Chapter 2
Governing Equations for Wave Propagation in a Fluid-Saturated Porous Medium

Abstract In this chapter the governing equations for wave propagation in a fluid-saturated porous medium are derived and the involved physical mechanisms and acoustic parameters are discussed. It is shown that the stress-strain relations associated with Biot's theory can be straightforward obtained from constitutive and continuity equations. Equations of motion are derived by combining these stress-strain relations with momentum equations. We present the equations of motion in the two different formulations that are known in the literature.

2.1 Introduction

In this thesis we study the wave propagation in fluid-saturated porous media. We investigate how various wavemodes can be described mathematically and detected experimentally (especially the interface wavemodes), and how the various waves can be used to characterize acoustic parameters of a porous medium. Therefore, in this chapter we give the theoretical framework for the description of the wave propagation in a fluid-saturated porous medium. Originally, this theory was developed by Biot (1956a). Here, we show the derivation of the governing equations and discuss the physical mechanisms and involved acoustic parameters.

First, we give the definitions of integral transforms and some notation conventions that we use in this chapter and throughout the thesis (Sect. 2.2). Then, we show that a fluid-saturated porous medium can be considered as a continuum and discuss the underlying assumptions (Sect. 2.3). In Sect. 2.4 we derive stress-strain relations associated with Biot's theory from straightforward constitutive and continuity equations following Kelder (1998) and Wisse (1999). This shows that the involved elastic constants are clearly related to physical quantities. Subsequently, we combine the stress-strain relations with the momentum equations to finally obtain the equations of motion (Sect. 2.5). We present the equations of motion in two different formulations that are known in the literature.

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2.2 Definitions

First, we define the integral transforms that are used in this thesis. For frequency-domain analysis we use the Fourier transform over time t defined as

$$\hat{\mathbf{u}}(\mathbf{x}, \omega) = \int_{-\infty}^{\infty} \mathbf{u}(\mathbf{x}, t) \exp(-\mathrm{i}\omega t)\, dt, \tag{2.1}$$

where ω denotes angular frequency, i is the imaginary unit and $\mathbf{u}$ is a displacement vector, but the Fourier transform can be applied to any other relevant field quantity. The vector $\mathbf{x} = (x_1, x_2, x_3)^T$ contains the spatial coordinates, where x_1 and x_2 are horizontal coordinates and x_3 is the vertical coordinate being positive in downward direction; the superscript T denotes the transpose. Because the time-domain signal $\mathbf{u}$ is real-valued it holds that $\hat{\mathbf{u}}(-\omega) = \hat{\mathbf{u}}^*(\omega)$, where the asterisk denotes complex conjugation. Hence it is sufficient to consider $\omega \geq 0$ only.

The Fourier transform over all spatial coordinates is defined as

$$\bar{\breve{\mathbf{u}}}(\mathbf{k}, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{\mathbf{u}}(\mathbf{x}, \omega) \exp(\mathrm{i}\mathbf{k} \cdot \mathbf{x})\, dx_1 dx_2 dx_3, \tag{2.2}$$

where $\mathbf{k} = (k_1, k_2, k_3)^T$ is the wavenumber vector. Throughout the thesis we often use slowness $\mathbf{p}$ which is related to the wavenumber according to $\mathbf{k} = \omega \mathbf{p}$ (Aki and Richards 1980). The hat ($\hat{\mathbf{u}}$) refers to the $(\mathbf{x}, \omega)$-domain and the combined bar/breve ($\bar{\breve{\mathbf{u}}}$) to the $(\mathbf{k}, \omega)$-domain. We use a single breve ($\breve{\mathbf{u}}$) to indicate the $(\mathbf{p}, \omega)$-domain.

Alternatively, when dealing with media that have discontinuities in x_3-direction (interfaces between layers), we apply the Fourier transform over horizontal coordinates only according to

$$\bar{\tilde{\mathbf{u}}}(\mathbf{k}_r, x_3, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{\mathbf{u}}(\mathbf{x}, \omega) \exp\left(\mathrm{i}\mathbf{k}_r \cdot \mathbf{r}\right)\, dx_1 dx_2, \tag{2.3}$$

where $\mathbf{k}_r = (k_1, k_2)^T$ is the horizontal wavenumber vector and $\mathbf{r} = (x_1, x_2)^T$ is the horizontal space vector. In the case we work with the slowness rather than the wavenumber we apply $\mathbf{k}_r = \omega \mathbf{p}_r$ (see above). The combined bar/tilde ($\bar{\tilde{\mathbf{u}}}$) refers to the $(\mathbf{k}_r, x_3, \omega)$-domain, and a single tilde ($\tilde{\mathbf{u}}$) refers to the $(\mathbf{p}_r, x_3, \omega)$-domain.

When using index notation we invoke the Einstein's summation convention for repeated indices. However, the summation convention does not apply to Greek symbols (e.g., α, β) because we use these to indicate different wavemodes. Further, the Kronecker delta is denoted δ_{ij} and is defined as

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases} \tag{2.4}$$

2.3 Continuum Description of a Porous Medium

As a basis for the derivation of the stress-strain relations associated with the theory for wave propagation in a fluid-saturated porous medium, in this section we give the underlying assumptions and we define stresses and strains.

The Biot theory describes porous materials as a medium consisting of two interpenetrating phases: the solid phase (porous frame) and the fluid phase (Biot 1956a). The original theory has been developed using a semi-phenomenological macroscopic approach, based on a set of physically realistic assumptions. This approach means that the microscopic dimensions of the individual constituents of the saturated porous medium are not considered, i.e., the medium is considered as a continuum. The following assumptions were made:

1. The fluid-saturated porous material is constituted in such a way that the fluid phase is fully interconnected. Any sealed void space is considered as a part of the solid.
2. A so-called representative elementary volume element is defined, which is small compared to the relevant wavelength but large compared to the individual grains and pores of the system. Each volume element is described by its averaged displacement of the solid parts $\mathbf{u}(\mathbf{x}, t)$ and of the fluid parts $\mathbf{U}(\mathbf{x}, t)$.
3. The deformation of the elementary volume element is assumed to be linearly elastic and reversible. This implies that displacements for both fluid and solid phases are small. The governing equations can be represented in their linearized form.
4. The solid is considered to have compressibility and shear rigidity, while the fluid only has compressibility as it is assumed to be a Newtonian fluid: the fluid does not sustain any shear force for static displacements.
5. The solid and fluid are assumed homogeneous and isotropic, and all possible dissipation mechanisms related to the solid itself are not taken into account. Only dissipation due to viscous relative fluid-solid motion is incorporated.
6. Thermoelastic and chemical reaction effects are assumed to be absent and the system behaves adiabatically.

