

[5] Krutilla (1967) and Barbier (2003).

Taking the derivative in (6) and rearranging one gets

$$\mathring{\lambda}/\lambda = -(1/\rho - n)\left\{n' H/\lambda - \left(\delta' + \rho - n\right)\right\}. \tag{8}$$

To eliminate λ, one can follow the usual procedure by taking the derivative of (5) in terms of (virtual) time. These derivatives are denoted by $\mathring{E} \equiv dE/d\Delta$ and $\mathring{L} \equiv dL/dT$. To simplify the analysis, let us adopt the *CIES* utility function $u(E/L) = [(E/L)^{1-\theta}]/(1-\theta)$, $\theta \neq 1$ with $u'' \cdot (E/L)/u' = -\theta$ to give

$$\mathring{\lambda}/\lambda = -\theta \mathring{E}/E + (\theta - 1)\mathring{L}/L, \tag{9}$$

which together with (8) gives $\mathring{E}/E = [1/\theta(\rho - n)]\{-n' H/\lambda - (\delta' + \rho - \theta n)\}$, where $\mathring{L}/L = n/(\rho - n)$ is applied. Substituting the expression $-n' H/\lambda = [n'/(\rho - n)][\theta E/(\theta - 1) - \delta]$ and noting $\mathring{E} = \dot{E}/(\rho - n)$ one finally derives

$$\frac{\dot{E}}{E} = \frac{1}{\theta}\left\{\frac{n'}{\rho - n}\left[\frac{\theta E}{\theta - 1} - \delta\right] - \left(\delta' + \rho - \theta n\right)\right\}. \tag{10}$$

The non-linear equations (3) and (10) supply the solution to the model. The phase lines become:

$$\frac{\dot{E}}{E} = 0 \quad \Leftrightarrow \quad E = \frac{\theta - 1}{\theta}\left\{\delta + \frac{\rho - n}{n'}\left(\delta' + \rho - \theta n\right)\right\}, \tag{11a}$$

$$\dot{S} = 0 \quad \Leftrightarrow \quad E = \delta. \tag{11b}$$

In the (S, E)-space, the shape of the $\dot{S} = 0$-line is that of δ, i.e., inverted U with $\delta(0) = \delta(\tilde{S}) = 0$ (Fig. 2). The shape of the $\dot{E} = 0$-line depends on the value of θ. Because Hall has argued that empirical elasticities tend to be large (Hall 1988), I assume $\theta > 1$, but nothing essential is changed if $\theta < 1$ is assumed instead. Even for $\theta > 1$, there is variety in the shape of the $\dot{E} = 0$-line. The following is the sufficient condition for the existence of at least one interior steady state:

Lemma 1 *If* $\delta'(0) + \theta n(0) > \rho$ *and* $\delta'(\tilde{S}) + \theta n(\tilde{S}) < \rho$ *then the problem has at least one steady state* $S^* \subset (0, \tilde{S})$.

Proof In the (S, E)-space the $\dot{S} = 0$-line hits the S-axis at $S = 0$ and at $S = \tilde{S}$. For $S = 0$ and $S = \tilde{S}$, (11a) then becomes $\dot{E} = 0 \Leftrightarrow E = \frac{\theta - 1}{\theta}\{\frac{\rho - n}{n'}(\delta' + \rho - \theta n)\}$. By assumption, $\theta - 1 > 0$, $\rho - n > 0$ and $n' < 0$. Graphically, if $\delta'(0) + \theta n(0) > \rho$ and $\delta'(\tilde{S}) + \theta n(\tilde{S}) < \rho$, the $\dot{E} = 0$-line lies below the $\dot{S} = 0$-line for $S = 0$ and above it for $S = \tilde{S}$ (Fig. 2). By continuity, the $\dot{E} = 0$-line intersects the $\dot{S} = 0$-line at least once. □

To comprehend, consider marginal emissions. If consumed tomorrow, emissions are discounted by ρ. If consumed today, it adds to the pollution stock S and produces a change in abatement $\delta'(S)$ and population $n(S)$. If the sum of the latter two is larger, consumption today pays. The first unit of emission is consumed if $\delta'(0) +$

