Chapter 2
The Outlook of the Sovereign Planner: The Linear Activity Model

The purpose of this chapter is to formulate a linear numerical general equilibrium model. The model is essentially a Leontief type of input–output model, extended with resource constraints. In this chapter the equilibrium model is developed and analysed under conditions of competitive market behaviour. To provide the reader with an understanding of the nature of this model and its link to economic theory, the concept of welfare optimum (Pareto efficiency) and its logical relation to competitive equilibrium is used as a connecting thread between the concept of economic equilibrium and the mathematical programming formulation. The following sections will highlight the major features of the model. At the same time, the assumptions necessary to make the model operational are made explicit.

2.1 Commodities and Activities

In this study we shall be considering an economy where there exists a finite number of commodities (commodity groups)[1] subject to production, consumption, or both. The commodity concept also includes services. A commodity is characterised by the property that two equal quantities of it are completely equivalent for each consumer and each producer. The commodities are here divided into two groups, according to whether they are produced within the production system or not. Commodities in the former group are called *produced* commodities, in the latter group, *primary* commodities.[2] Thus, total supply within the economic system specified in this study is a result of the domestic production system.

[1] Generally, a commodity is defined by its physical characteristics, its location, and the date of its delivery. Commodities differing in any of these characteristics will be regarded as different. However, in this model a commodity is synonymous with the industry supplying the commodity (sector classification principle).

[2] Thus, there is only use of primary commodities, not production of them.

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2.2 Producers

The n producers (industries) execute the production programs represented by the n nonnegative multiples Z_j of a_{ij}. The extent to which the activity is utilised must be feasible, i.e. to say the produced amount Z_j must be an element of the production set Y_j.

For any producer j there exists a given quantity of capital commodities, previously produced commodities, and in the short run specific for each produced commodity, and hence, each producer. In other words, capacities are assumed immobile. For the producer each activity implies a given transformation of primary commodities into produced commodities, and to make this transformation possible, a given quantity of capacities available. By this specification, the capacities are considered as primary commodities. Hence, the primary commodities can in the short run be partioned in two kinds of commodities. On one hand, capacities, which in the current point of time are fixed to the existent establishments and on the other hand resources (labour), which the different producers (industries) are competing for in the market.

Closely related to the assumptions given above is the assumption of irreversibility of production, i.e. the production process cannot reversed, thus, excluding negative activity levels from the solution. Further, free disposal is assumed, i.e. it is possible for all producers together to dispose of all commodities. Finally the assumption of free disposal together with the assumption of irreversibility implies the impossibility of free production, i.e., it requires inputs to produce outputs.[3]

2.3 Consumers

The s consumers are the only owners and final users of commodities. Each consumer, denoted i owns the supplied quantity r_{ih} of the primary commodity, denoted h, and a share, denoted θ_{ij}, of the industry j. By this specification a special economy is then considered, namely the *private ownership economy* where consumers own the resources and control the producers. The rents may be assumed to be distributed following a certain rule, such as a fixed proportion. It should be noted that no matter how the rents are distributed, all the rents must be paid to consumers.

The set of consumption which enables consumer i to survive is his attainable set X_i, defined for all combinations of demand of desired commodities x_{ij}, and supplies of his initial endowment of primary commodities (labour service) r_{ih}, which he can sell to obtain income. Thus, each consumer is assumed to have an endowment of leisure, a portion which can be sold as labour service, and the leisure remaining is a component (nonnegative) in his attainable set.

The consumer's preferences among different vectors x_{ij} and r_{ih} are represented by a utility function $S_i(x_{ij}, -r_{ih})$ defined for all nonnegative quantities of desired

[3] See further Debreu G. (1959), p. 42.

commodities x_{ij} and quantities of primary commodities r_{ih}, represented as a non-positive quantity.[4] Under the conditions of a private ownership economy, where primary commodities and capital commodities are owned by individual consumers, the i:th consumer's income R_i will be the sum of the value of the supplied quantities of primary commodities and the shares θ_{ij}, of the rents (returns of capital as a factor of production) of the producers.