Following these assumptions, we can now define porosity, stresses and strains unambiguously. Considering a fluid-filled elastic porous matrix with a statistical distribution of interconnected pores, the porosity is usually defined by

$$\phi = \frac{V_f}{V_b}, \tag{2.5}$$

where V_f is the volume of the pores contained in a sample of bulk volume V_b, and the term "porosity" refers to the effective porosity (see assumption 1 above).

Within the restrictions of the linearized theory the (macroscopic) deformation of solid and fluid are described by the small-strain tensors, e_{ij} and ε_{ij}, respectively, according to

$$e_{ij} = \tfrac{1}{2}(\partial_i u_j + \partial_j u_i), \tag{2.6}$$

$$\varepsilon_{ij} = \tfrac{1}{2}(\partial_i U_j + \partial_j U_i), \tag{2.7}$$

where $\partial_j = \partial/\partial x_j$. It is evident that $e_{ij} = e_{ji}$ and $\varepsilon_{ij} = \varepsilon_{ji}$.

If we consider a cube of unit size of the bulk material (solid and fluid), the total stress tensor can be defined as

$$\tau_{b,ij} = \begin{pmatrix} \tau_{11} + \tau & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} + \tau & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} + \tau \end{pmatrix}, \tag{2.8}$$

where τ represents the total normal tension force per unit **bulk** area A_b applied to the fluid part of the faces of the cube. The total stress tensor is symmetric, i.e., $\tau_{b,ij} = \tau_{b,ji}$, which can be shown using the balance of angular momentum (Achenbach 1973). Denoting the pressure of the fluid in the pores by p_f we can write

$$\tau = -\phi p_f, \tag{2.9}$$

where p_f is defined positive in compression. The remaining components τ_{ij} of the total stress tensor are the forces per unit **bulk** area applied to that portion of the cube faces occupied by the solid. They are a result of both the fluid pressure p_f and the additional intergranular stresses σ_{ij},

$$\tau_{ij} = -\sigma_{ij} - (1 - \phi) p_f \delta_{ij}, \tag{2.10}$$

where the Kronecker delta reflects the assumption that the pore fluid cannot sustain any shear forces. The intergranular stresses are also defined positive in compression, and are called "additional" because they add up to the stresses in the solid induced by the fluid pressure.

For later use, we also define the forces per unit **solid** area A_s applied to that portion of the cube faces occupied by the solid

$$\tau_{ij} A_b / A_s = -\sigma_{ij}/(1 - \phi) - p_f \delta_{ij}. \tag{2.11}$$

Obviously, the total normal tension force per unit **fluid** area A_f applied to the fluid part of the faces of the cube can be written as

$$\tau A_b / A_f = -p_f, \tag{2.12}$$

where $A_f / A_b = V_f / V_b$ and Eq. (2.5) have been used.

Using now Eqs. (2.9) and (2.10) the total stress tensor (Eq. (2.8)) in the bulk material can be written as

$$\tau_{b,ij} = \begin{pmatrix} -\sigma_{11} - p_f & -\sigma_{12} & -\sigma_{13} \\ -\sigma_{21} & -\sigma_{22} - p_f & -\sigma_{23} \\ -\sigma_{31} & -\sigma_{32} & -\sigma_{33} - p_f \end{pmatrix}, \tag{2.13}$$

where $\sigma_{ij} = \sigma_{ji}$ (see Eq. (2.8)). This expression for the total stress tensor is also given by Verruijt (1982), where it must be noted that he has denoted the total stress tensor as σ_{ij} and the intergranular stress as $\bar{\sigma}_{ij}$.

2.4 Stress-Strain Relations

We now derive the stress-strain relations for a fluid-saturated porous medium and relate the elastic coefficients of the model to physical quantities.

Following the assumptions and definitions as mentioned in the previous section, and by a generalization of the procedure followed in the classical theory of elasticity (Love 1944), the elastic potential energy density E_p for a fluid-saturated porous medium can be written as Biot (1955)

$$E_p = \tfrac{1}{2}(\tau_{11}e_{11} + \tau_{22}e_{22} + \tau_{33}e_{33} + 2\tau_{12}e_{12} + 2\tau_{13}e_{13} + 2\tau_{23}e_{23} + \tau\varepsilon), \tag{2.14}$$

where $\varepsilon = \varepsilon_{kk}$. In Eq. (2.14) the symmetry property of the stresses and strains has been used, $\tau_{ij} = \tau_{ji}$ and $e_{ij} = e_{ji}$, respectively. Following the generalized Hooke's law, here the number of independent elastic coefficients is twenty eight, which is known as general anisotropic poroelasticity. When the material is isotropic, i.e., when there are no preferred directions in the material which also means that the principal stress and strain directions coincide, this is reduced to four distinct elastic coefficients. Introducing the elastic constants A, Q, R and G, the stress-strain relations for an isotropic porous medium can be written as Biot (1956a)

$$\tau_{ij} = 2Ge_{ij} + Ae_{kk}\delta_{ij} + Q\varepsilon\delta_{ij}, \tag{2.15}$$

$$\tau = Qe_{kk} + R\varepsilon. \tag{2.16}$$

The elastic constants A, Q, R are generalized elastic coefficients that can be related to physical quantities such as porosity ϕ, the fluid bulk modulus K_f, the bulk modulus of the grains K_s, the bulk of the drained matrix K_b, and the (drained) composite shear modulus G.

