Surface Viscoelasticity and Effective Properties of Materials and Structures

Holm Altenbach and Victor A. Eremeyev

Abstract In this paper we discuss the influence of surface viscoelasticity on the effective properties of materials such as effective bending stiffness of plates or shells. Viscoelasticity in the vicinity of the surface can differ from the properties of the bulk material, in general. This difference influences the behavior of nanosized thin elements. In particular, the surface viscoelastic stresses are responsible for the size-depended dissipation of nanosized structures. Extending of the Gurtin-Murdoch model and using the correspondence principle of the linear viscoelasticity we derive the expressions of the stress resultant tensors for shear deformable plates and shells.

1 Introduction

The surface effects play an important role for such nanosized materials as films, nanoporous materials, etc., while in this case the influence of surface is more significant. The mechanics of solids which takes into account explicitly the phenomenon of surface stresses was proposed by Gurtin and Murdoch [1]. Within the framework of the theory of surface stresses an elastic body can be considered as a "usual" elastic body with elastic membrane glued on its surface. Unlike to classical mechanics of materials where the surface stresses can be neglected in most cases, at the micro- and nanoscale the surface stresses play an important role. For example, they influence the effective or apparent properties of very thin specimens and predict the so-called

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size effect, that is dependence of apparent material properties on a specimen size. Hence, the classical continuum mechanics can be extended at the nanoscale taking into account surface stresses acting on the boundary of a nanosized body, see [2, 3]. In the literature are presented various applications of the Gurtin-Murdoch model in nanomechanics, see, for example, the review [4]. In particular, the theory of elasticity with surface stresses is used in the two-dimensional theories of nanosized plates and shells, see [5–13]. Let us note that in most of papers the elastic medium is considered. On the other hand, inelastic behavior analysis is also important in micro- and nanomechanics. Dissipative processes in the vicinity of the surface are related to the higher mobility of molecules, surface imperfections, absorbates, etc., see [14] among others. For the description of surface dissipation of nanosized beams, Ru [15] was proposed one-dimensional constitutive law that is similar to the model of the standard viscoelastic solids but formulated for the two-dimensional surface stresses.

Following [16] in this paper we consider the influence of surface viscoelasticity on the effective or apparent properties of nanosized thin-walled structures. We recall the basic equations of the continuum with surface stresses and use the more general constitutive viscoelastic model for the surface stresses than the proposed by Ru [15]. Using the correspondence principle, we present the governing equations of plates and shells with viscoelastic surface stresses. Here we assume that the bulk material is elastic while the surface has viscoelastic properties. We formulate the two-dimensional (2D) constitutive equations and obtain the 2D relaxation functions for plates and shells. Finally, we compare the proposed model of shells with viscoelastic surface stresses with the model of a sandwich plate with viscoelastic faces.

2 Basic Equations of Linear Elasticity with Viscoelastic Surface Stresses

Let us consider the problem for a deformable body with surface stresses. Let $V \in \mathbb{R}^3$ is the volume of the body with the boundary $\Omega = \partial V$. For quasistatic deformations of solids with surface stresses the boundary-value problem is given by

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{f} = \mathbf{0}, \quad \mathbf{x} \in V, \tag{1}$$

$$\mathbf{u}|_{\Omega_1} = \mathbf{0}, \quad \mathbf{n} \cdot \boldsymbol{\sigma}|_{\Omega_2} = \mathbf{t}, \quad \mathbf{x} \in \Omega, \tag{2}$$

where $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{u}$ the displacement vector, ∇ the 3D gradient operator (3D nabla operator), ρ the density, $\mathbf{f}$ the density of the volume forces, and $\mathbf{n}$ the external unit normal to $\Omega = \Omega_1 \bigcup \Omega_2$, $\Omega_1 \bigcup \Omega_2 = \emptyset$. The surface stress vector $\mathbf{t}$ is expressed through a given load $\boldsymbol{\varphi}$ and the stress vector due the surface stresses $\mathbf{t}_S$ by the formula [1, 2, 17]

$$\mathbf{t} = \boldsymbol{\varphi} + \mathbf{t}_S, \quad \mathbf{t}_S = \nabla_S \cdot \boldsymbol{\tau}.$$

Here τ is the surface stress tensor on Ω, ∇_S is the surface nabla operator on Ω given by $\nabla_S = \nabla - \mathbf{n}\partial/\partial z$, where z is the coordinate along the normal to Ω.