2.4 Feasible Activities

For each process actually carried out within the economic system outlined above, the variable Z_j will take specific value. This seems agreeable to common sense. Any feasible state of supply, i.e. the ability of the economy to achieve an allocation within the limits of its resources, may be stated more formally. Thus, the commodity balance constraint (Eq. 2.1 below) states that each feasible allocation must contain at least one production activity.

Final supply is made up of the total supply of a commodity minus the amount of the commodity used within the production system (intermediate demand), where a_{ij} denote the intermediate requirements of commodity i per unit of output of sector j. On the other hand, use outside of the production system is called final demand, here denoted D_j, represents *domestic* final demand, i.e. the sum of private consumption, investment and government expenditures.

$$Z_j - \sum_{j'} a_{ij} Z_j \geq \sum_i D_{ij} \tag{2.1}$$

$$Z_j \geq 0, \qquad D_j \geq 0$$

Equation 2.2, the primary commodity constraint, further restricts the feasible set. The primary commodity constraint represents here labour, supplied by the households. This specification distinguishes different skill categories of labour, where b_{hj} denote the input coefficient of each primary commodity h. in each sector j. Despite different individuals will be of different productivities, the labour input in each sector is assumed to be an aggregation of labour of different skill categories. Hence, there is only one aggregate, and homogenous, primary commodity supplied by the households. This implies that labour is assumed perfectly mobile across sectors.

$$\Sigma_{j'}\, b_{hj} Z_j \;\leq\; \Sigma_{i'}\, r_{ih} \tag{2.2}$$

$$r_{ih} \geq 0$$

[4] In mathematical language, the utility function S, is continuous and increasing, twice continuously differentiable, strictly quasi-concave and its first derivatives are not all simultaneously equal to zero.

Empirically, *labour* is measured in unit wage costs, which refer to all wage payments including collective payroll charges. This implies that factor payments data is used as observations on physical quantities of factors for use in the determination of parameters for the model. The total supply of *labour resources* is given exogenously, calculated on the basis of total labour force (minus employed in the government sector) and we measure it in terms of wages (and salaries). Thus, the labour balance requirement is stated in value terms and not in physical terms. In all experiments, the labour resource constraint will be binding, i.e. our model solutions requiring full employment of labour. However, it is necessary to note that computed market equilibrium (model solution) may, in principle, permit unemployment of labour.

Equation 2.3 represents the *capital stock* by sector. At each point of time it is assumed that the supply of these commodities is given and specific for each production unit. With these characteristics we must have a restriction for each capital commodity i and each sector j.[5] This is also the reason for classifying these commodities as primary commodities in the short run.

$$c_{ij} Z_j \quad \leq \quad K_{ij} \tag{2.3}$$

$$K_{ij} \geq 0$$

The real capital stock is a composite commodity and the commodity composition of capital differs across sectors. Consequently, the real capital stock is impossible to measure with any real precision. In this model the capital stock in each sector is aggregated into a single commodity and no difference is made between the two definitions, the real and the utilised. Recapitulating, the total supply of commodities in the economic system is partly a result of the activity within the domestic production system. Since each process implies use of primary commodities, and production and use of produced commodities, the possibility to carry on these processes are therefore dependent on the given quantities of primary commodities, the produced amount of produced commodities.

2.5 The Programming Formulation

The point of departure for the programming model presented below is an economic system where an excess demand for any commodity implies an increase of the corresponding commodity price without any upper limit, and an excess supply of any commodity that the corresponding commodity prices decreases, given the restriction that the price will not take any negative value. Thus, while we would

[5] This forms a matrix with capacity input coefficients in its principal diagonal and zero elements everywhere else. Hence, $i = j$ for all c_{ij}.

never accept a situation with positive excess demand in some market as equilibrium, an excess supply in a market where the price is zero is quite consistent with our notion of equilibrium. An economic system with these characteristics is compatible with a market economy. A state of equilibrium in this market economy is a situation where no individual. Given the price system and the actions of the other individuals, has any incentive to choose a different allocation of commodities.