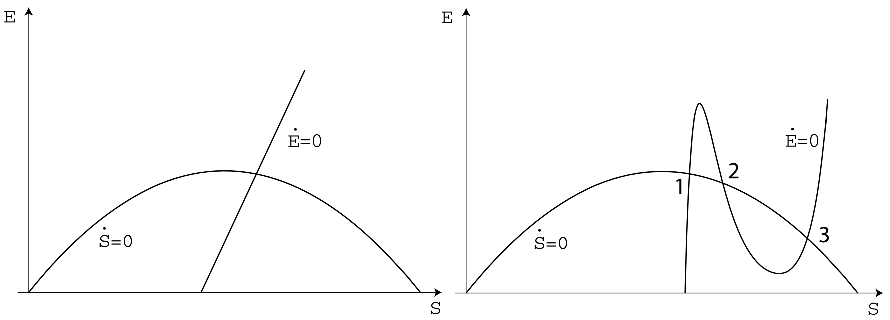

Fig. 2 The phase diagrams of the model

$\theta n(0) > \rho$. On the other hand, if $\delta'(\tilde{S}) + \theta n(\tilde{S}) < \rho$ it never pays to pollute until the carrying capacity $\tilde{S}$.

Depending upon the properties of the population growth function (1), the $\dot{E} = 0$-line may be non-linear and the model may have several steady states; I assume that the number of the steady states is either one or three as shown in Fig. 2. The local stability analysis in Appendix shows that the single steady state is a saddle with stable manifolds running from the North-West and South-East, as the left panel of Fig. 2 illustrates. If the number of the steady states is three (the right panel of Fig. 2), then the first and third are saddles but the second is an unstable focus or node. The following lemma characterizes all saddle-stable steady states:

Lemma 2 *Inefficient under-accumulation of the pollutant is not possible.*

Proof Equations (11a) and (11b) imply that in a steady state

$$\frac{\theta - 1}{\theta}\left\{\delta + \frac{\rho - n}{n'}\left[\delta' + \rho - \theta n\right]\right\} = \delta. \tag{12}$$

The transversality condition is $\lim_{\Delta\to\infty}\{\lambda(\Delta)e^{-\Delta}S(\Delta)\} = 0$. Because the model tends to the steady state, S and $n(S)$ go to constants S^* and $n(S^*)$. In a steady state, $\mathring{E} = 0$ so that (9) implies $\mathring{\lambda}/\lambda = (\theta - 1)\mathring{L}/L$, which is a constant in the steady state. The transversality condition then requires $(\theta - 1)\mathring{L}/L - 1 < 0$. Because $\mathring{L}/L = n/(\rho - n)$, we get $(\theta - 1)n(S^*)/(\rho - n(S^*)) - 1 < 0$ and further

$$\rho - \theta n\left(S^*\right) > 0. \tag{13}$$

Arranging and using (12) we get $\rho - \theta n = \frac{n'}{(\rho-n)(\theta-1)}\delta - \delta' > 0$. Because $\frac{n'}{(\rho-n)(\theta-1)}\delta < 0$, it must be $\delta'(S^*) < 0$. Therefore, the steady state is located on the downwards sloping part of the $\dot{S} = 0$-line. □

4 Sustainability and Technical Progress

The Brundtland Commission 1987 defines sustainable development as a development that "meets the needs of the present without compromising the ability of future generations to meet their own needs" (WCED 1987). This definition refers to non-decreasing consumption or non-decreasing utility, concepts also used by most economists (for a review, see Pezzey 1992). With the positive check present, the concept of sustainability needs a redefinition:

Definition An optimal path is sustainable if it provides non-decreasing consumption for a non-decreasing population.

Thus, an optimal path can lose sustainability either because per capita consumption decreases or because population decreases.

Consider first a steady state. Recall that $E = C$. The growth rate of the per capita consumption is $\gamma_{C/L} = \dot{E}/E - \dot{L}/L$. In the steady state, E is constant so that $\gamma_{C/L} = -\dot{L}/L = -n(S^*)$. Three alternatives are possible. For $n(S^*) > 0$, the population keeps increasing and per capita consumption decreasing. For $n(S^*) = 0$, both the population and per capita consumption are constants. For $n(S^*) < 0$, an ever-decreasing population enjoys ever-increasing per capita consumption. Note that this steady state implies $\lim_{t\to\infty} L(t) = 0$ so that, asymptotically, the size of the population vanishes to zero. Thus, of the above alternatives, only $n(S^*) = 0$ is sustainable.

Which of the above cases realizes? First note that the a priori assumptions $\rho > 0$ and $\rho - n(S) > 0$ pose no explicit limit to $\operatorname{sign} n(S^*)$. Another candidate that would limit $\operatorname{sign} n(S^*)$ is the transversality condition in (13) but for the suitable values of ρ and θ it can hold for positive and negative values of $n(S^*)$. Thus, in the steady state S^* the optimal population may be constant, increasing, or decreasing because the utilitarian objective functional $\int_0^\infty u(E/L)Le^{-\rho t}dt$ may take its maximum both at high E/L and low L or vice versa. Therefore, it may well be optimal to increase consumption at the cost of population.