For the sake of simplicity, we restrict ourselves to an isotropic material. We also assume that the bulk material is elastic but the surface stresses are viscoelastic. Hence, we have the Hooke law for the bulk material

$$\boldsymbol{\sigma} = 2\mu\boldsymbol{\varepsilon} + \lambda\mathbf{I}\mathrm{tr}\,\boldsymbol{\varepsilon} \quad \text{with} \quad \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\mathbf{u}) \equiv \frac{1}{2}\big(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\big), \tag{3}$$

where $\boldsymbol{\varepsilon}$ is the strain tensor, λ and μ are Lamé's moduli, and $\mathbf{I}$ is the three-dimensional unit tensor, respectively.

For the surface stresses we assume the following constitutive equation

$$\boldsymbol{\tau} = 2\int\limits_{-\infty}^{t} \mu_S(t-\tau)\dot{\mathbf{e}}(\tau)\,d\tau + \int\limits_{-\infty}^{t} \lambda_S(t-\tau)\mathrm{tr}\,\dot{\mathbf{e}}(\tau)\,d\tau\mathbf{A}, \tag{4}$$

$$\mathbf{e} = \mathbf{e}(\mathbf{v}) \equiv \frac{1}{2}\big(\nabla_S\mathbf{v}\cdot\mathbf{A} + \mathbf{A}\cdot(\nabla_S\mathbf{v})^T\big),$$

where $\mathbf{e}$ is the surface strain tensor, $\mathbf{v}$ the displacement of the surface point $\mathbf{x}$ of Ω_2, $\mathbf{A} \equiv \mathbf{I} - \mathbf{n}\otimes\mathbf{n}$ the two-dimensional unit tensors, the overdot denotes differentiation with respect to time t, and λ_S and μ_S are the relaxation functions of the surface film Ω_2, respectively.

Following [1, 17], we state that the displacements of the surface film Ω_2 coincide with the body displacements on the boundary $\mathbf{v} = \mathbf{u}|_{\Omega_2}$.

The integral constitutive law (4) contains the viscoelastic constitutive equation of [15] as the special case. If μ_S and λ_S are constants then (4) reduces to the elastic constitutive equations used in [2].

The system of Eqs. (1)–(4) constitute the boundary-value problem (BVP) for the elastic body with viscoelastic surface stresses. In what follows we use this BVP to derive two-dimensional (2D) equations of shear-deformable shells.

3 Reduction to the Two-Dimensional Theory

In the literature there are known various approaches of derivation of 2D equations of plates and shells using the reduction procedure of the equations of 3D continuum mechanics. Here we apply to the nonclassical BVP (1)–(4) the through-the-thickness integration procedure described, for example, in [18].

In the case of viscoelastic material we use the correspondence principle which establishes that if an elastic solution of the problem is known, the corresponding viscoelastic solution can be obtained by substituting for the elastic quantities the Laplace transform of the unknown functions [19, 20]. In other words, one can use the solution of BVP for elastic material as the solution of BVP for viscoelastic material but given in terms of Laplace transform. According to this principle we use the results of 3D to 2D reduction procedure for the elastic shell-like body given by [6, 7].

In fact, using the Laplace transform one can write (4) as it follows

$$\overline{\boldsymbol{\tau}} = 2s\overline{\mu}_S(s)\overline{\mathbf{e}} + s\overline{\lambda}_S(s)(\mathrm{tr}\,\overline{\mathbf{e}})\mathbf{A}, \quad \overline{(\ldots)}(s) = \int\limits_0^\infty (\ldots)(t)e^{-st}\,\mathrm{d}t, \tag{5}$$

which coincides formally with the surface Hooke's law assumed in [6, 7].

