Chapter 2
Frequency Estimation on Power System Using Recursive-Least-Squares Approach

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Abstract An approach based on recursive-least-squares (RLS) algorithm is applied to the frequency estimation of the instantaneous power system. The three-phase voltage signal is transformed to a complex form which is easy to be handled by the proposed approach. When compared with other algorithms, the RLS algorithm is more suitable for online frequency estimation due to its rapid convergence rate. An arccosine function-free technique is applied to the frequency estimation approach to reduce computational complexity. The effect of noise, convergence rate, harmonics and dynamic frequency variation on the performance of the approach is discussed.

Keywords Frequency estimation · Power system · Recursive-least-squares

2.1 Introduction

In a power system, frequency is a quite important parameter that its fast and precise estimation is vitally necessary. The rapid development of signal processing technology makes modern frequency measurement flourishing, and many approaches have been applied to it [1–3]. The Least Mean Square (LMS) algorithm is a typical representative of an adaptive algorithm. Its inherent disadvantage is slow convergence rate. The LS algorithm is also widely used for frequency

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estimation while it must re-compute every time. To solve these problems, the Recursive-Least-Squares (RLS) algorithm is applied to frequency estimation.

The RLS algorithm is used extensively in signal processing area. It is an algorithm which recursively finds the coefficients that minimize a weighted linear least squares cost function related to the input signals [4]. When compared with other algorithms, the RLS algorithm exhibits the feature of rapid convergence rate. However, this benefit comes at the cost of high computational complexity. In this paper, an arccosine function-free technique is applied to the frequency estimation to reduce the burden of computation with little decline in frequency estimation accuracy. Moreover, a complex signal model derived from the three-phase voltage signal with the transform [5] is also suitable for RLS algorithm. With its fast convergence and good robustness, the algorithm is fit for online frequency estimation.

This paper is organized as follows. In Sect. 2.2, we present the RLS algorithm (one-phase case). In Sect. 2.3, inspired by the algorithm of one-phase case, we extend it to three-phase case. In Sect. 2.4, the performances of these approaches in different situations are discussed. Finally, in Sect. 2.5, we conclude this paper.

2.2 Proposed Algorithm

In this section, we present the RLS algorithm for online frequency estimation. A power system signal is sampled by an Analog to Digital Converter (ADC). Hence, the voltage signal can be described in discrete form as follows:

$$v(n) = A\cos(\omega t_n + \varphi_0) = A\cos(\omega n T_s + \varphi_0) \tag{2.1}$$

where A is the amplitude of fundamental component, ω is the actual angular frequency, φ_0 is the initial phase, t_n is the time, and T_s is the sampling clock period.

Similarly, $v(n-1)$ and $v(n+1)$ can be described as follows:

$$\begin{aligned} v(n-1) &= A\cos(\omega(n-1)T_s + \varphi_0) \\ v(n+1) &= A\cos(\omega(n+1)T_s + \varphi_0) \end{aligned} \tag{2.2}$$

Then

$$v(n-1) + v(n+1) = 2A\cos(\omega n T_s + \varphi_0)\cos(\omega T_s) = 2v(n)\cos(\omega T_s) = \eta v(n) \tag{2.3}$$

where $\eta = 2\cos(\omega T_s)$. The actual frequency f can be directly estimated with the relationship below.

$$\hat{f} = \frac{\arccos\left(\frac{\eta}{2}\right)}{2\pi T_s} \tag{2.4}$$

Although this frequency estimation approach frequently presents high precision and speed, it is still unreliable because of two deficiencies. First, derivation of equation of this algorithm does not take noise effect into account. Second, when $v(n)$ is around zero, the precision of the approach would become poor. Given $v(n) = 0$, the frequency f cannot be obtained from Eq. (2.3) [6]. Therefore, the algorithm must be modified to improve the performance. The recursive-least-squares (RLS) algorithm can be applied to Eq. (2.3) to estimate η and calculate the frequency f by Eq. (2.4). In Eq. (2.3), $v(n)$ is regarded as the input vector, and $v(n-1) + v(n+1)$ is the desired signal, and $\eta(n)$ is the weight vector.