The elastic coefficients were related to physical quantities by Gassmann (1951), Biot and Willis (1957), Geertsema and Smit (1961), Stoll (1974), Brown and Korringa (1975) and Berryman (1981), using so-called static "Gedanken" experiments on jacketed and unjacketed porous samples. Here, we discuss these tests following (Kelder 1998), who derived the stress-strain relations from straightforward continuity and constitutive relations. In the gedanken experiments the volume effects caused by the stresses in the porous medium are investigated. As these stresses can be expressed in terms of fluid pressure and intergranular stresses (see Eqs. (2.9) and (2.10)), we discuss two experiments in which the influences of the two stresses are studied separately. By superposition of the results, and in combination with continuity equations, we arrive at stress-strain relations in the form equivalent to Eqs. (2.15) and (2.16).

2.4.1 Effect of Fluid Pressure (Unjacketed Test)

The first experiment is the so-called unjacketed test in which the influence of the fluid pressure is studied. When a porous sample is fully submerged in a watertank (pressure change dp_e) and the sample is assumed to be fully water-saturated, it is immediately clear that the fluid pressure must be continuous over the interface (Deresiewicz and Skalak 1963),

$$dp_f = dp_e. \tag{2.17}$$

For the intergranular stresses at the interface we can write

$$d\sigma_{11} = d\sigma_{22} = d\sigma_{33} = 0. \tag{2.18}$$

As there are no changes in the intergranular stresses, the unjacketed test is used to study the volume effects caused by the pore pressure changes. Defining the bulk modulus K_a, the bulk volume change dV_b is measured in this test,

$$dV_b = -\frac{V_b}{K_a} dp_f. \tag{2.19}$$

In the case of homogeneous media, either isotropic or not, the application of an incremental pressure dp_e means applying this increment both to the outer and inner pore surface, which leads to a linear mapping and does not change the porosity ϕ ($d\phi = 0$). Therefore we may write for the volume change of the matrix grains

$$dV_s = (1 - \phi) dV_b = -\frac{1}{K_a} V_s dp_f. \tag{2.20}$$

This means that for homogeneous media K_a can also be interpreted as the bulk modulus of the individual grains, which we denote by K_s. Hence, in Eqs. (2.19) and (2.20) K_a can be replaced by K_s.

2.4.2 Effect of Intergranular Stresses (Jacketed Test)

The second experiment is the so-called jacketed test in which the influence of intergranular stresses is studied. In this case, a porous sample is jacketed and fully submerged in a watertank (pressure change dp_e) and the inside of the jacket is made to communicate with the atmosphere through a tube to ensure constant internal fluid pressure. Now we can write Deresiewicz and Skalak (1963)

$$dp_e = d\sigma_{11} = d\sigma_{22} = d\sigma_{33}, \tag{2.21}$$

see also Eq. (2.13) and $dp_f = 0$. As there are no pore pressure changes, the jacketed test is used to study the volume effects caused by intergranular stresses. Defining the matrix bulk modulus K_b, the bulk volume change dV_b,

$$dV_b = -\frac{V_b}{K_b} d\sigma. \tag{2.22}$$

is measured in this test, where σ is the isotropic component of the intergranular stress ($\sigma = \frac{1}{3}\sigma_{kk}$). In the literature, it is often assumed that a dry specimen exhibits the same properties as a fully saturated one and therefore the conventional jacketed test is usually performed on a dry specimen. Assuming that the response of the solid particles to a unit increase of the average stress induced by the intergranular forces equals the response to a unit increase of the uniform stress induced in these particles by the fluid pressure, we can write for the volume change of the particles (see Eqs. (2.11) and (2.20))

$$dV_s = -\frac{1}{(1-\phi)} \frac{1}{K_s} V_s d\sigma. \tag{2.23}$$

The associated change in porosity $d\phi$ can be found using the relation $dV_s = d[(1-\phi)V_b)]$ (cf. Eq. (2.20))

$$d\phi = -\left(\frac{1-\phi}{K_b} - \frac{1}{K_s}\right) d\sigma. \tag{2.24}$$

It can be argued that a small increase of the intergranular stress must result in a decrease of the porosity, so $\partial\phi/\partial\sigma < 0$. From Eq. (2.24), we then find that $(1-\phi)K_s > K_b$, which was also previously stated by Verruijt (1982).

2.4.3 Combination of Effects

Now, the bulk volume change dV_b can be described as a function of both the pore pressure change and the change of the intergranular stresses, and thus as a summation of the effects discussed in both experiments (see Eqs. (2.19) and (2.22))

$$\frac{dV_b}{V_b} = -\frac{1}{K_b}d\sigma - \frac{1}{K_s}dp_f. \tag{2.25}$$

Introducing $de = de_{kk} = dV_b/V_b$, integrating Eq. (2.25) and ignoring the integration constants, which is allowed because we only consider varying (dynamic) quantities, we obtain

$$-\sigma = K_b e + \frac{K_b}{K_s}p_f. \tag{2.26}$$

Next, we want to include the effect of shear strain. When we measure the shear modulus of a dry sample, i.e., $p_f = 0$, the shear modulus G of the matrix can be incorporated following Hooke's law for an isotropic solid. As only the intergranular stress σ_{ij} can produce shear strain, it can be seen from Eq. (2.26) that the stress-strain relation for the bulk can be written as

$$-\sigma_{ij} = \left(K_b - \frac{2}{3}G\right)e\delta_{ij} + 2Ge_{ij} + \frac{K_b}{K_s}p_f\delta_{ij}. \tag{2.27}$$

This relation does not yet have the final form of Eqs. (2.10) and (2.15). Therefore, we proceed with the derivation below. In the literature, the effective stress σ'_{ij} is often introduced in such a way that the deformation of the matrix is fully determined by that stress (Verruijt 1982)

$$-\sigma'_{ij} = -\sigma_{ij} - \frac{K_b}{K_s}p_f\delta_{ij} = \left(K_b - \frac{2}{3}G\right)e\delta_{ij} + 2Ge_{ij}. \tag{2.28}$$