Stated more formal, the equilibrium conditions state that there will be no excess demand for any commodity and market pricing of each commodity. Thus, the equilibrium conditions state that each commodity has only one price throughout the economy, and specifies that when the market equilibrium price for the commodity is positive, there is no excess supply or demand. Since the consumers in spite of the positive commodity prices demand all supplied quantities of Z_j, and supplies the sum of r_{ih} up to the quantity demanded by the producers, commodities with a positive price are regarded as *desired* commodities.[6]

The objective of our allocation problem is to find the set of supply activities that result in a bundle of desired commodities, in the sense that given the specified resources (resource constraints) it is impossible to increase the net amount of any desired commodity without decreasing the net amount of some other desired commodity. Such a bundle is called an efficient final commodity point, and the collection of all such efficient points traces the efficient supply frontier where each point is a possible efficient (Pareto efficient) state of allocation. In this framework the well known concept of Pareto optimality, i.e. a state in which no one's satisfaction can be raised without lowering someone else's, is translated to efficiency, and a term like 'allocation efficiency' is a more accurately descriptive of the concept.[7] A state of Pareto efficiency thus defined expresses a concept of allocative efficiency in converting resources into satisfactions. By the use of the concept of allocation efficiency, we can formulate the equilibrium model specified above within a mathematical programming format. Given the objective function and the constraint set the problem takes the following form, i.e. maximise:

$$W(x_i; r_h) \equiv \Sigma_i,\ S_i(x_{ij}, -r_{ih}) \tag{2.4}$$

Subject to

$$Z_j - \Sigma_j,\ a_{ij} Z_j \quad \geq \quad \Sigma_i D_{ij} \tag{2.5}$$

$$\Sigma_j,\ b_{hj} Z_j \quad \leq \quad \Sigma_i,\ r_{ih} \tag{2.6}$$

$$c_{ij} Z_j \quad \leq \quad K_{ij} \tag{2.7}$$

$$Z_j \geq 0, \quad D_{ij} \geq 0, \quad r_{ih} \geq 0, \quad K_{ij} \geq 0$$

[6] A commodity is *desirable* if any increase in its consumption, ceteris paribus, increases utility.

[7] Koopmans T.C. (1957), p. 84.

This is a typical programming problem and we use the Kuhn-Tucker theorem[8] to derive the optimality conditions. If the assumptions regarding the objective function and the constraint set are satisfied, then a necessary and sufficient condition that $\left(x_j^{\mathrm{o}},\ r_h^{\mathrm{o}}\right)$ is the optimum solution to (x_j, r_h), is that there exists $p_j^{\mathrm{o}} \geq 0,\ w_h^{\mathrm{o}} \geq 0,$ $v_{ij}^{\mathrm{o}} \geq 0$ such that the Lagrangean:

$$\begin{aligned} \mathrm{L}\{x_{ij}, r_{ih}, Z_j,\ p_j,\ w_h,\ v_{ij}\} &= \Sigma_i,\ S_i\,(x_{ij}, -r_{il})+ \\ &+ p_j\ (Z_j - \Sigma_j,\ a_{ij}Z_j - \Sigma_i,\ D_{ij}) + w_h(\Sigma_i,\ r_{ih} - \Sigma_j,\ b_{hj}\,Z_j) \\ &+ \Sigma_i,\ \Sigma_j,\ v_{ij}(K_{ij} - c_{ij}\,Z_j) \end{aligned}$$

forms a saddle point at $\left\{x_{ij}^{\mathrm{o}},\ r_{ih}^{\mathrm{o}},\ Z_j^{\mathrm{o}},\ p_j^{\mathrm{o}},\ w_h^{\mathrm{o}},\ v_{ij}^{\mathrm{o}}\right\}$.

We identify the Lagrangean multipliers p_j^o, w_h^o, and v_{ij}^o associated with the commodity constraints, as efficiency prices and rents. These efficiency prices or shadow prices of the mathematical program incorporate the effect of the constraints upon the activity level in the model, so that resources are allocated most efficiently. Supply choices open to this model are to supply each commodity by domestic production.