Based on above discussion, the RLS algorithm for frequency estimation can be described as
Initialization:

$$\eta(1) = 0 \qquad P(1) = \delta^{-1} I = \delta^{-1} \tag{2.5}$$

Computation: for $n = 2, 3, \ldots$

$$\begin{aligned} k(n) &= \frac{P(n-1)v(n)}{\lambda + v^2(n)P(n-1)} \\ \xi(n) &= v(n) + v(n-2) - \eta(n-1)v(n) \\ \eta(n) &= \eta(n-1) + k(n)\xi(n) \\ P(n) &= \lambda^{-1}P(n-1) - \lambda^{-1}k(n)v(n)P(n-1) \end{aligned} \tag{2.6}$$

where λ is forgetting factor, δ is the value to initialize $P(1)$. The frequency is obtained by the Eq. (2.4).

In this algorithm above, there is a high computational complexity so that it will have a negative effect on the speed of online frequency estimation. Especially, the arccosine is a transcendental function which needs a time-consuming computation. Therefore, the arccosine function should be modified for speed as

$$\begin{aligned} v(n-1) + v(n+1) &= 2v(n)\cos(\omega T_s) = 2v(n)\cos((\omega_0 + \Delta\omega)T_s) \\ &= 2v(n)\cos(\omega_0 T_s)\cos(\Delta\omega T_s) - 2v(n)\sin(\omega_0 T_s)\sin(\Delta\omega T_s) \end{aligned} \tag{2.7}$$

where ω_0 is the normal frequency, $\Delta\omega$ is the frequency drift. As we know, the frequency drift is small in a real power system. And the sampling time interval T_s is also small. Consequently, $\Delta\omega T_s$ can be regarded as a number that approaches to zero.

Then

$$v(n-1) + v(n+1) \approx 2v(n)\cos(\omega_0 T_s) - 2v(n)\sin(\omega_0 T_s)\Delta\omega T_s \tag{2.8}$$

Using the relationship of $\omega_0 = 2\pi f_0$, $T_s = 1/f_s$, $\Delta\omega = 2\pi\Delta f$, $f_s = Nf_0$, we can obtain

$$v(n)\Delta f = \frac{2\cos\left(\frac{2\pi}{N}\right)v(n) - v(n+1) - v(n-1)}{4\pi T_s \sin\left(\frac{2\pi}{N}\right)} \tag{2.9}$$

Similarly, the RLS algorithm can be applied to Eq. (2.9), just like Eq. (2.3). We recursively find the frequency drift. Finally, the estimated frequency is equal to

$$f(n) = f_0 + \Delta f(n) \tag{2.10}$$

So by this method the arccosine function is removed from the algorithm. Moreover, in Eq. (2.9) $4\pi T_s \sin(2\pi/N)$ and $2\cos(2\pi/N)$ can be calculated offline. It effectively reduces the computational complexity and improves the speed of the algorithm.

2.3 Three-Phase Case

In Sect. 2.2, we have discussed the proposed algorithm in one-phase case of a power system. Now, inspired by [7], we extend the algorithm to three-phase case. Similarly, the three-phase signal of a power system can be described as follows:

$$\begin{aligned} v_a(n) &= A\cos(\omega t_n + \varphi_0) \\ v_b(n) &= A\cos(\omega t_n - \frac{2\pi}{3} + \varphi_0) \\ v_c(n) &= A\cos(\omega t_n + \frac{2\pi}{3} + \varphi_0) \end{aligned} \tag{2.11}$$

Then we apply the $\alpha\beta$ transform to Eq. (2.11).

$$\begin{bmatrix} v_\alpha(n) \\ v_\beta(n) \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_a(n) & v_b(n) & v_c(n) \end{bmatrix}^T \tag{2.12}$$

Finally, a complex voltage $V(n)$ can be obtained

$$V(n) = v_\alpha(n) + \mathrm{i}v_\beta(n) \tag{2.13}$$

Equation (2.3) is a classical relationship that is suitable for sinusoidal signal. Therefore, it can be proved that Eq. (2.3) is also suitable for the complex signal $V(n)$.

$$V(n-1) + V(n+1) \; = \; 2V(n)\cos(\omega T_s) \tag{2.14}$$

With the same principle as one-phase signal, the RLS algorithm can be applied to a complex form based on Eq. (2.14). The complex form of RLS algorithm for frequency estimation is described as