Some optimists argue, however, that technical progress ultimately warrants sustainability (Neumayer 1999, for example). To see whether this optimism is supported by the model, let $A(t)$ be the available technology at time t and assume that technical progress is exogenously running at rate x so that $A(t) = e^{xt}$ for $A(0) = 1$. Further, let technical progress be consumption augmenting in the meaning that, at every instant of time t, we have $C = e^{xt}E$ implying that for given emissions it is possible to consume more than before (Krautkraemer 1985). Per capita consumption then becomes

$$C/L = e^{xt}E/L. \tag{14}$$

Per capita consumption C/L grows at rate $\gamma_{C/L} = \dot{E}/E + x - n$. In a steady state, $\dot{E}/E = 0$, so that $\gamma_{C/L}$ is positive if $x > n(S^*)$. It is thus *possible* to have growing per capita consumption and growing population together. However, positive popu-

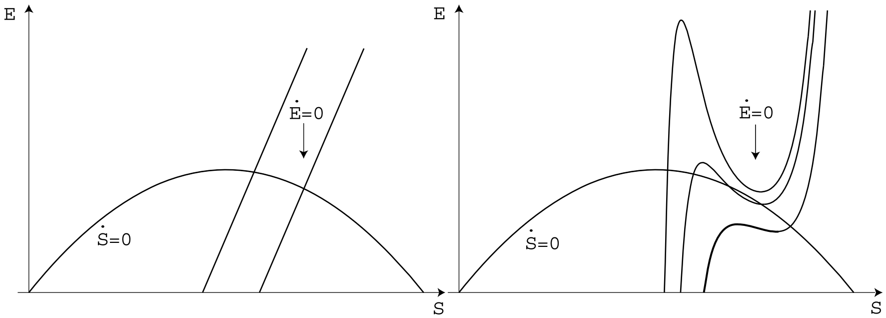

Fig. 3 Technical progress shifts the $\dot{E}=0$-*line* down and increases the steady state pollution S^*

lation growth is by no means warranted. To see why, apply (14) to (4)–(9) to derive

$$\frac{\dot{E}}{E}=0 \quad \Leftrightarrow \quad E=\frac{\theta-1}{\theta}\left\{\delta+\frac{\rho-n}{n'}\left[\delta'+(\theta-1)x+(\rho-\theta n)\right]\right\}. \qquad (15)$$

The derivative of (15) in terms of x is:

$$\left.\frac{\partial E}{\partial x}\right|_{\dot{E}=0}=\frac{(\theta-1)^2(\rho-n)}{\theta n'}<0.$$

Therefore, the $\dot{E}=0$-line shifts down as the pace of technical progress increases (Fig. 3).

To comprehend, note that in the equilibrium, the *negative* utility effect of a marginal emission through an increase in S and a decrease in population growth, and its *positive* utility effect through an increase in consumption are equal and further emissions are rejected. Technical progress increases the positive consumption effect and larger emission are accepted. Given the biologically determined $\hat{S}$, it is then more likely that $\hat{S}<S^*$ and $n(S^*)$ is negative. Therefore, contrary to conventional wisdom, we find that technical progress does not necessarily save us because it makes extra consumption and emission pay.

To stipulate $\gamma_{C/L}=\dot{E}/E+x-n$ during the transitional period, write

$$\frac{\dot{E}}{E}=\frac{n'}{(\rho-n)(\theta-1)}\left\{\dot{S}+\frac{1}{\theta}\left[\delta-\frac{(\rho-n)(\theta-1)}{n'}\left[\delta'+\rho-\theta n+(\theta-1)x\right]\right]\right\},$$

where the leftmost element is the positive difference of the $\dot{S}=0$ and $E=0$-lines indicating $\dot{E}/E<0$ along the north-western saddle path. Further, as earlier, we have $\lim_{S\to S^*}\dot{E}/E=0$. Therefore, the sign of $\lim_{S\to S^*}\gamma_{C/L}$ depends on the sign of $x-n(S^*)$. In particular, for $n(S^*)<0$ we have $x-n(S^*)>0$ for all x and $\lim_{S\to S^*}\gamma_{C/L}=\dot{E}/E+x-n>0$ implying that per capita consumption increases as the economy approaches the steady state $n(S^*) < 0$.[6] The following proposition summarizes the results:

Proposition *If the optimal population growth in the steady state is negative, then per capita consumption increases as the economy approaches the steady state. A high rate of technical progress increases the probability for negative steady state population growth.*

5 Parametric Examples

Consider the current pollution-population situation. The population on our planet is larger than ever and increasing, and many specialists argue that we are running out of food supply, that air pollution increases, and that global warming is already on its way. The evidence in Sect. 2 indicates that some signals of the positive check are already available. The parametric examples of this Section try to illustrate the this situation and to give some ideas how our demographic and environmental future looks like.

