

# Chapter 2

## Research on Conformal Phased Array Antenna Pattern Synthesis

Guoqi Zeng, Siyin Li and Zhimian Wei

**Abstract** Phased array antenna has many technical advantages: high power efficiency, shaped beam, fast tracking by electric scanning, high stealth performance. If the array elements distribute on the airframe surface, the shape of the array is the same as the airframe contour, then the array is a conformal phased array. Conformal phased array has many advantages compared to planar phased array: smaller volume, no effect to aerodynamic performance of aircrafts, wider scanning range. In a conformal array, antenna elements are not placed in one plane because of its conformal structure. So the array elements and array factors cannot be separated, and the polarization direction of each element is different from each other, which will cause severe cross-polarization component. A field vector synthesis method is used in this paper to analyze the pattern of conformal phased array antenna, which can avoid dealing with the array element and array factor. This method is suitable for conformal array of any shape. This method is verified by calculating the pattern of a conformal array of truncated cone shape.

**Keywords** Conformal phased array · Pattern synthesis · Coordinate transformation

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## 2.1 Introduction

Due to the importance of conformal phased array, USA, Japan and European Countries attach great importance to the development in this area. In recent years, conformal phased array antenna has been more and more widely applied due to the requirement of kinds of advanced aircrafts. The American “Advanced Anti-Radiation Guided Missile (AARGM)” Project focuses on the research on the technique of wide band passive receiving conformal phased array [1]. There is a special academic annual conference in Europe every other year since 1999 [2]. The Raytheon company promote a conformal phased array scheme used in a dual mode seeker, which contains a dielectric frustum radome structure (the open slot on it provides the space to place the infrared sensor), and an electronically steerable slot array in the radio-frequency range [3].

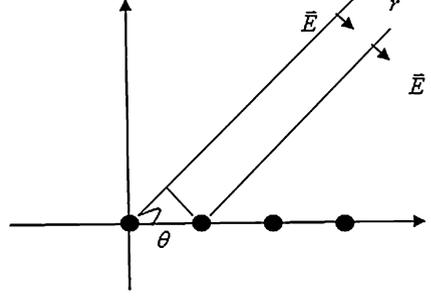
As for conformal phased array antenna, antenna elements are not placed in one plane because of its conformal structure. So the array elements and array factors cannot be separated, and the polarization direction of each element is different from each other, which will cause severe cross-polarization component. Thus, we have to find a new method to solve the pattern synthesis problem of a conformal structure.

Pattern synthesis of conformal phased array has been widely researched around the world. Reference [4] uses moment method, Ref. [5] uses numerical methods such as finite element method to analyze pattern of conformal phased array. Reference [6] uses the least squares method to optimize the in-phase polarization component and cross-polarization component of the spherical conformal array pattern. Reference [7] uses the iterative least square method to analyze pattern of spherical conformal array by change the incentive phase only. Reference [8] uses the least squares method and changes the phase only to analyze pattern of parabolic conformal structure. The numerical calculating method can only calculate a certain size conformal array, and computation speed is very slow. The methods used in Ref. [6–8] are only suitable for specific conformal structure, don’t have versatility. In this paper, the field vector synthesis method is used to analyze pattern of conformal array. This method can be used in any shape of conformal structure and time cost is acceptable.

## 2.2 Planar Array Pattern Synthesis

$$\overrightarrow{E}(\theta, \varphi) = \sum_{n=0}^{N-1} A_n e^{j\varphi_n + jkl \cos \theta} F(\theta, \varphi) = F(\theta, \varphi) \sum_{n=0}^{N-1} A_n e^{j\varphi_n + jkl \cos \theta} = F(\theta, \varphi) f(\theta, \varphi) \quad (2.1)$$

**Fig. 2.1** One-dimensional antenna array



$$\begin{aligned}
 \vec{E}(\theta, \phi) &= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{m,n} e^{j\varphi_{m,n} + jk(ml \sin \theta \cos \phi + nl \sin \theta \sin \phi)} F(\theta, \phi) \\
 &= F(\theta, \phi) \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{m,n} e^{j\varphi_{m,n} + jk(ml \sin \theta \cos \phi + nl \sin \theta \sin \phi)} = F(\theta, \phi) * f(\theta, \phi)
 \end{aligned} \tag{2.2}$$