For any given objective function the *i*:th shadow price measures the opportunity cost of the last unit of the *i*:th resource or commodity employed in a binding constraint. The fact that the shadow prices are computed and measured in terms of the objective function (all efficiency concepts in our model is measured in terms of the objective function) implies that the objective function is crucial in determining and interpreting the shadow price system.[9] If the constraint is not binding, i.e. carries the $<$ or $>$ sign at the optimum, the shadow price will be zero implying that the resource or commodity is free. In this context, it is worth mentioning that any resource omitted from the specification of the model is considered as free and having an opportunity cost of zero. Given this behaviour, it is natural to interpret the Lagrangean multipliers as equilibrium prices. Thus

$$\frac{\partial L^{\mathrm{o}}}{\partial x_{ij}} = S'_{ij} - p_j^{\mathrm{o}} = 0 \tag{2.8}$$

$$\frac{\partial L^{\mathrm{o}}}{\partial r_{ih}} = -S'_{ih} + w_h^{\mathrm{o}} = 0 \tag{2.9}$$

[8] Kuhn H. W. and A. W. Tucker (1950). The Kuhn-Tucker theorem for con-strained optimisation tells us that the necessary conditions for the solution of the primal are equivalent to finding the solution of the dual. It does not in itself provide us with a practical solution method for the problem.

[9] The shadow prices of the model cannot be considered as "ideal", because this interpretation would be valid only if the specification of the objective function quantitatively embodied all goals of the economy.

$$\begin{aligned}\frac{\partial L^{o}}{\partial Z_{j}} &= p_{j}^{o} - \Sigma_{j},\, p_{j}^{o} a_{ij} - \Sigma_{j},\, w_{h}^{o} b_{hj} - \Sigma_{i},\, \Sigma_{j},\, v_{ij}^{o} c_{ij} \leq 0 \\ -'' - &< 0 \;\Rightarrow\; Z_{j}^{o} = 0\end{aligned} \tag{2.10}$$

$$\begin{aligned}\frac{\partial L^{o}}{\partial p_{j}} &= Z_{j}^{o} - \Sigma_{j},\, a_{ij} Z_{j}^{o} - \Sigma_{i},\, D_{ij} \geq 0 \\ -'' - &> 0 \;\Rightarrow\; p_{j}^{o} = 0\end{aligned} \tag{2.11}$$

$$\begin{aligned}\frac{\partial L^{o}}{\partial w_{h}} &= \Sigma_{i},\, r_{ih}^{o} - \Sigma_{j},\, b_{hj} Z_{j}^{o} \geq 0 \\ -'' - &> 0 \Rightarrow w_{h}^{o} = 0\end{aligned} \tag{2.12}$$

$$\begin{aligned}\frac{\partial L^{o}}{\partial v_{ij}} &= K_{ij} - c_{ij} Z_{j}^{o} \geq 0 \\ -'' - &> 0 \;\Rightarrow\; v_{kj}^{o} = 0\end{aligned} \tag{2.13}$$

Thus, the conditions (2.8), (2.9), (2.10), (2.11), (2.12), and (2.13) spell out the characteristics of the market pricing and rent system at the optimum that is consistent with an efficient supply and allocation program.

By the assumption that the utility function is differentiable, the equalities above, equality (2.8) and (2.9), establish certain classical relations between prices and marginal rates of substitution relating to consumer equilibrium x_{ij}^{o} and r_{ih}^{o}. These equalities imply that the marginal rate of substitution of any pair of commodities is equal to the ratio between any corresponding pair of prices.

Condition (2.10) states that, at the optimum, total profits must be zero in all production activities actually used and no activity may show a positive profit, i.e. production costs will exactly equal the shadow prices p_{j}^{o} for all commodities that are actually produced. The produced commodity is exhausted (Euler's theorem is met) by paying to each of the contributing factor its full marginal product. If the strict inequality holds, then the production costs exceed the shadow price p_{j}^{o} and the commodity will not be produced.