The abstract style of the model naturally makes its parametric presentation difficult but not impossible.[7] Let us start with the assumption that the carrying capacity of the environment $\tilde{S}$ takes some arbitrary value, say $\tilde{S} = 1000$. Since this value refers to a complete disappearance of life, it seems that, in spite of some alarming signals, this situation is not very close yet. Thus, let the current pollution stock be $S(0) = 250$ which is one quarter of $\tilde{S} = 1000$. Further, let $n(S(0)) = 0.005$, indicating that the current (initial) population growth rate is 0.5%. Next, assume that the environmental mortality is high enough to push the population growth below zero if pollution reaches three quarters of $\tilde{S} = 1000$, implying that the critical value is $\hat{S} = 750$.

Other parameters of the model are adapted such that they are in line with the benchmark values above. Consider the population function given in (1) and Fig. 1. The parametric examples provided here concentrate on cases A and B which refer to linear and threshold population function respectively (Fig. 1). These functions are specified as

$$n(S) = \beta - \eta S, \tag{16a}$$

$$n(S) = \beta - \frac{\alpha}{1 + (\mu S)^{-\gamma}}, \tag{16b}$$

[6]The slope of the entire time path for $\gamma_{C/L}$ depends on $\lim_{S\to 0} \gamma_{C/L}$. This and the cases $n(S^*) > 0$ and $n(S^*) = 0$ are not considered for shortness.

[7]The critical obstacle, preventing a full calibration on real data is that, on order to focus on population and pollution, no production function is specified in the model. The main simplification is that the stock of capital (another state variable) is left away, which makes the optimization procedure much simpler and the phase portrait much more intuitive.

where the latter is one of the simplest expressions to produce a threshold function. In the linear case (16a), the demographic response to pollution is given by a single parameter, $\eta > 0$, whereas this response is more complicated in the threshold case as $\beta - \alpha$ gives the lowest population growth reached, $\mu > 0$ multiplies the effect of pollution such that a large value of μ leads to negative population growth at low concentrations and $\gamma > 0$ gives the curvature of the threshold function with high values referring to a highly curved shape and severity of the mortality crisis. In both (16a) and (16b), the parameter β gives the autonomous population growth, i.e., the population growth rate which prevails in a complete the absence of pollution ($S = 0$).

To meet the benchmark values $n(S(0)) = n(250) = 0.005$ and $\tilde{S} = 750$, the parameters of the linear case (16b) must be $\beta = 0.0075$ and $\eta = 0.00001$. In the non-linear case (16b), the autonomous population growth rate $\beta \approx 0.005$ directly warrants $n(S(0)) = n(250) = 0.005$. The choice $\alpha \approx 0.020$ indicates that the lowest population growth reached is -0.15%, a value that seems reasonable even though it can not be derived from the benchmark values above. Further, if $\gamma = 8$ then $\mu \approx 0.00116$ warrants the property $\tilde{S} = 750$ for the threshold case. The top panel in Fig. 4 illustrates.

The abatement function $\delta(S)$ takes the standard logistic formula

$$\delta(S) = rS\left(1 - \frac{S}{\tilde{S}}\right), \tag{17}$$

in which r is the intrinsic rate of annual decay with an assumed value of $r = 0.175$. Conventional values $\theta = 4$ and $\rho = 0.03$ describe the preferences (see Barro and Sala-i-Martin 1995, for example). Two rates of technical progress are assumed, namely $x = 0.00$ and $x = 0.02$. All parameters are collected to Table 1. To summarize, there will be four cases, the linear case without and with technical progress, referred to as A00 and A02, and the threshold case without and with technical progress, referred to as B00 and B02 respectively.[8]

Table 2 reports the main steady-state results of the parameterized model and Fig. 4 illustrates, showing that all cases have a single steady state S^*. Table 2 shows that, in the absence of technical progress we have $S^* < \hat{S}$ both in linear and threshold cases while, in the presence of technical progress, the opposite is true, i.e., $S^* > \hat{S}$. Thus, technical progress makes the steady state population to decrease both in the linear and threshold case, the half-life times being 3623 and 181 years, respectively (Table 2). On the other hand, the steady state population increases by 0.17% or 0.45% if there is no technical progress, doubling in 409 and 153 years (Table 2).