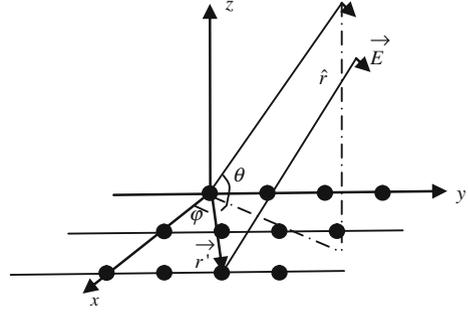
From Eqs. (2.1) and (2.2), it can be found that all the elements are in one plane. So the electromagnetic field in the far field region in direction  $(\theta, \phi)$  is the superposition of field of each element in direction  $(\theta, \phi)$ . The right side of the equations above can be divided into two parts, the field of each element in direction  $(\theta, \phi)$  is called the element component and the rest is called array factor (Figs 2.1, 2.2).

### 2.3 Spatial Field Calculation of Conformal Structure

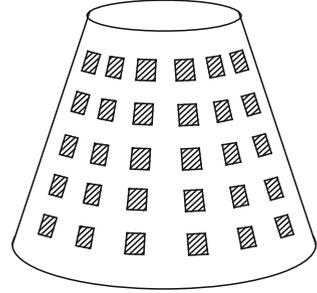
According to the pattern synthesis method of a planar array, total field can be calculated by superimposing the contribution field of each element. We have to transform the pattern in local coordinate system of each element into global coordinate system by coordinate transformation. The field in direction  $(\theta, \phi)$  in global coordinate system is the superposition of the field  $\vec{E}(\theta_{m,n}, \varphi_{m,n})$  of element  $(m, n)$  at the coordinate  $(\theta_{m,n}, \varphi_{m,n})$  in local coordinate system. (Fig. 2.3)

$$\begin{aligned}
 \vec{F}_W(\theta, \varphi) &= Z_L^W \vec{F}_L(\theta', \varphi') \\
 \vec{E}(\theta, \varphi) &= \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} A_{m,n} e^{j\varphi_{m,n} + jk(\vec{r}' \cdot \hat{r})} [\vec{F}_W(\theta, \varphi)] \\
 &= \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} A_{m,n} e^{j\varphi_{m,n} + jk(\vec{r}' \cdot \hat{r})} [Z_L^W \vec{F}_L(\theta'_{mn}, \varphi'_{mn})]
 \end{aligned} \tag{2.3}$$

**Fig. 2.2** Two-dimensional antenna array



**Fig. 2.3** Element distribution of conformal array antenna



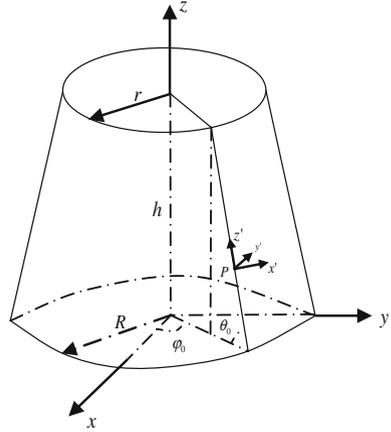
In which,  $Z_L^W$  is the transform matrix from local coordinate system to global coordinate system,  $\vec{F}_L(\theta', \varphi')$  is the pattern of each element in local coordinate system,  $\vec{F}_W(\theta, \varphi)$  is the pattern of each element in global coordinate system,  $\vec{E}(\theta, \varphi)$  is the field in global coordinate system,  $A_{m,n}$  is the amplitude of element  $(m, n)$ ,  $\varphi_{m,n}$  is the phase of element  $(m, n)$ .