The through-the-thickness integration procedure applied to shell-like bodies with surface stresses leads to the following 2D equations, see [16],

$$\nabla_S \cdot \mathbf{T} + \mathbf{q} = \mathbf{0}, \quad \nabla_S \cdot \mathbf{M} + \mathbf{T}_\times + \mathbf{m} = \mathbf{0}, \tag{6}$$

where $\mathbf{T}$ is the stress resultant tensor, $\mathbf{M}$ the couple stress tensor, $\mathbf{T}_\times$ denotes the vectorial invariant of second-order tensor $\mathbf{T}$, see [18], $\mathbf{q}$ and $\mathbf{m}$ are the surface force and couple vector fields defined as in [6, 7].

Tensors $\mathbf{T}$ and $\mathbf{M}$ can be represented each as the sums of two terms, see [5–7, 16],

$$\mathbf{T} = \mathbf{T}_b + \mathbf{T}_s, \quad \mathbf{M} = \mathbf{M}_b + \mathbf{M}_s. \tag{7}$$

Here $\mathbf{T}_b$ and $\mathbf{M}_b$ are the stress and couple stress resultant tensors related to the bulk material while $\mathbf{T}_s$ and $\mathbf{M}_s$ are the stress and couple stress resultant tensors related to the surface stresses. With the accuracy of $O(h/R)$ where h is the shell thickness and R is the maximum of the curvature radius of the shell base surface, one can use the following formulae for $\mathbf{T}_b$, $\mathbf{M}_b$, $\mathbf{T}_s$, and $\mathbf{M}_s$

$$\mathbf{T}_b = \langle \mathbf{A} \cdot \boldsymbol{\sigma} \rangle, \quad \mathbf{M}_b = -\langle \mathbf{A} \cdot z\boldsymbol{\sigma} \times \mathbf{n} \rangle, \qquad \langle (\ldots) \rangle = \int\limits_{-h/2}^{h/2} (\ldots)\,\mathrm{d}z, \tag{8}$$

$$\mathbf{T}_s = \boldsymbol{\tau}_+ + \boldsymbol{\tau}_-, \quad \mathbf{M}_s = -\frac{h}{2}(\boldsymbol{\tau}_+ - \boldsymbol{\tau}_-) \times \mathbf{n}, \tag{9}$$

where $\boldsymbol{\tau}_\pm$ are the surface stresses acting at the shell faces, i.e. $\boldsymbol{\tau}_\pm = \boldsymbol{\tau}\big|_{z=\pm h/2}$. Equation (8) result in the following component representations

$$\mathbf{T}_b = T_{\alpha\beta}\boldsymbol{\rho}^\alpha \otimes \boldsymbol{\rho}^\beta + T_{\alpha 3}\boldsymbol{\rho}^\alpha \otimes \mathbf{n}, \quad \mathbf{M}_b = -M_{\alpha\beta}\boldsymbol{\rho}^\alpha \otimes \boldsymbol{\rho}^\beta \times \mathbf{n}, \qquad \alpha, \beta = 1, 2, \tag{10}$$

$$T_{\alpha\beta} = \langle \sigma_{\alpha\beta} \rangle, \quad T_{\alpha 3} = \langle \sigma_{\alpha 3} \rangle, \quad M_{\alpha\beta} = \langle z\sigma_{\alpha\beta} \rangle,$$

where $\sigma_{\alpha\beta} = \boldsymbol{\rho}_\alpha \cdot \boldsymbol{\sigma} \cdot \boldsymbol{\rho}_\beta$, $\sigma_{\alpha 3} = \boldsymbol{\rho}_\alpha \cdot \boldsymbol{\sigma} \cdot \mathbf{n}$, $\boldsymbol{\rho}_\alpha$ and $\boldsymbol{\rho}^\beta$ are the main and reciprocal bases on the shell base surface ω with the unit normal vector $\mathbf{n}$.