The depicted off-steady-state paths for population in Fig. 5 (left) show that population first rises from the initial value $L(0) = 1$ in all cases, continues to rise for A00 and B00, almost levels-off for A02 and starts to decrease for B02. In

[8]Calculations are performed by Mathematica 7.0. Time-elimination method is used to derive the saddle paths (Mulligan and Sala-i-Martin 1991).

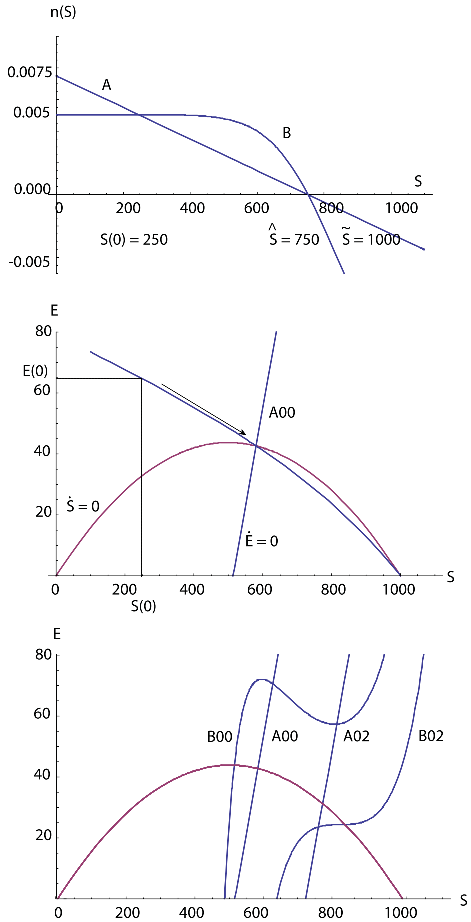

Fig. 4 The parametric population growth functions A and B (*top*), the phase diagram for the case $A00$ (*middle*), and a combined phase diagram for all cases (*bottom*)

the latter case, the initial population $L = 1$ is reached after 160 years. The difference between the population projections is prominent, indeed. The per capita emission (per capita consumption) paths, instead, are rather similar initially. But after some hundred years, per capita emissions along B02 start to rise as the number of people decreases, meeting the proposition in Sect. 4. Thus, in B02, it is optimal to choose higher and higher per capita consumption at the cost of lower and lower number of people. Note also that per capita consumption almost levels-off in A02. Given that population levels-off as well, A02 almost meets the sustainability as defined in Sect. 4, but only by change.[9] To summarize, the parametric example provided here imply that the utility-maximizing path with positive check may take a large variety of consumption-population combinations depending upon

[9]Note, that in the presence of technical progress, however, the case with rising per capita consumption and rising population in the steady state is possible, see Sect. 4.

Table 1 The parameters of the model

Parameter	Linear A	Threshold B
$\tilde{S}$	1000	1000
r	0.175	0.175
β	0.0075	0.005
η	0.00001	
α		0.020
μ		0.00116
γ		8
θ	4	4
ρ	0.03	0.03
x	0.00 or 0.02	0.00 or 0.02

the parameters of the population growth functions and on the rate of technical progress.

Several extensions of the current model are both necessary and possible. Maybe the first of them would be to include a realistic production function in order to see how the role of population (labor) as a factor of production changes the results. A more realistic version of technical progress would take this progress as a response to environmental degradation and overpopulation. This paper assumes that all technical progress is consumption-augmenting, but technical progress may also

Table 2 The results of the model

Results	A00	A02	B00	B02
x	0.00	0.02	0.00	0.02
S^*	580.7	769.1	545.8	833.8
$n(S^*)$	0.17%	−0.019%	0.45%	−0.38%
Doubling/half-life (years)	409	3623	153	181