Different from the planar array,  $\vec{F}_W(\theta, \varphi)$  in Eq. (2.3) is various. Thus, the pattern cannot be divided into two parts just like Eqs. (2.1) and (2.2). In Eq. (2.3),  $\vec{F}_L(\theta'_{mn}, \varphi'_{mn})$  is the known component,  $Z_L^W$  and  $(\theta'_{mn}, \varphi'_{mn})$  are unknown. And  $(\theta'_{mn}, \varphi'_{mn})$  in local coordinate system is corresponding to  $(\theta, \varphi)$  in global coordinate system.

### 2.3.1 Spatial Field Calculation of Truncated Cone Shape Conformal Array

As showed in Fig. 2.4,  $r$  is the radius of top surface of truncated cone and  $R$  is the radius of the bottom surface,  $h$  is height. So the dip angle is  $\theta_0 = a \tan\left(\frac{h}{R-r}\right)$ . In Fig. 2.4,  $xyz$  is the global coordinate system and  $x''y''z''$  is the local coordinate system.

**Fig. 2.4** Coordinate system of truncated cone shape conformal structure



The transform process from  $xyz$  to  $x'y'z'$  is: firstly, rotate  $xyz$  around  $z$  axis by  $\varphi_0$  to get  $x'y'z'$ , then rotate  $x'y'z'$  around  $y'$  axis by  $\theta_0$  to get the  $x''y''z''$  coordinate system. Oppositely, rotate  $x''y''z''$  around  $y''$  axis by  $-\theta_0$  to get  $x'y'z'$ , and then rotate around  $z'$  axis by  $-\varphi_0$  to get the  $xyz$  coordinate system.

The transformation matrix from global coordinate system to local coordinate system is:

$$\begin{aligned}
 Z_L^{W'} &= Z_W^L = Z_Y Z_Z \\
 &= \begin{bmatrix} \cos \theta_0 & 0 & -\sin \theta_0 \\ 0 & 1 & 0 \\ \sin \theta_0 & 0 & \cos \theta_0 \end{bmatrix} \begin{bmatrix} \cos \varphi_0 & \sin \varphi_0 & 0 \\ -\sin \varphi_0 & \cos \varphi_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos \theta_0 \cos \varphi_0 & \cos \theta_0 \sin \varphi_0 & -\sin \theta_0 \\ -\sin \varphi_0 & \cos \varphi_0 & 0 \\ \sin \theta_0 \cos \varphi_0 & \sin \theta_0 \sin \varphi_0 & \cos \theta_0 \end{bmatrix} \quad (2.4)
 \end{aligned}$$

Assume that  $(\theta, \varphi)$  is the angle in global coordinate system, and  $(\theta', \varphi')$  is the corresponding angle in local coordinate system of a certain array element.

$$\begin{bmatrix} x_L \\ y_L \\ z_L \end{bmatrix} = Z_W^L \begin{bmatrix} x_W \\ y_W \\ z_W \end{bmatrix} = \begin{bmatrix} \cos \theta_0 \cos \varphi_0 & \cos \theta_0 \sin \varphi_0 & -\sin \theta_0 \\ -\sin \varphi_0 & \cos \varphi_0 & 0 \\ \sin \theta_0 \cos \varphi_0 & \sin \theta_0 \sin \varphi_0 & \cos \theta_0 \end{bmatrix} \begin{bmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{bmatrix}$$

So,

$$\varphi' = a \tan \left( \frac{\sin \theta \sin(\varphi - \varphi_0)}{\cos \theta_0 \sin \theta \cos(\varphi - \varphi_0) - \sin \theta_0 \cos \theta} \right) \quad (2.5)$$

$$\theta' = a \cos(\sin \theta_0 \sin \theta \cos(\varphi_0 - \varphi) + \cos \theta_0 \cos \theta)$$

For the circular column structure,  $\theta_0 = 90^\circ$ . Then,

$$\begin{aligned}\varphi' &= a \tan(\tan \theta \sin(\varphi_0 - \varphi)) \\ \theta' &= a \cos(\sin \theta \cos(\varphi_0 - \varphi))\end{aligned}\quad (2.6)$$

### 2.3.2 Direction Determination of Spatial Field

In a planar array, in far field region in direction  $(\theta, \varphi)$ , the direction of pattern of each element is also  $(\theta, \varphi)$ . And the field direction of each element is the same. So the amplitude of each element can be added together directly. But in a conformal phased array, elements are not in one plane, thus the field direction of each element is different from each other in far field region in direction  $(\theta, \varphi)$ . So the vector superposition method has to be taken.