2.4.4 Relation of Biot's Elastic Constants to Physical Quantities

We continue with the derivation of stress-strain relations for a fluid-saturated porous medium by combining the constitutive equations with continuity equations. The constitutive equation for the solid is found by combination of Eqs. (2.20) and (2.23), and using $dV_s/V_s = -d\rho_s/\rho_s$. For the fluid, the bulk modulus K_f is introduced. The constitutive equations read

$$\frac{1}{\rho_s}\partial_t\rho_s = \frac{1}{K_s}\partial_t p_f + \frac{1}{(1-\phi)}\frac{1}{K_s}\partial_t\sigma, \tag{2.29}$$

$$\frac{1}{\rho_f}\partial_t\rho_f = \frac{1}{K_f}\partial_t p_f. \tag{2.30}$$

The linearized continuity equations read (Smeulders 1992)

$$(1-\phi)\partial_t\rho_s - \rho_s\partial_t\phi + (1-\phi)\rho_s\nabla\cdot\mathbf{v} = 0, \tag{2.31}$$

$$\phi\partial_t\rho_f + \rho_f\partial_t\phi + \phi\rho_f\nabla\cdot\mathbf{V} = \phi\partial_t\theta_m, \tag{2.32}$$

where $\mathbf{v} = \partial_t\mathbf{u}$ and $\mathbf{V} = \partial_t\mathbf{U}$ are the averaged velocities of the solid and fluid, respectively. For later use we include a source term in the equation for the fluid; θ_m denotes the volume density of (fluid) mass injection having dimensions [kgm^{-3}] (Wapenaar and Berkhout 1989). We do not include a similar source term for the solid because it is found in the literature only for the fluid (Bonnet 1987). As the physical meaning of this (fluid) source may not be immediately clear we further discuss its nature in Chap. 3.

As we are dealing with a linearized theory, in Eqs. (2.31) and (2.32) and all subsequent equations the products of quantities (e.g., $\phi\partial_t\rho_f$) are understood as follows: the quantity preceding the derivative (ϕ) denotes the unperturbed (background) value, and the quantity to which the derivative is applied ($\partial_t\rho_f$) denotes the wave-induced variation of that quantity.

By combining the solid relations, Eqs. (2.29) and (2.31), and the fluid equations, Eqs. (2.30) and (2.32), respectively, we eliminate the factors $\partial_t\rho_s$ and $\partial_t\rho_f$ and obtain

$$\frac{1-\phi}{K_s}\partial_t p_f + \frac{1}{K_s}\partial_t\sigma - \partial_t\phi + (1-\phi)\nabla\cdot\mathbf{v} = 0, \tag{2.33}$$

$$\frac{\phi}{K_f}\partial_t p_f + \partial_t\phi + \phi\nabla\cdot\mathbf{V} = \frac{\phi}{\rho_f}\partial_t\theta_m. \tag{2.34}$$

Elimination of the porosity term ($\partial\phi$) by adding the equations yields

$$\left(\frac{1-\phi}{K_s} + \frac{\phi}{K_f}\right)\partial_t p_f + \frac{1}{K_s}\partial_t\sigma + (1-\phi)\nabla\cdot\mathbf{v} + \phi\nabla\cdot\mathbf{V} = \frac{\phi}{\rho_f}\partial_t\theta_m, \tag{2.35}$$

which is usually called the "storage equation"; it forms a basic relationship in consolidation problems (Verruijt 1982).

Now we eliminate either σ or p_f from the combination of Eqs. (2.26) and (2.35). Using the identity $\partial_t e = \nabla\cdot\mathbf{v}$ this yields

$$\phi'\partial_t\sigma + \phi K_b\nabla\cdot\mathbf{v} - \phi K_f\frac{K_b}{K_s}\nabla\cdot\mathbf{V} = \phi K_f\partial_t\theta_m, \tag{2.36}$$

$$\phi'\partial_t p_f + K_f\left(1-\phi-\frac{K_b}{K_s}\right)\nabla\cdot\mathbf{v} + \phi K_f\nabla\cdot\mathbf{V} = -\phi K_f\frac{K_b}{K_s}\partial_t\theta_m, \tag{2.37}$$

where we have introduced

$$\phi' = \phi + \frac{K_f}{K_s}\left(1-\phi-\frac{K_b}{K_s}\right). \tag{2.38}$$

Then, by combining Eqs. (2.27) and (2.37), and Eqs. (2.26) and (2.36), respectively, we obtain the following set of stress-strain relations for a fluid-saturated porous medium in a form similar to Eqs. (2.10) and (2.15), and (2.9) and (2.16) (except for the volume injection source), respectively,

$$-\sigma_{ij} - (1-\phi)p_f\delta_{ij} = G(\partial_i u_j + \partial_j u_i) + A\partial_k u_k \delta_{ij} + Q(\partial_k U_k - \theta)\delta_{ij}, \quad (2.39)$$

$$-\phi p_f = Q\partial_k u_k + R(\partial_k U_k - \theta). \quad (2.40)$$

Here, we note that ϕ is the unperturbed value of the porosity; cf. Eqs. (2.31) and (2.32). Further, we have used $\rho_f^{-1}\partial_t\theta_m = \partial_t(\rho_f^{-1}\theta_m) = \partial_t\theta$, which is possible because ρ_f denotes the unperturbed fluid density; θ denotes the volume density of volume injection (Wapenaar and Berkhout 1989), which is a dimensionless quantity. Indefinite integration over time has been applied to obtain Eqs. (2.39) and (2.40), where the integration constants are ignored because we only consider varying (dynamic) quantities. In the above derivation of Eqs. (2.39) and (2.40) the generalized elastic constants A, Q and R (cf. Eqs. (2.15) and (2.16)) are found to be related to the physical quantities ϕ, K_b, K_f, K_s and G according to

$$A = K_b - \frac{2}{3}G + \frac{K_f(1-\phi-\frac{K_b}{K_s})^2}{\phi'}, \quad (2.41)$$

$$Q = \frac{\phi K_f(1-\phi-\frac{K_b}{K_s})}{\phi'}, \quad (2.42)$$

$$R = \frac{\phi^2 K_f}{\phi'}. \quad (2.43)$$

In the limit case in which the porous matrix and the fluid are much more compressible than the grains themselves (i.e., K_b/K_s, $K_f/K_s \to 0$), the expressions reduce to

$$A = K_b - \frac{2}{3}G + \frac{K_f(1-\phi)^2}{\phi}, \quad (2.44)$$

$$Q = K_f(1-\phi), \quad (2.45)$$

$$R = \phi K_f. \quad (2.46)$$

From Eqs. (2.41)–(2.43) it can be derived that

$$K_b = A - \frac{Q^2}{R} + \frac{2}{3}G, \quad (2.47)$$

which shows a similarity with the elastic case, where the well-known Lamé constants λ and G are related to the bulk modulus according to (Achenbach 1973)

$$K_b = \lambda + \frac{2}{3}G. \quad (2.48)$$

Obviously, the Lamé constant λ of a porous material, under condition of constant fluid pressure (see Sect. 2.4.2), is found as

$$\lambda = A - \frac{Q^2}{R}. \tag{2.49}$$

2.5 Equations of Motion

Next, we derive the equations of motion by combination of the stress-strain relations with momentum equations. We derive two different formulations of the equations of motion and we show that a viscous mechanism can be incorporated, describing the frequency-dependent interaction between fluid and solid.