Initialization:

$$\eta(1) = 0 \qquad P(1) = \delta^{-1} I = \delta^{-1} \tag{2.15}$$

Computation: for $n = 2, 3, \ldots$

$$\begin{aligned} k(n) &= \frac{P(n-1)V(n)}{\lambda + V^*(n)P(n-1)V(n)} \\ \xi(n) &= V(n) + V(n-2) - \eta^*(n-1)V(n) \\ \eta(n) &= \eta(n-1) + k(n)\xi^*(n) \\ P(n) &= \lambda^{-1}P(n-1) - \lambda^{-1}k(n)V^*(n)P(n-1) \end{aligned} \tag{2.16}$$

Moreover, due to Eq. (2.14), we can also obtain the complex form of frequency drift as follows:

$$V(n)\Delta f = \frac{2\cos\left(\frac{2\pi}{N}\right)V(n) - V(n+1) - V(n-1)}{4\pi T_s \sin\left(\frac{2\pi}{N}\right)} \tag{2.17}$$

The RLS algorithm similarly can be applied to this equation to estimate the frequency drift Δf, and the unknown frequency can be obtained in Eq. (2.10).

2.4 Simulation Results

In this paper, four approaches for frequency estimation are proposed, i.e., one-phase arccosine, one-phase arccosine free, three-phase arccosine and three-phase arccosine free. In this section, we discuss the effect of noise, convergence rate, harmonics and dynamic frequency variation on the performance of the proposed approaches. In the simulation, the sampling rate is 1 KHz and the normal frequency is 50 Hz. By default, the parameters of the RLS algorithm in this paper are $\delta = 0.01$ and $\lambda = 0.98$. Zero-mean Gaussian noise is added to the normal frequency signal to simulate a power system.

Case 1 Noise effect: We study the performance of these approaches by adding zero-mean Gaussian noise to the fundamental signal. As an index of accuracy, we use normalized mean square error (MSE), which is define as $\sum_n (f - \hat{f}_n)^2 / \left(f^2 \sum_n 1\right)$, where f is the exact and $\hat{f}_n$ the estimated frequency at time t_n [2]. In this simulation, the real frequency is 49.5 Hz. The initial weight vector of the arccosine approach is $\eta(1) = 2\cos(2\pi \times 50 \times T_s)$, while that of the arccosine free approach is $\eta(1) = 0$. Figure 2.1 shows the relationship between SNRs and the MSE. It shows that the performance of these approaches badly declines in highly noisy environment. This is an inherent defect of many algorithms, which are not inherently filter based [2]. Moreover, the arccosine-free approach can decrease the computation complexity at the cost of little increasing in the mean of square error of frequency estimation.

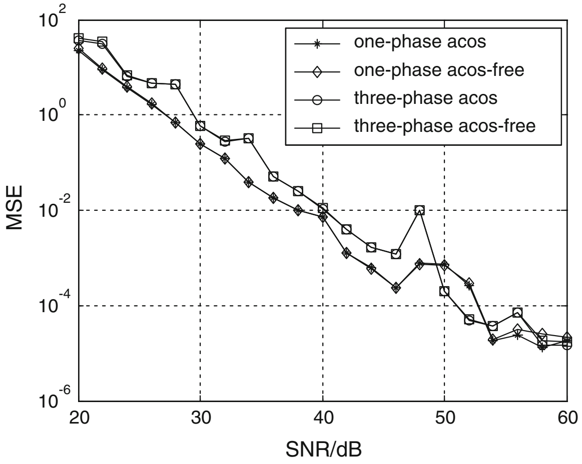

Fig. 2.1 MSE of different SNRs

Case 2 Convergence rate: When compared with the LMS algorithm, the RLS algorithm shows its good convergence rate. Figure 2.2 describes the comparison. In this simulation, the SNR of the input signal is 40 dB and the initial weight vectors of both algorithms are zeros. The step size of the LMS algorithm is 0.1.

Case 3 Harmonics effect: In our simulation, the fundamental signal, the SNR and the initial weight vectors are the same as Case 2. We compare the performance of the harmonics signal and the filtered signal with the one-phase arccosine-use approach. The harmonics signal contains 10 % third harmonics, 5 % fifth harmonic, and 3 % seventh harmonic besides the fundamental signal. A three-order low-pass Butterworth filter with a cut-off frequency of 100 Hz is performed. It is observed from Fig. 2.3 that the estimated frequency is quite close to the fundamental frequency after pre-filtering.