In what follows we use the linear approximation of the translation vector $\mathbf{u}$

$$\mathbf{u}(z) = \mathbf{w} - z\boldsymbol{\vartheta}, \quad \mathbf{n}\cdot\boldsymbol{\vartheta} = 0. \tag{11}$$

This approximation is used in the theories of shear-deformable plates and shells, see, e.g., [18], $\mathbf{w}$ is the translation vector of the shell base surface ω and $\boldsymbol{\vartheta}$ is the rotation vector of the shell normal. Both are kinematically independent each other.

For the isotropic shell dependence of $\mathbf{T}_b$ and $\mathbf{M}_b$ on strain measures is given by

$$\mathbf{T}_b = C_1\boldsymbol{\varepsilon} + C_2\mathbf{A}\operatorname{tr}\boldsymbol{\varepsilon} + \varGamma\boldsymbol{\gamma}\otimes\mathbf{n}, \quad \mathbf{M}_b = -\left[D_1\boldsymbol{\kappa} + D_2\mathbf{A}\operatorname{tr}\boldsymbol{\kappa}\right]\times\mathbf{n}, \tag{12}$$
$$C_1 = 2C_{22}, \quad C_2 = C_{11} - C_{22}, \quad D_1 = 2D_{22}, \quad D_2 = D_{33} - D_{22}.$$

where $\boldsymbol{\varepsilon}$, $\boldsymbol{\kappa}$, and $\boldsymbol{\gamma}$ are strain measures introduced by

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla_S\mathbf{w}\cdot\mathbf{A} + \mathbf{A}\cdot(\nabla_S\mathbf{w})^T\right), \quad \boldsymbol{\kappa} = \frac{1}{2}\left(\nabla_S\boldsymbol{\vartheta}\cdot\mathbf{A} + \mathbf{A}\cdot(\nabla_S\boldsymbol{\vartheta})^T\right),$$
$$\boldsymbol{\gamma} = \nabla_S(\mathbf{w}\cdot\mathbf{n}) - \boldsymbol{\vartheta},$$

and the components C_{11}, C_{22}, D_{22}, D_{33}, and $\varGamma$ are given by

$$C_{11} = \frac{Eh}{2(1-\nu)}, \qquad C_{22} = \frac{Eh}{2(1+\nu)},$$
$$D_{22} = \frac{Eh^3}{24(1+\nu)}, \qquad D_{33} = \frac{Eh^3}{24(1-\nu)}, \quad \varGamma = k\mu h,$$
$$E = 2\mu(1+\nu), \qquad \nu = \frac{\lambda}{2(\lambda+\mu)},$$
$$C \equiv C_{11} + C_{22} = \frac{Eh}{1-\nu^2}, \quad D \equiv D_{11} + D_{22} = \frac{Eh^3}{12(1-\nu^2)},$$

where E and ν are the Young modulus and Poisson ratio of bulk material. C and D are the tangential and bending stiffness of the shell, $\varGamma$ is the transverse shear stiffness, and k the transverse shear factor, respectively.

Let us consider the constitutive equations for $\mathbf{T}_s$ and $\mathbf{M}_s$. For simplicity we assume the same viscoelastic behaviour of both shell faces. From (11) it follows the relations

$$\boldsymbol{\tau}_\pm = \int_{-\infty}^{t} \lambda_S(t-\tau)\operatorname{tr}\dot{\boldsymbol{\varepsilon}}(\tau)\,d\tau\mathbf{A} + 2\int_{-\infty}^{t} \mu_S(t-\tau)\dot{\boldsymbol{\varepsilon}}(\tau)\,d\tau$$
$$\mp\frac{h}{2}\left(\int_{-\infty}^{t} \lambda_S(t-\tau)\operatorname{tr}\dot{\boldsymbol{\kappa}}(\tau)\,d\tau\mathbf{A} + \int_{-\infty}^{t} 2\mu_S(t-\tau)\dot{\boldsymbol{\kappa}}(\tau)\,d\tau\right).$$