The momentum equations for a porous medium have been derived by Biot (1956a) using Lagrange's equations. Starting from the linearized Navier-Stokes equations and the linearized equations of elasticity, Burridge and Keller (1981) arrived at the same result using a two-space method of homogenization for the case that the viscosity of the saturating fluid is relatively small. In this section we summarize the derivation by Biot.

For both the solid and the fluid phase, Lagrange's equation including dissipation can be formulated as (Achenbach 1973; Graff 1975; Davis 1988; Allard 1993; Pierce 2007)

$$\partial_t \left(\frac{\partial E_k}{\partial v_i} \right) + \frac{\partial E_d}{\partial v_i} = T_{s,i} + F_{s,i}, \tag{2.50}$$

$$\partial_t \left(\frac{\partial E_k}{\partial V_i} \right) + \frac{\partial E_d}{\partial V_i} = T_{f,i} + F_{f,i}, \tag{2.51}$$

where E_k is the kinetic energy density of the porous medium, E_d denotes the dissipation function, $T_{s,i}$ is the elastic force (due to stresses) acting on the solid per unit volume, $T_{f,i}$ is the elastic force acting on the fluid per unit volume, and $F_{s,i}$ and $F_{f,i}$ are the external volume forces acting on the solid and fluid phase, respectively. The expression for the kinetic energy density reads (Biot 1956a)

$$E_k = \tfrac{1}{2}(\rho_{11} v_i v_i + \rho_{22} V_i V_i + 2\rho_{12} v_i V_i). \tag{2.52}$$

The density terms ρ_{11}, ρ_{22} and ρ_{12} are related to the density of the solid ρ_s and that of the fluid ρ_f according to

$$\rho_{11} = (1-\phi)\rho_s - \rho_{12}, \tag{2.53}$$

$$\rho_{22} = \phi\rho_f - \rho_{12}, \tag{2.54}$$

$$\rho_{12} = -(\alpha_\infty - 1)\phi\rho_f. \tag{2.55}$$

The latter density term represents a mass coupling parameter between the solid and the fluid, which exists due to the (infinite-frequency) tortuosity α_∞ of the porous network: $\alpha_\infty \geq 0$, and hence $\rho_{12} \leq 0$. We discuss α_∞ more extensively in Sect. 2.5.1.

Dissipation depends only on the relative motion of the fluid and the solid phases. Like Eq. (2.52), the dissipation function can be expressed in terms of six velocity components. For the isotropic case it reads (Biot 1956a)

$$E_d = \tfrac{1}{2} b_0 (v_i - V_i)(v_i - V_i), \tag{2.56}$$

where the coefficient b_0 is related to the Darcy flow permeability k_0 and the dynamic viscosity η of the saturating fluid as

$$b_0 = \frac{\eta \phi^2}{k_0}. \tag{2.57}$$

The nature of the dissipation mechanism is discussed in more detail in Sect. 2.5.1.

The forces $T_{s,i}$ and $T_{f,i}$ are related to spatial derivatives of the stresses. Using Eqs. (2.39) and (2.40) the expressions read

$$\begin{aligned} T_{s,i} &= -\partial_j \sigma_{ij} - (1-\phi)\partial_i p_f \\ &= G\partial_i\partial_j u_j + G\partial_j^2 u_i + A\partial_i\partial_j u_j + Q(\partial_i\partial_j U_j - \partial_i\theta), \end{aligned} \tag{2.58}$$

$$\begin{aligned} T_{f,i} &= -\phi\partial_i p_f \\ &= Q\partial_i\partial_j u_j + R(\partial_i\partial_j U_j - \partial_i\theta). \end{aligned} \tag{2.59}$$

Now, by combining Eqs. (2.50), (2.52), (2.56) and (2.58), and by combining Eqs. (2.51), (2.52), (2.56) and (2.59), we obtain the following equations of motion

$$\begin{aligned} \rho_{11}\partial_t^2\mathbf{u} + \rho_{12}\partial_t^2\mathbf{U} + b_0\partial_t(\mathbf{u}-\mathbf{U}) &= P\nabla\nabla\cdot\mathbf{u} - G\nabla\times\nabla\times\mathbf{u} \\ &\quad + Q\nabla\nabla\cdot\mathbf{U} + \mathbf{f}, \end{aligned} \tag{2.60}$$

$$\rho_{12}\partial_t^2\mathbf{u} + \rho_{22}\partial_t^2\mathbf{U} - b_0\partial_t(\mathbf{u}-\mathbf{U}) = Q\nabla\nabla\cdot\mathbf{u} + R\nabla\nabla\cdot\mathbf{U} + \mathbf{F}, \tag{2.61}$$

where $P = A + 2G$ and we have used the vector identity $\nabla^2\mathbf{u} = \nabla\nabla\cdot\mathbf{u} - \nabla\times\nabla\times\mathbf{u}$ to separate dilatation and rotation terms. The source terms are defined as

$$\mathbf{f} = \mathbf{F}_s - Q\nabla\theta, \tag{2.62}$$

$$\mathbf{F} = \mathbf{F}_f - R\nabla\theta. \tag{2.63}$$

Eqs. (2.60) and (2.61) are the equations of motion for wave propagation in a fluid-saturated porous medium, as originally derived by (Biot 1956a) (without source terms). The incorporated dissipation mechanism being frequency-independent, however, simplifies reality too much. Therefore, in the next section we modify the associated terms in Eqs. (2.60) and (2.61).