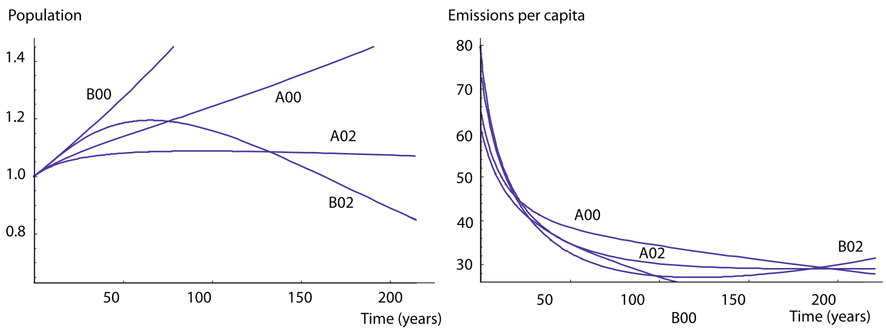

Fig. 5 The parametric time paths for population and per capita consumption

save lives and increase longevity. It may be "dirty" or "clean" as it may either increase the number of polluting product variants or rise the consumer's utility value of constant-pollution goods (Palokangas 2012; Smulders et al. 2011). Furthermore, since new ideas and $R\&D$ are positively related to the population size (Kremer 1993), the economies should be better equipped to tackle the environmental problems in the future. Public policies to support technical progress should be modeled as well. On the consumer's side, the option to choose between dirty and clean goods and between lower and higher birth rates should enrich the model. All these extensions should be made in the future. Nevertheless, it seems that they will not change the basic implication of the model, namely that there is a fundamental trade-off between per capita consumption and population in the long run optimization. This trade-off arises because per capita consumption and population are both valuable to man and because emissions increase the former but decrease the latter. The trade-off implies that sustainable paths may appear, but are not warranted, indicating that optimality and sustainability may conflict for the reasonable parameters and functional specifications of the model.

6 Discussion

Any article on sustainable growth is, more or less, a wake-up call. Broadly speaking, one wants to predict what happens if the currently shown disturbing behavior continues and if environmental concerns are not taken seriously. Currently, some people suffer and die for environmental reasons but the vast majority consumes ever more. If, however, pollution-related mortality remains tolerable, the worrisome conclusion is that, in the real world as well as in the model, the incentives for a change in economic behavior may not be sufficient.

The long run consumer optimization with endogenous pollution and endogenous population implies that utility maximization may take place at the cost of human lives. Solow has suggested that "The theory of optimal growth… is thoroughly utilitarian in conception. It is also utilitarian in the narrow sense that social welfare is (usually) defined as the sum of the utilities of different individuals or generations" (Solow 1974). In the case of endogenous pollution and endogenous population, this utilitarianism may take an extreme expression: a path that ultimately leads to self-imposed extinction may still be optimal. Naturally, a different result would have been derived if positive population were posed as an a priori constraint on optimization. However, an emerging empirical evidence suggests that there already is an increase in mortality because of environmental reasons. Therefore, as a description of the current situation, the utilitarian approach may not be so distorted after all.

Appendix: Local Stability of the Steady States

Lets write $\dot{S} = \varphi(S, E)$ and $\dot{E} = \phi(S, E)$. In a steady state it hods $\dot{S} = \dot{E} = 0$ implying

$$\delta + \delta' S + \rho - \theta n = \frac{n'}{(\rho - n)(\theta - 1)} \delta S. \tag{18}$$

The Jacobian of the model is

$$J = \begin{bmatrix} \varphi_S & \varphi_E \\ \phi_S & \phi_E \end{bmatrix}.$$

As evaluated around a steady state, its elements become

$$\begin{aligned}
\varphi_S &= -\big(\delta + \delta' S\big), \\
\varphi_E &= 1, \\
\phi_S &= \frac{E}{\theta} \left\{ \begin{array}{c} \frac{-n''(\rho-n)-(n')^2}{(\rho-n)^2} [\frac{\theta E}{1-\theta} + \delta S] \\ -\frac{n'}{\rho-n}[\delta + \delta' S] - [2\delta' + \delta'' S - \theta n'] \end{array} \right\}, \\
\phi_E &= \frac{1}{\theta} \left\{ \frac{n'}{\rho - n} \left[\frac{\theta E}{\theta - 1} + \delta S \right] - \big[\delta + \rho + \delta' S - \theta n\big] \right\} + \frac{E}{\theta} \left\{ \frac{n'}{\rho - n} \frac{\theta}{\theta - 1} \right\} \\
&= \frac{n' E}{(\rho - n)(\theta - 1)},
\end{aligned}$$

in which the last row is derived by using (12) and (11b). Because ϕ_E contains the undefined second derivative $n(S)$, we write

$$\begin{aligned}
\text{DET}\, J &= \varphi_S \cdot \phi_E - \phi_S \cdot \varphi_E \\
&= \left[\left(-\frac{\varphi_S}{\varphi_E} \right) - \left(-\frac{\phi_S}{\phi_E} \right) \right] (-\varphi_E) \cdot \phi_E.
\end{aligned}$$