Finally we have, see [16],

$$\mathbf{T}_s = \int\limits_{-\infty}^{t} \left[C_1^S(t-\tau)\dot{\boldsymbol{\varepsilon}}(\tau) + C_2^S(t-\tau)\mathbf{A}\operatorname{tr}\dot{\boldsymbol{\varepsilon}}(\tau) \right] d\tau, \tag{13}$$

$$\mathbf{M}_s = -\int\limits_{-\infty}^{t} \left[D_1^S(t-\tau)\dot{\boldsymbol{\kappa}}(\tau) + D_2(t-\tau)^S\mathbf{A}\operatorname{tr}\dot{\boldsymbol{\kappa}}(\tau) \right] d\tau \times \mathbf{n}, \tag{14}$$

$$C_1^S = 4\mu_S, \quad C_2^S = 2\lambda_S, \quad D_1^S = h^2\mu_S, \quad D_2^S = h^2\lambda_S/2.$$

As a result from (7), (13), and (14) we derive the constitutive equations of the shell with viscoelastic surface stresses in the form

$$\mathbf{T} = \int\limits_{-\infty}^{t} [C_1(t-\tau)\dot{\boldsymbol{\varepsilon}}(\tau) + C_2(t-\tau)\mathbf{A}\operatorname{tr}\dot{\boldsymbol{\varepsilon}}(\tau)]\, d\tau + \Gamma\boldsymbol{\gamma}\otimes\mathbf{n},$$

$$\mathbf{M} = -\int\limits_{-\infty}^{t} [D_1(t-\tau)\dot{\boldsymbol{\kappa}}(\tau) + D_2(t-\tau)\mathbf{A}\operatorname{tr}\dot{\boldsymbol{\kappa}}(\tau)]\, d\tau \times \mathbf{n},$$

$$C_1(t) = 2C_{22} + 4\mu_S(t), \quad C_2(t) = C_{11} - C_{22} + 2\lambda_S(t),$$

$$D_1(t) = 2D_{22} + h^2\mu_S(t), \quad D_2(t) = D_{33} - D_{22} + \frac{h^2}{2}\lambda_S(t).$$

The tangential and bending relaxation functions are given by

$$C = \frac{Eh}{1-\nu^2} + 4\mu_S + 2\lambda_S, \quad D = \frac{Eh^3}{12(1-\nu^2)} + \frac{h^2}{2}(2\mu_S + \lambda_S). \tag{15}$$

Let us note that the surface stresses do not influence the transverse shear stiffness.

4 Plate with Surface Stresses as Three-Layered Plate

The presented above model of plates and shells with surface stresses is similar to the theories of three-layered plates and shells that are widely presented in the literature, see [7] for the elastic case and [16] for viscoelastic faces. We consider the symmetric three-layered plate (sandwich plate) with the thickness $h = h_c + 2h_f$, where h_c is the thickness of core, h_f the thickness of faces, and $h_c \gg h_f$. We assume that the core is made of elastic material with the Young modulus E or the shear modulus μ, and Poisson ratio ν while the faces are viscoelastic with the relaxation function $E_f(t)$ and the constant Poisson ratio ν_f.

Using the approach suggested in [7, 21], for the viscoelastic sandwich plate we obtain the constitutive equations in the form similar to (13) and (14) but with different expressions for relaxation functions. The tangential and bending relaxation functions of the three-layered plate are given by

$$\widetilde{C} = \widetilde{C}_{11} + \widetilde{C}_{22} = \frac{2E_\mathrm{f}h_\mathrm{f}}{1-\nu_\mathrm{f}^2} + \frac{Eh_\mathrm{c}}{1-\nu^2}, \tag{16}$$

$$\widetilde{D} = \widetilde{D}_{22} + \widetilde{D}_{33} = \frac{1}{12}\left[\frac{E_\mathrm{f}(h^3-h_\mathrm{c}^3)}{1-\nu_\mathrm{f}^2} + \frac{E_\mathrm{c}h_\mathrm{c}^3}{1-\nu_\mathrm{c}^2}\right], \tag{17}$$