2.5.1 *Incorporation of Frequency-Dependent Permeability or Tortuosity*

In this section we discuss the behavior of the dissipation mechanism which describes the frequency-dependent interaction between the solid and the fluid. Starting from the low- and high-frequency limits, the behavior in the intermediate frequency band is obtained for the rigid-frame limit ($\mathbf{u} = \mathbf{0}$). We clarify the relation between frequency-dependent permeability and tortuosity, illustrate their behavior using a numerical example, and finally we show how the frequency-dependent dissipation mechanism can be included in the general equations of motion Eqs. (2.60) and (2.61).

In the rigid-frame limit, after application of the Fourier transform over time (Eq. (2.1)), the equation of motion for the fluid reduces to

$$- \nabla \hat{p}_f = \left(-\omega^2 \alpha_\infty \rho_f + \mathrm{i}\omega \frac{\eta\phi}{k_0} \right) \hat{\mathbf{U}}, \tag{2.64}$$

which is obtained from Eq. (2.61) by expressing $\nabla\nabla \cdot \mathbf{U}$ in terms of p_f using Eq. (2.40).

In the low-frequency limit the acceleration term tends to zero and the viscous forces are dominant. Hence, Eq. (2.64) reduces to

$$\lim_{\omega\to 0} (-\nabla \hat{p}_f) = \mathrm{i}\omega \frac{\eta\phi}{k_0} \hat{\mathbf{U}}, \tag{2.65}$$

which is the well-known Darcy's law for flow through porous media. In the high-frequency limit the acceleration term dominates the viscous forces, and we obtain

$$\lim_{\omega\to \infty} (-\nabla \hat{p}_f) = -\omega^2 \alpha_\infty \rho_f \hat{\mathbf{U}}. \tag{2.66}$$

In this equation the tortuosity α_∞ appears as a modification of the acceleration term of the fluid. To understand this, it is important to realize that we are dealing with a macroscopic (continuum) theory. The macroscopic length scale is related to the wavelength L at which measurable, continuous and differentiable quantities can be identified. The microstructure of a random porous medium is generally characterized by a length scale proportional to the pore size (Smeulders et al. 1992). The direction of the acceleration on the microscale may very well differ from the macroscopic acceleration direction. For instance, when the macroscopic flow is one-dimensional, the microscopic flow is at least two-dimensional. Smeulders et al. (1992) relate the microscopic flow field to the macroscopic flow field using an averaging technique of homogenization. In the high-frequency limit they obtain

$$\alpha_\infty = \frac{\langle |\mathbf{v}_p|^2 \rangle}{|\mathbf{v}_0|^2}, \tag{2.67}$$

where $\langle\rangle$ denotes the averaging operator, $\mathbf{v}_p$ is the microscopic potential-flow solution and $\mathbf{v}_0$ is the macroscopic velocity of the fluid. In this way, one can imagine that the local variations of the flow contribute to the inertia term on the macroscopic level. In a cylindrical duct the averaged microscopic velocity equals the macroscopic velocity and, consequently, $\alpha_\infty = 1$.

Considering now Eq. (2.64), we observe that the momentum equation of the fluid is constituted by superposition of the low- and high-frequency limits described above. This simple superposition is, however, too simplified a description of the frequency-dependent dissipation process. A more realistic description has been proposed by Biot (1956b) and by Johnson et al. (1987). Here, we follow the latter model where either the concept of dynamic permeability $\hat{k}(\omega)$ is introduced, or the concept of dynamic tortuosity $\hat{\alpha}(\omega)$, by reformulations of Eq. (2.64) according to

$$-\nabla \hat{p}_f = \mathrm{i}\omega \frac{\eta\phi}{\hat{k}(\omega)}\hat{\mathbf{U}}, \tag{2.68}$$

$$-\nabla \hat{p}_f = -\omega^2 \hat{\alpha}(\omega)\rho_f \hat{\mathbf{U}}. \tag{2.69}$$

Obviously, Eqs. (2.68) and (2.69) are alternative descriptions of the same physical reality and therefore, $\hat{k}(\omega)$ and $\hat{\alpha}(\omega)$ are related as

$$\hat{\alpha}(\omega) = -\frac{\mathrm{i}\eta\phi}{\omega\rho_f \hat{k}(\omega)}. \tag{2.70}$$

In the low-frequency limit, the dynamic permeability approaches the stationary value (see Eq. (2.65))

$$\lim_{\omega\to 0} \hat{k}(\omega) = k_0, \tag{2.71}$$

and, consequently, using Eq. (2.70) for the dynamic tortuosity it follows that

$$\lim_{\omega\to 0} \hat{\alpha}(\omega) = -\frac{\mathrm{i}\eta\phi}{\omega\rho_f k_0}. \tag{2.72}$$

In this limit the fluid follows a Stokes flow pattern on the pore scale (i.e., the flow is described by the linearized Navier-Stokes equation with intertia terms neglected). In the high-frequency limit the fluid obeys a potential flow pattern (i.e., the flow described by the linearized Navier-Stokes equation with viscosity terms neglected) on the pore scale, except for a very thin boundary layer $\delta = \sqrt{2\eta/(\omega\rho_f)}$ at the pore walls; hence, tortuosity and permeability are given as

$$\lim_{\omega\to\infty} \hat{\alpha}(\omega) = \alpha_\infty, \tag{2.73}$$

$$\lim_{\omega\to\infty} \hat{k}(\omega) = -\frac{\mathrm{i}\eta\phi}{\omega\rho_f \alpha_\infty}, \tag{2.74}$$

where we have again used Eq. (2.70). For the intermediate frequency range Johnson et al. (1987) postulated a branching function connecting the two limiting situations based on the ratio of the viscous skin depth δ and the characteristic length scale of the pores Λ according to

$$\hat{k}(\omega) = k_0 \left[\left(1 + \mathrm{i}\frac{M}{2}\frac{\omega}{\omega_c}\right)^{\frac{1}{2}} + \mathrm{i}\frac{\omega}{\omega_c} \right]^{-1}, \tag{2.75}$$

$$\hat{\alpha}(\omega) = \alpha_\infty \left[1 - \mathrm{i}\frac{\omega_c}{\omega} \left(1 + \mathrm{i}\frac{M}{2}\frac{\omega}{\omega_c}\right)^{\frac{1}{2}} \right], \tag{2.76}$$

where $\mathrm{Re}(1 + \mathrm{i}\frac{M}{2}\frac{\omega}{\omega_c})^{\frac{1}{2}} \geq 0$ for $\omega \geq 0$, and

$$\omega_c = \frac{\eta\phi}{k_0 \rho_f \alpha_\infty}, \quad M = \frac{8\alpha_\infty k_0}{\phi \Lambda^2}. \tag{2.77}$$

The rollover frequency ω_c denotes the frequency where the inertia effects and the viscous effects are of the same order of magnitude. The pore-shape factor M is often close to 1 (Johnson et al. 1987; Smeulders et al. 1992).