The expression in the square brackets is the difference in the slopes of the phase lines $\dot{S} = 0$ and $\dot{E} = 0$ and $(-\varphi_E) \cdot \phi_E = -\frac{n'E}{(\rho-n)(\theta-1)}$ is positive for all $E > 0$. In steady states 1 and 3 the slope of the $\dot{E} = 0$-line is steeper than that of the $\dot{S} = 0$-line (see Fig. 2) making the square brackets negative. Thus, DET $J < 0$ and these steady states are saddles. In steady state 2 the slope of the $\dot{E} = 0$-line is smaller (possibly negative) than the slope of the $\dot{S} = 0$-line and the value of the square brackets is positive. The trace of the Jacobian is

$$\begin{aligned}
\text{TR}\, J &= \varphi_S + \phi_E \\
&= -\big(\delta + \delta' S\big) + \frac{n' E}{(\rho - n)(\theta - 1)} \\
&= -\big(\delta + \delta' S\big) + \frac{n' \delta S}{(\rho - n)(\theta - 1)}
\end{aligned}$$

$$
\begin{aligned}
&= -\left(\delta + \delta' S\right) + \frac{n'}{(\rho - n)(\theta - 1)} \cdot \frac{(\rho - n)(\theta - 1)}{n'} \cdot \left(\delta + \delta' S + \rho - \theta n\right) \\
&= \rho - \theta n > 0.
\end{aligned}
$$

Therefore, this steady state is an unstable node or focus.

References

Barbier, E. B. (2003). The role of natural resources in economic development. *Australian Economic Papers*, *42*(2), 253–272.

Barro, R. J., & Sala-i-Martin, X. (1995). *Economic growth* (1st edn.). New York: McGraw-Hill.

Benveniste, L., & Scheinkman, J. (1982). Duality theory of dynamic optimization models of economics: the continuous time case. *Journal of Economic Theory*, *27*, 1–19.

Brunekreef, B., & Holgate, S. T. (2002). Air pollution and health. *Lancet*, *360*(9341), 1233–1242.

Chay, K. Y., & Greenstone, M. (2003). The impact of air pollution on infant mortality: evidence from geographic variation in pollution shocks induced by a recession. *Quarterly Journal of Economics*, *118*, 1121–1167.

Currie, J., & Neidell, M. (2004). Air pollution and infant health: what can we learn from California's recent experience? *Quarterly Journal of Economics*, *120*, 1003–1030.

Dasgupta, P. S., & Heal, G. (1974). The optimal depletion of exhaustible resources. *Review of Economic Studies*, *41*, 3–28.

Deschênes, O., & Greenstone, M. (2011). Climate change, mortality, and adaptation: evidence from annual fluctuations in weather in the U.S. *American Economic Journal: Applied Economics*, *3*(4), 152–185.

Evans, M. F., & Smith, K. V. (2005). Do new health conditions support mortality—air pollution effects? *Journal of Environmental Economics and Management*, *50*, 496–518.

Foster, B. A. (1973). Optimal consumption planning in a polluted environment. *Economic Record*, *49*(128), 534–545.

Gosling, S. N., Lowe, J. A., McGregor, G. R., Pelling, M., & Malamud, B. D. (2009). Associations between elevated atmospheric temperature and human mortality: a critical review of the literature. *Climatic Change*, *92*, 299–341.

Hall, R. E. (1988). Intertemporal substitution in consumption. *Journal of Political Economy*, *96*(2), 339–357.

IPPC (2003). *Climate change, impacts, adaption, and vulnerability. Intergovernmental panel on climate change, the third assessment*. Cambridge: Cambridge University Press.

Keeler, E., Spence, M., & Zeckhauser, R. (1971). The optimal control of pollution. *Journal of Economic Theory*, *4*, 19–34.

Krautkraemer, J. A. (1985). Optimal growth, resource amenities and the preservation of natural environments. *Review of Economic Studies*, *52*, 153–170.

Kremer, M. (1993). Population growth and technological change: one million B.C. to 1990. *The Quarterly Journal of Economics*, *108*(3), 681–716.

Krutilla, J. V. (1967). Conservation reconsidered. *American Economic Review*, *57*(4), 777–786.

Logan, W. P. D. (1953). Mortality in London fog incident, 1952. *Lancet*, *261*(6755), 336–338.

Lutz, W., O'Neill, F., & MacKellar, L. (2005). *Population and climate change*. New York: Cambridge University Press.

Malthus, T. R. (1914). *An essay on the principle of population*. London: Dent. 1973 reprint.

Meadows, D., Meadows, D., Zahn, E., & Milling, P. (1972). *The limits to growth*. London: Universe Books.