Each element radiates and produces spatial field. The direction of spatial field is  $\hat{\theta}$  when field is vertical polarization. And its direction is  $\hat{\varphi}$  for horizontal polarization. For circular polarization field, the amplitude of each field can be added together directly.

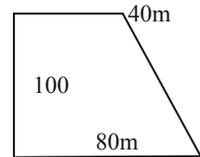
The transform formula from rectangular coordinate to spherical coordinate is as follows:

$$\begin{cases} \vec{i}_{rs} = \vec{i}_x \sin \theta \cos \varphi + \vec{i}_y \sin \theta \sin \varphi + \vec{i}_z \cos \theta \\ \vec{i}_\theta = \vec{i}_x \cos \theta \cos \varphi + \vec{i}_y \cos \theta \sin \varphi - \vec{i}_z \sin \theta \\ \vec{i}_\varphi = -\vec{i}_x \sin \varphi + \vec{i}_y \cos \varphi \end{cases} \begin{cases} \vec{i}_x = \vec{i}_{rs} \sin \theta \cos \varphi + \vec{i}_\theta \cos \theta \cos \varphi - \vec{i}_\varphi \sin \varphi \\ \vec{i}_y = \vec{i}_{rs} \sin \theta \sin \varphi + \vec{i}_\theta \cos \theta \sin \varphi + \vec{i}_\varphi \cos \varphi \\ \vec{i}_z = \vec{i}_{rs} \cos \theta - \vec{i}_\theta \sin \theta \end{cases}$$

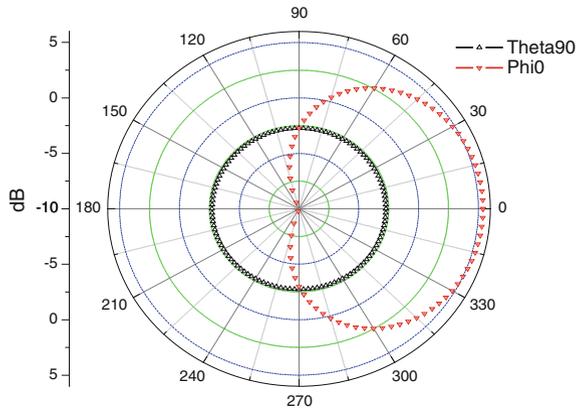
## 2.4 Example of Field Calculating

The longitudinal section of a truncated cone shape conformal structure is as shown in Fig. 2.5. The radius of top line is 40 mm, bottom line is 80 mm and the height is 100 mm. Set frequency as 35 GHz, thus the wave length is 8 mm. Assume that the pattern of array element  $\vec{F}_L(\theta'_{mn}, \varphi'_{mn})$  in Eq. (2.3) is as shown in Fig. 2.6. So just the scalar sum of fields should be considered in calculation, and the summation is the pattern of the conformal array. Consider two kinds of uniform distribution of

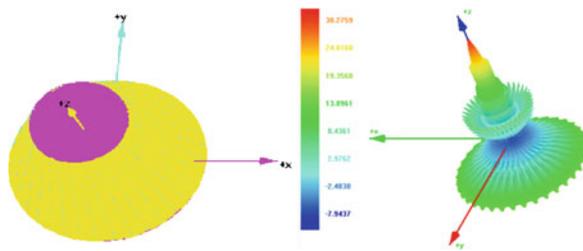
**Fig. 2.5** Longitudinal section of truncated cone shape conformal structure