Before incorporating the dissipation mechanism in the equations of motion, we visualize the frequency-dependent behavior of $\hat{k}(\omega)$ and $\hat{\alpha}(\omega)$. For material properties related to Sand of Mol (Degrande et al. 1998) (the parameter values are given in Table 2.1), which is representative of a water-saturated shallow subsurface situation of loosely packed sand, we show the behavior in Fig. 2.1. We observe that the magnitude of the dynamic permeability $|\hat{k}|$ reduces to the Darcy permeability k_0 in the low-frequency limit, which agrees with Eq. (2.65). In the low-frequency limit, $|\hat{\alpha}|$ tends to infinity as ω^{-1} (see Eq. (2.72)), which can be understood from Eq. (2.69). In the high-frequency limit, the magnitude of the dynamic tortuosity $|\hat{\alpha}|$ goes to α_∞, which agrees with Eq. (2.66). The value of $|\hat{k}|$ tends to zero (see Eq. (2.74)), which is because the pressure variation is too fast for the fluid to react (cf. Eq. (2.68)). For the intermediate frequency range, the behavior is described by the branching functions

Table 2.1 Material parameters as used for water-saturated Sand of Mol (Degrande et al. 1998). We assume that $M = 1$ (see Eq. (2.77))

Solid (frame) density ρ_s [kgm^{-3}]	2650
Fluid density ρ_f [kgm^{-3}]	1000
Tortuosity α_∞	1.789
Porosity ϕ	0.388
Permeability k_0 [μm^2]	10.214
Dynamic fluid viscosity η [Pa·s]	0.001
Shear modulus G [MPa]	111.86
Frame bulk modulus K_b [MPa]	298.3
Grain bulk modulus K_s [GPa]	36.5
Fluid bulk modulus K_f [GPa]	2.22

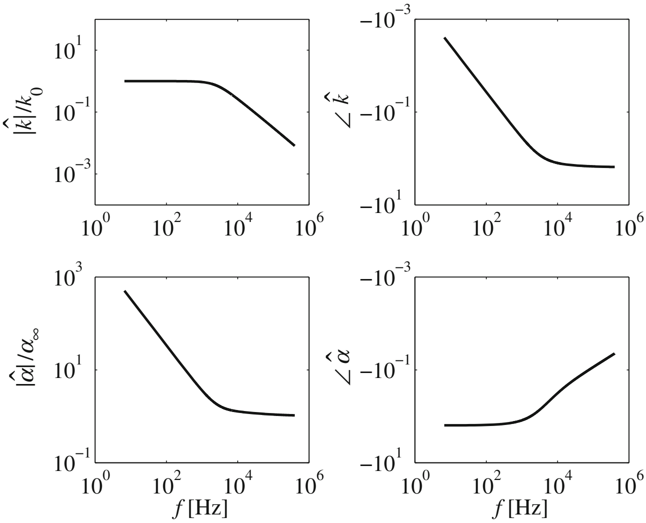

Fig. 2.1 Frequency-dependent permeability and tortuosity according to the viscous attenuation mechanism of Johnson et al. (1987) (Eqs. (2.75) and (2.76)). Both magnitudes and phases are shown

of Johnson et al. (1987) (Eqs. (2.75) and (2.76)), showing a point of inflection at approximately the rollover frequency $f_c = \omega_c/(2\pi) = 3387$ Hz. The phases $\angle\hat{k}$ and $\angle\hat{\alpha}$ (Fig. 2.1) are also consistent with the low- and high-frequency limits (see Eqs. (2.71)–(2.74)).

The expression for dynamic permeability (Eq. (2.75)) can be substituted in Eq. (2.68), and that of the dynamic tortuosity (Eq. (2.76)) in Eq. (2.69). One of these should be used to incorporate the frequency-dependent dissipation mechanism in the equations of motion for deformable (non-rigid) porous media (Eqs. (2.60) and (2.61)). In this thesis we choose to work with dynamic permeability. By rewriting of the expression Eq. (2.68) to a form comparable with Eq. (2.64), we find that the effect can be incorporated by simply replacing b_0 by $\hat{b}(\omega)$ according to

$$\hat{b}(\omega) = b_0\,(1 + \mathrm{i}\omega\tau_c)^{\frac{1}{2}}\,, \tag{2.78}$$

where $\tau_c = M/(2\omega_c)$ and $\mathrm{Re}(\hat{b}(\omega)) \geq 0$ for $\omega \geq 0$. Then, the $(\mathbf{x}, \omega)$-domain representation of the equations of motion (Eqs. (2.60) and (2.61)) can be written as

$$-\omega^2\hat{\rho}_{11}\hat{\mathbf{u}} - \omega^2\hat{\rho}_{12}\hat{\mathbf{U}} = P\nabla\nabla\cdot\hat{\mathbf{u}} - G\nabla\times\nabla\times\hat{\mathbf{u}} + Q\nabla\nabla\cdot\hat{\mathbf{U}} + \hat{\mathbf{f}}, \tag{2.79}$$

$$-\omega^2\hat{\rho}_{12}\hat{\mathbf{u}} - \omega^2\hat{\rho}_{22}\hat{\mathbf{U}} = Q\nabla\nabla\cdot\hat{\mathbf{u}} + R\nabla\nabla\cdot\hat{\mathbf{U}} + \hat{\mathbf{F}}, \tag{2.80}$$