Condition (2.11) states that if the shadow prices p_{j}^{o} are zero at the optimum, then there exists excess supply of final commodities, and if the shadow prices are positive, there exists no excess supply of any final commodity.

Condition (2.12) states that if the optimum shadow factor price w_{h}^{o} is positive, the primary commodity r_{h} must be used to the maximum availability, and if the shadow price is zero, then a part of the commodity is left unused.

Condition (2.13) states that rent v_{ij}^{o}, the shadow price of each sector's capacity constraint, on processing plants may at the optimum exceed zero only if the capacities in each case are fully utilized. Since we are concerned with a short run model where capital is sectorally fixed, the rent concept can be viewed only within the context of scarcity, which implies that each sector has a sector-specific scarce factor with its own shadow price. Therefore, as noted, rents may be greater than zero only if the capacity is used to the limit. The rents represent the marginal return (measured in terms of the objective function) of capital employed in a particular sector and is therefore the marginal product (rate of return) of capital in this sector. The rents have significance for decision making because they will provide an estimate to the profitability of investments directed toward capacity expansion.

The optimality conditions, conditions (2.10), (2.11), (2.12), and (2.13), are thus consistent with the requirements of a price and allocation equilibrium, and the allocation which maximizes the objective function subject to the constraints, is a welfare optimum. In the following section it will be shown that the optimality conditions not only are consistent with the requirements of a price and allocation equilibrium, but also are consistent with the conditions for a competitive equilibrium.

In order to establish conditions compatible with the characteristics of a competitive equilibrium, equilibrium must prevail, not only on the market, but also for each producer and each consumer. For each producer in the sense that they cannot increase their profits by a change in the structure of production, and for each consumer in the sense that they cannot increase their utility by choosing a new combination of commodities specified in the utility function. Thus, a market equilibrium satisfying the system constraints consistent with the assumptions of competitive equilibrium must be characterised by the existence of a set of prices[10] such that profit maximising producers and utility maximising consumers, subject to their constraints, will generate production and consumption decisions such that the choices together constitute a balanced allocation of commodities, i.e. excess demands are non-positive.

The *producer equilibrium* stipulates that each producer (industry) is assumed to maximise its profits Π_h at given prices p_j^{o}, w_h^{o} subject to the technological and institutional constraints. The producer's profit is the difference between the total revenue from the sale of its commodity i and the expenditure upon all inputs.

Thus, the programming solution guarantees zero profits, equality of supply and demand for every commodity with non-zero prices, and equality of price and marginal costs for every producer in every commodity he actually produces. Consequently, it is clear that a decentralised decision-making process would lead to the same aggregate production pattern identical to the one which is provided by the solution of the programming, provided that each producer faces the same set of prices and strives to maximise profits.

$$\prod_h = p_j^{\mathrm{o}} Z_j - \Sigma_j,\ p_j^{\mathrm{o}} a_{ij} Z_j - \Sigma_h,\ \Sigma_j,\ w_h^{\mathrm{o}} b_{hj} Z_j \tag{2.14}$$

[10] These prices carry to each producer and each consumer a summary of information about the supply possibilities, resource availabilities and preferences of all other decision makers.

Subject to:

$$c_{ij} Z_j \leq K_{ij} \tag{2.15}$$

$$Z_j \geq 0, \quad K_{ij} \geq 0$$

Stated mathematically, each producer chooses Z_j among the points of Y_j so as to maximize:

$$\begin{aligned} \text{Max L}\{Z_j, v_{ij}\} = p_j^{\text{o}} Z_j - \Sigma_j, p_j^{\text{o}} a_{ij} Z_{hj} - \Sigma_h, \Sigma_j, w_h^{\text{o}} b_{hj} Z_j + \\ + \Sigma_i, \Sigma_j, v_{ij} \left(K_{ij} - c_{ij} Z_j\right) \end{aligned} \tag{2.16}$$