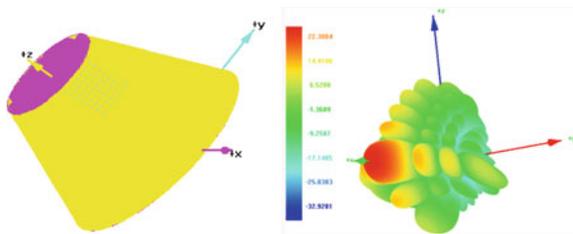
**Fig. 2.6** Pattern of array element



**Fig. 2.7** Elements distribution of truncated cone conformal array and its 3-D pattern (36 × 20 elements)

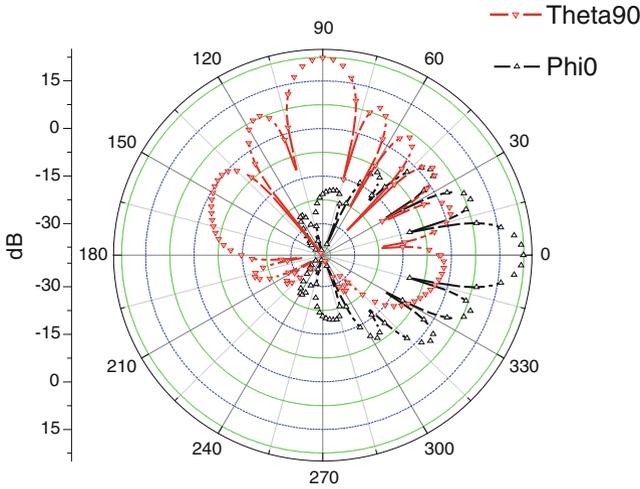


**Fig. 2.8** Elements distribution of truncated cone conformal array and its 3-D pattern (8 × 8 elements)



array elements with equal angle difference. The first one, elements cover the whole inclined plane of the truncated cone. And the next one, elements cover part of the inclined plane. Assume that all elements have the same amplitude in calculation. The synthesized pattern are as shown in Figs. 2.7, 2.8 and 2.9 (Table 2.1).

As shown in Fig. 2.7, the number of element is  $36 \times 20 = 720$ , and the output calculation result has 361 points in azimuth and 361 points in elevation; it takes 240 s to calculate, totally. As for Fig. 2.8, with the number of element  $8 \times 8 = 64$ , it takes 22 s to calculate. And the computer configuration is: IBM T61: CPU Intel Core2 Duo 2.2 GHz, RAM 1.96 GHz.



**Fig. 2.9** Pattern section in main lobe direction  $\theta = 90^\circ$  and  $\varphi = 0^\circ$

**Table 2.1** Pattern parameters of truncated cone conformal structure

Section	Main lobe (dB)	3 dB band-width ( $^\circ$ )	First side lobe (dB)	Maximum side lobe (dB)
$\varphi = 0^\circ$	22.30	13.72	9.79	9.79
$\theta = 90^\circ$	22.30	13.10	9.16	9.16

## 2.5 Conclusions

In a conformal array, all the elements are not in one plane. When synthesizing the far field pattern, the field amplitude and phase of each element are different from each other. So the elements and array factors cannot be separated. At the same time the polarization direction of each element is different from each other, which will cause severe cross-polarization component. In this paper, the field vector synthesis method is introduced; establish global coordinate system based on pattern of the conformal array and establish local coordinate system based on the location of each element. According to the relationship between global and local coordinate system, there is the field in direction  $(\theta_{mn}, \varphi_{mn})$  in local coordinate system of each element responding to the field in direction  $(\theta, \varphi)$  in global coordinate system, from which the amplitude and phase contribution to the total field by each element can be calculated. And then the pattern of the array can be synthesized. In this paper the pattern of a truncated cone conformal array with different element distribution situation is analyzed: all covered by array element and partly covered by array element.

However, when synthesizing the pattern of a real conformal array antenna, the mutual coupling among elements should be taken into consideration. Also the amplitude or phase of each element should be optimized to synthesis the pattern, which needs further studying.

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