where $\hat{b}(\omega)$ shows up in the frequency-dependent density terms that read

$$\hat{\rho}_{11} = \rho_{11} - \mathrm{i}\hat{b}/\omega, \quad (2.81)$$

$$\hat{\rho}_{22} = \rho_{22} - \mathrm{i}\hat{b}/\omega, \quad (2.82)$$

$$\hat{\rho}_{12} = \rho_{12} + \mathrm{i}\hat{b}/\omega. \quad (2.83)$$

The $(\mathbf{x}, t)$-domain equivalents of these terms are time-dependent convolution operators. Their expressions can be found using a standard inverse Laplace transform (Prudnikov et al. 1992) and read

$$\rho_{11}(t) = \rho_{11}\delta(t) + b_0\varrho(t), \quad (2.84)$$

$$\rho_{22}(t) = \rho_{22}\delta(t) + b_0\varrho(t), \quad (2.85)$$

$$\rho_{12}(t) = \rho_{12}\delta(t) - b_0\varrho(t), \quad (2.86)$$

where $\delta(\ldots)$ denotes the Dirac delta function (Abramowitz and Stegun 1972) and

$$\varrho(t) = \left(\frac{\exp(-t/\tau_c)}{\sqrt{\pi t/\tau_c}} + \operatorname{erf}\left(\sqrt{t/\tau_c}\right) \right) H(t). \quad (2.87)$$

Here, $H(t)$ denotes the Heaviside step function, i.e., $H(t) = \{0, \frac{1}{2}, 1\}$ for $\{t < 0, t = 0, t > 0\}$, and $\operatorname{erf}(\ldots)$ denotes the error function (Abramowitz and Stegun 1972).

Now we have arrived at the general form of the equations of motion that incorporate the frequency-dependent dissipation mechanism, and which we will often use in this thesis. We refer to it as the $(\mathbf{u}, \mathbf{U})$-formulation because the equations of motion are expressed in the field quantities $\mathbf{u}$ and $\mathbf{U}$. For completeness, a less well-known but more compact representation of Biot's equations of motion is given in the next section.

2.5.2 Alternative Formulation of the Equations of Motion

An alternative to the $(\mathbf{u}, \mathbf{U})$-formulation of the equations of motion is the so-called $(\mathbf{u}, p_f)$-formulation in which the equations of motion are expressed in terms of $\hat{\mathbf{u}}$ and $\hat{p}_f$ (Bonnet 1987; Wiebe and Antes 1991; van Dalen et al. 2008; Schanz 2009). The equations are obtained by rewriting of Eqs. (2.79) and (2.80), and using Eqs. (2.58) and (2.59) to eliminate $\mathbf{U}$. The result is

$$\omega^2\hat{\rho}_{eq}\hat{\mathbf{u}} + (\lambda + 2G)\nabla\nabla \cdot \hat{\mathbf{u}} - G\nabla \times \nabla \times \hat{\mathbf{u}} = \frac{\phi H_S}{R}\nabla\hat{p}_f - (\hat{\mathbf{f}} + \beta_S\hat{\mathbf{F}}), \quad (2.88)$$

$$\omega^2\hat{\rho}_{22}\hat{p}_f + R\nabla^2\hat{p}_f = -\omega^2\hat{\rho}_{22}\frac{H_S}{\phi}\nabla \cdot \hat{\mathbf{u}} + \frac{R}{\phi}\nabla \cdot \hat{\mathbf{F}}. \quad (2.89)$$

The Lamé parameter λ has already been defined in Eq. (2.49). Further, the following definitions hold

$$\hat{\rho}_{eq} = d_0/\hat{\rho}_{22}, \tag{2.90}$$

$$d_0 = \hat{\rho}_{11}\hat{\rho}_{22} - \hat{\rho}_{12}^2, \tag{2.91}$$

$$H_S = Q + R\beta_S, \tag{2.92}$$

$$\beta_S = -\hat{\rho}_{12}/\hat{\rho}_{22}, \tag{2.93}$$

where $\hat{\rho}_{eq}$ reduces to $\rho_{11} + \rho_{22} + 2\rho_{12} = \rho$ (bulk density) for $\omega \to 0$; the physical meaning of β_S is given later (below Eq. (3.23)).

The similarity of the equations of motion Eqs. (2.88) and (2.89) with those of an elastic solid and an acoustic medium, respectively, is obvious (Achenbach 1973; de Hoop 1995). The difference lies in the definition of the specific density and elastic constants, and in the coupling terms of the equations that can be interpreted as source terms.

Bonnet (1987) showed that only four out of the seven field variables ($\mathbf{u}$, $\mathbf{U}$, p_f) are independent. Therefore, the ($\mathbf{u}$, p_f)-formulation provides a set of independent equations governing wave propagation in a fluid-saturated porous medium. The ($\mathbf{u}$, $\mathbf{U}$)-formulation, which is the original form of Biot's equations (Biot 1956a), is used more often but only four of the six equations are independent.

In wave propagation problems either the ($\mathbf{u}$, U)-formulation or the ($\mathbf{u}$, p_f)-formulation can be used. In Chap. 3 we derive Green's tensors for both sets of equations to illustrate the basic properties of the wave propagation process in a porous medium.

2.6 Conclusions

In this chapter we derived the equations of motion for wave propagation in a fluid-saturated porous medium. First, we illustrated that the stress-strain relations associated with Biot's theory can be obtained from constitutive and continuity equations, by considering the porous medium as a two-phase continuum. This shows that the involved elastic constants are clearly related to physical quantities, i.e., the bulk moduli of the grains, the porous frame and the fluid, to the shear modulus and to the porosity. By combination of the stress-strain relations with Lagrange's momentum equations for the solid and fluid, the equations of motion were found. We presented the equations of motion in two different formulations that are known in the literature, i.e., the ($\mathbf{u}$, $\mathbf{U}$)-formulation (solid and fluid particle displacements) and the ($\mathbf{u}$, p_f)-formulation (solid particle displacement and fluid pressure). The latter formulation shows that an arbitrary wavefield in a fluid-saturated porous medium has only four independent field quantities.

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