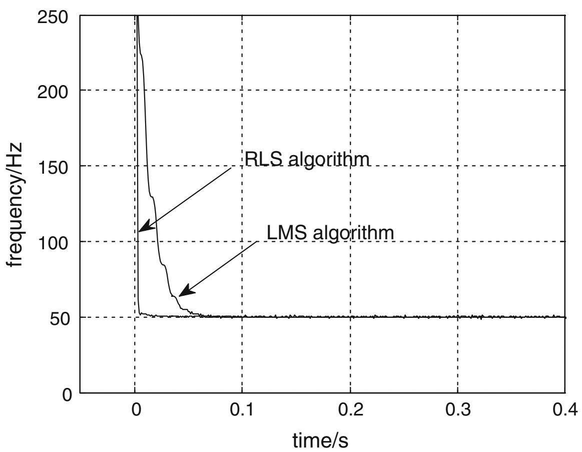

Fig. 2.2 Convergence rate of LMS and RLS

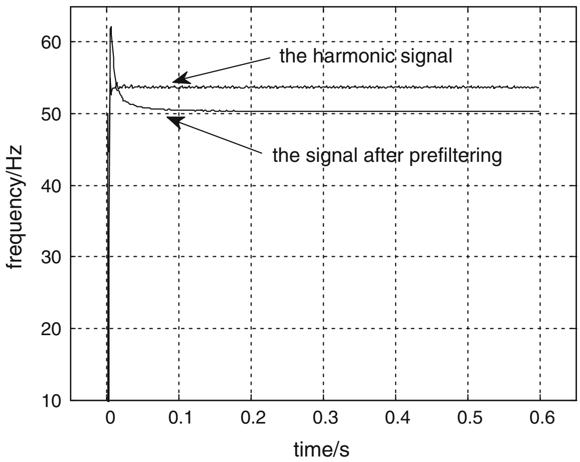

Fig. 2.3 Harmonics effect

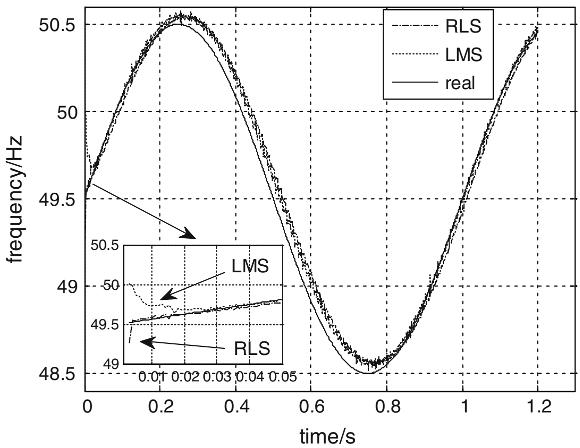

Fig. 2.4 Estimation during dynamic variation

Case 4 Dynamic frequency estimation: Voltage signal with ± 1 Hz frequency oscillation starting from 49.5 Hz are exposed to the first proposed approach. In this simulation, the SNR is 40 dB, the forgetting factor is $\lambda = 0.95$, The step size of the LMS algorithm is 0.1 and the initial weight vector of both algorithms is $2\cos(2\pi \times 50 \times T_s)$. We learn from Fig. 2.4 that the estimated frequency which is calculated in real time is the same as real frequency. Moreover, the performance of the RLS algorithm is better than that of the LMS algorithm in [6] because of its faster convergence rate.

2.5 Conclusion

A new frequency estimating approach based on the RLS algorithm is proposed in this paper. The approach is derived from a classical formula which holds for every three consecutive samples. An arccosine-free version of the approach is proposed for reducing the computational complexity. Moreover, we extend the three-phase signal to a complex form, and apply the RLS algorithm to it. Simulation results show that the accuracy and speed of estimation is satisfactory even in the presence of noise, harmonics, and frequency variation. When compared with other approaches, this approach shows rapid convergence rate. Therefore, the proposed approach is suitable for application in online frequency estimation.

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