A necessary and sufficient condition that $\left\{Z_j^{\text{o}}, \; v_{ij}^{\text{o}}\right\}$ is a nonnegative saddle point, is:

$$\begin{aligned} \frac{\partial L^{\text{o}}}{\partial Z_j} &= p_j^{\text{o}} - \Sigma_j, p_j^{\text{o}} a_{ij} - \Sigma_h, w_h^{\text{o}} b_{hj} - \Sigma_i, \Sigma_j, v_{ij} c_{ij} \leq 0 \\ -'' - \quad & < \quad 0 \Rightarrow Z_j = 0 \end{aligned} \tag{2.17}$$

$$\begin{aligned} \frac{\partial L^{\text{o}}}{\partial v_{ij}} &= K_{ij} - c_{ij} Z_j \quad \geq \quad 0 \\ -'' - \quad & > \quad 0 \Rightarrow v_{ij} = 0 \end{aligned} \tag{2.18}$$

Condition (2.17) states that if production takes place at a positive level at the optimum, then the shadow price of the commodity must be equal to the cost of producing the commodity, where costs have two components, the explicit market costs of inputs and economic rents, which accrue to the use of the fixed capacities. Given our assumption of constant returns to scale, the unit cost equals the selling price, meaning that total profits must be zero on all production activities used and no activity may show a positive profit. Condition (2.18) state, that the rents are positive only when the capacity of the available capital stock is exhausted. These conditions are exactly the same as condition (2.10) and (2.13). This implies that the equilibrium situation outlined in this model forms for each of the individual producers a competitive profit maximizing equilibrium. Thus, the programming solution guarantees zero profits, equality of supply and demand for every commodity with non-zero prices, and equality of price and marginal costs for every producer in every commodity he actually produces. Consequently, it is clear that a decentralized decision-making process would lead to the same aggregate production pattern identical to the one which is provided by the solution of the programming model, provided that each producer faces the same set of prices and strives to maximize profits.

In a parallel way, *consumer equilibrium* is equivalent to the problem that each consumer maximises his utility $S_i(x_{ij}, -r_{ih})$ subject to his income constraint. Given this specification, the consumer derives utility from the consumed quantities of the desired commodities and the quantities of the primary factors he retains. When the consumer has an initial endowment of primary commodities, rather than a fixed income, he may be willing to supply his endowment in the competitive market, and then choose a bundle of desired commodities to maximise his preferences in the budget set, defined by the income he receives from his sale of labour plus his profit earnings. Since a producer optimum is attained, the p_j^{o}, w_h^{o} respective v_{ij}^{o} are known constants, and consequently the individual's income is fixed at R_i, where R_i is the maximum income attainable to him evaluated at the equilibrium point. Thus, the i:th consumer's income R_i will be the sum of the values $w_h^{\mathrm{o}} r_{ih}$ of the supplied quantities of r_{ih} and the shares θ_{ij} of the rents v_{ij}^{o} of the producers.[11] Mathematically:

$$\Sigma_j,\, p_j^{\mathrm{o}} x_{ij} \leq \Sigma_h,\, w_h^{\mathrm{o}} r_{ih} + \Sigma_i,\, \Sigma_j,\, \theta_{ij} v_{ij}^{\mathrm{o}} \equiv R_i \tag{2.19}$$

Given that each consumer maximizes his utility $S_i(x_{ij}, -r_{ih})$ subject to his income R_i, we form the Lagrangean:

$$\mathrm{L}\{x_{ij}, -r_i, \lambda_i\} = \lambda_i \left(\Sigma_h,\, w_h^{\mathrm{o}} r_{ih} + \Sigma_i,\, \Sigma_j,\, \theta_{ij} v_{ij}^{\mathrm{o}} - \Sigma_j,\, p_j^{\mathrm{o}} x_{ij}\right) \tag{2.20}$$

$$x_{ij} \geq 0,\; r_i \geq 0,\; \lambda_i > 0$$

A necessary and sufficient condition that $\left\{x_{ik}^{\mathrm{o}}, r_{il}^{\mathrm{o}}, \lambda_i\right\}$ is a non-negative saddle point, is:

$$\frac{\partial L^{\mathrm{o}}}{\partial x_i} = S_i' - \lambda_i p_i^{\mathrm{o}} = 0 \tag{2.21}$$

$$\frac{\partial L^{\mathrm{o}}}{\partial r_i} = -S_i' + \lambda_i w_l^{\mathrm{o}} = 0 \tag{2.22}$$

$$\frac{\partial L^{\mathrm{o}}}{\partial \lambda_i} = \Sigma_h,\, w_h^{\mathrm{o}} r_{ih} + \Sigma_i,\, \Sigma_j,\, \theta_{ij} v_{ij}^{\mathrm{o}} - \Sigma_j,\, p_j^{\mathrm{o}} x_{ij} = 0 \tag{2.23}$$

[11] Following Jaffe (1980),: "When Walras defined his entrepreneur as a fourth per-son, entirely distinct from the landowner, the worker and the capitalist, whose role it is to lease land from the landowner, hire personal faculties from the labourer, and borrow capital from the capitalist, in order to combine the three productive services in agriculture, industry and trade." Thus, then he (Walras) said in a state of equilibrium, *les entrepreneurs ne font ni bénéfices ni pertes'* (entrepreneurs make neither profit nor loss), he did not mean that there are no returns to capital in state of equilibrium, but only that there is nothing left over for the entrepreneur, *qua entrepreneur*, when selling price equal all cost of production including the cost of capital-services for payment is made to capitalists. "See further Jaffe W. and Morishima M. (1980).

In the equations above, S_i' denotes the partial derivatives of S_i with respect to x_{ij} and r_{ih}. The shadow price λ_i is the marginal utility of money, or the marginal utility of income. By the assumption that the utility function is differentiable, the equalities above establish certain classical relations between prices and marginal rates of substitution relating to consumer equilibrium x_{ij}^{o} and r_{ih}^{o}. These equalities imply that the marginal rate of substitution of any pair of commodities is equal to the ratio between any corresponding pair of prices. The condition (condition 2.23), which specifies that each individual spends all of his income to purchase x_j seems to be trivial. However, the consumer efficiency condition does not stipulate that R_i must be equal to the sum of $p_j x_{ij}$' i.e. the expenditures of each household exhaust its income, but from a general competitive equilibrium point of view income and expenditures must balance.[12]

Thus, market equilibrium would be a more precise concept here. If such market equilibrium is consistent with profit maximisation and utility maximisation on the part of each producer and each consumer, then market equilibrium and competitive equilibrium are consistent. Clearly, a competitive equilibrium is a special case of a market equilibrium and the programming problem whose solution if it exists is a competitive equilibrium for the economy stipulated by this model.

2.6 Concluding Remarks

In the equilibrium model presented and discussed so far, competitive behaviour has been specified for all participants, and competitive equilibrium has been taken as the norm. Capital commodities are assumed to be given and sector-specific. By relaxing this restriction the model could be made applicable in a dynamic context. The relationship between optimum theory and competitive equilibrium has been made explicit in this model. The chapter follows a classical approach, first the search for the optimum, and then competitive equilibrium.

However, we treat the aggregate demand and factor supply functions as if they could be generated by a single representative individual. In other words, the central planner is assumed to be the only maximising actor. Theoretically, that conflicts with the market equilibrium price system, where the demand and supply decisions are made separately and independently by various economic actors. Moreover, the demand for commodities and supply of factors are assumed to remain constant no matter what happens to prices. In other words, the shadow prices result as a by-product of the solution as equilibrium prices. Thus, these prices cannot be interpreted as market-clearing prices of general equilibrium theory because

[12] Assuming that each consumer is on his budget constraint, the system as a whole must satisfy Walras's Law, i.e. the value of market demands must equal the value of market endowments at all prices.

endogenous prices and general equilibrium interaction to simulate competitive market behaviour cannot be achieved using this specification.

A technique which removes any of the shortcomings mentioned above will greatly improve the applicability of the model. For this purpose the quadratic programming model, a straightforward extension of the linear programming model, have been developed. That model is presented in the next chapter.

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