Mulligan, C. B., & Sala-i-Martin, X. (1991). *A note on the time-elimination method for solving recursive dynamic economic models* (Working Paper 116). National Bureau for Economic Research.

Nemery, B., Hoet, P. H. M., & Nemmar, A. (2001). The Meuse Valley fog 1930: an air pollution disaster. *Lancet*, *357*, 704–708.

Neumayer, E. (1999). *Weak versus strong sustainability*. Cheltenham Glos: Edward Elgar.

Palokangas, T. (2012). *Clean versus dirty economic growth* (Discussion Paper 648:2012). Department of Economics, University of Helsinki.

Pezzey, J. C. (1992). *Sustainable development concepts* (Working Paper 11425). World Bank.

Pezzey, J. C., & Withagen, C. A. (1998). The rise, fall, and sustainability of capital-resource economies. *Scandinavian Journal of Economics*, *100*(2), 513–527.

Pitcher, H., Ebi, K., & Brenkert, A. (2008). Population health model for integrated assessment models. *Climatic Change*, *88*(1), 35–57.

Plourde, C. (1972). A model of waste accumulation disposal. *Canadian Journal of Economics*, *5*(1), 119–125.

Pope, C. A., & Dockery, D. W. (2006). Health effects of fine particulate air pollution: lines that connect. *Journal of Air Waste Management Association*, *56*(6), 709–742.

Pope, C. A., Burnett, R. T., Thun, M. J., Calle, E. E., Krewski, D., Ito, K., & Thurston, G. D. (2002). Lung cancer, cardiopulmonary mortality, and long-term exposure to fine particulate air pollution. *Journal of the American Medical Association*, *287*(9), 1132–1141.

Raaschou-Nielsen, O., Andersen, Z. J., Hvidberg, M., Jensen, S. S., Ketzel, M., Sorensen, M., & Tjonneland, A. (2011). Lung cancer incidence and long-term exposure to air pollution from traffic. *Environmental Health Perspective*, *119*(6), 860–865.

Samet, J. M., Dominici, F., Curriero, F. C., Coursac, I., & Zeger, S. L. (2000). Fine particulate air pollution and mortality in 20 U.S. cities, 1987–1994. *New England Journal of Medicine*, *343*, 1742–1749.

Scheers, H., Mwalili, S. M., Faes, C., Fierens, F., Nemery, B., & Nawrot, T. S. (2011). Does air pollution trigger infant mortality in western Europe? A case-crossover study. *Environmental Health Perspectives*, *119*, 1017–1022.

Smulders, S., & Gradus, R. (1996). Pollution abatement and long-term growth. *European Journal of Political Economy*, *12*, 505–532.

Smulders, S., Bretschger, L., & Egli, H. (2011). Economic growth and the diffusion of clean technologies: explaining environmental Kuznets curves. *Environmental and Resource Economics*, *49*(1), 79–99.

Solow, R. (1974). Intergenerational equity and exhaustive resources. *Review of Economic Studies*, *41*, 29–45.

Stiglitz, J. (1974). Growth with exhaustible natural resources: efficient and optimal growth paths. *Review of Economic Studies*, *41*, 123–137.

Tahvonen, O., & Salo, S. (1996). Nonconvexities in optimal pollution accumulation. *Journal of Environmental Economics and Management*, *31*, 160–177.

Tanser, F. C., Sharp, B., & Le Sueur, D. (2003). Potential effect of climate change on malaria transmission in Africa. *Lancet*, *362*, 1792–1798.

Uzawa, H. (1968). Time preference, the consumption function, and optimum asset holdings. In J. Wolfe (Ed.), *Value, capital, and growth* (Chap. 21). Chicago: Aldine.

WCED (1987). *Our common future*. New York: Oxford University Press.

WHO (2003). *Climate change and human health, risks and responses*, Geneva: World Health Organization.

WHO (2004). *Meta-analysis of time-series studies and panel studies of particulate matter (PM) and ozone (O3)*. Copenhagen: World Health Organization, Regional Office for Europe.

WHO (2011). *Air quality and health* (Fact Sheet Number 313). Updated September 2011. http://www.who.int/mediacentre/factsheets/fs313/en/index.html.

WHO (2012). Health and environment linkages initiative. http://www.who.int/heli/risks/climate/climatechange/en/.

http://www.springer.com/978-3-642-34353-7

Green Growth and Sustainable Development
Crespo Cuaresma, J.; Palokangas, T.; Tarasyev, A. (Eds.)
2013, XII, 228 p., Hardcover
ISBN: 978-3-642-34353-7