$$\widetilde{C}_{11} = \frac{1}{2}\left(\frac{2E_\mathrm{f}h_\mathrm{f}}{1-\nu_\mathrm{f}} + \frac{Eh_\mathrm{c}}{1-\nu}\right), \quad \widetilde{C}_{22} = \frac{1}{2}\left(\frac{2E_\mathrm{f}h_\mathrm{f}}{1+\nu_\mathrm{f}} + \frac{Eh_\mathrm{c}}{1+\nu}\right),$$

$$\widetilde{D}_{22} = \frac{1}{24}\left[\frac{E_\mathrm{f}(h^3-h_\mathrm{c}^3)}{1+\nu_\mathrm{f}} + \frac{Eh_\mathrm{c}^3}{1+\nu}\right], \quad \widetilde{D}_{33} = \frac{1}{24}\left[\frac{E_\mathrm{f}(h^3-h_\mathrm{c}^3)}{1-\nu_\mathrm{f}} + \frac{Eh_\mathrm{c}^3}{1-\nu}\right]$$

Comparing (16) with $(15)_1$ we conclude that the surface relaxation functions λ_S and μ_S can be expressed through the relaxation function of faces E_f, Poisson ratio ν_f, and the thickness h_f. With accuracy of $O(h_\mathrm{f}^2)$ we obtain that

$$\mu_S \approx \frac{E_\mathrm{f}h_\mathrm{f}}{2(1+\nu_\mathrm{f})} \equiv \mu_\mathrm{f}h_\mathrm{f}, \quad \lambda_S \approx \frac{\nu_\mathrm{f}E_\mathrm{f}h_\mathrm{f}}{1-\nu_\mathrm{f}^2} \equiv \lambda_\mathrm{f}h_\mathrm{f}\frac{1-2\nu_\mathrm{f}}{1-\nu_\mathrm{f}}, \tag{18}$$

where λ_f is the second relaxation function of faces. Let us note that the comparison of $(15)_2$ with (17) results in the same formulae. Hence, we get

$$\mu_S = \lim_{h_\mathrm{f}\to 0} \mu_\mathrm{f}h_\mathrm{f}, \quad \lambda_S = \lim_{h_\mathrm{f}\to 0} \lambda_\mathrm{f}\frac{1-2\nu_\mathrm{f}}{1-\nu_\mathrm{f}}h_\mathrm{f}. \tag{19}$$

The latter equations give us the interpretation of the surface viscoelastic functions μ_S and λ_S through the relaxation functions of plate faces and their thickness.

5 Conclusions

Here we discuss the extension of the constitutive relations of elastic thin-walled structures with surface stresses taking into account the surface viscoelasticity. As in the Gurtin-Murdoch model of surface elasticity the linear surface viscoelasticity contains the surface stresses which depend on the surface the prehistory of strains. In the linear isotropic case these dependencies are given by the relation (4). Using the correspondence principle and the through-the-thickness integration technique of reduction of 3D equations to 2D ones we derive the constitutive equations for stress resultants and analyzed the dependence of the effective properties on bulk and surface material behaviour.

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References

1. Gurtin, M.E., Murdoch, A.I.: A continuum theory of elastic material surfaces. Arch. Ration. Mech. Anal. **57**(4), 291–323 (1975)
2. Duan, H.L., Wang, J., Karihaloo, B.L.: Theory of elasticity at the nanoscale. Adv. Appl. Mech. **42**, 1–68 (2008)
3. Wang, J., Duan, H.L., Huang, Z.P., Karihaloo, B.L.: A scaling law for properties of nano-structured materials. Proc. Royal Soc. Lond. A **462**(2069), 1355–1363 (2006)
4. Wang, J., Huang, Z., Duan, H., Yu, S., Feng, X., Wang, G., Zhang, W., Wang, T.: Surface stress effect in mechanics of nanostructured materials. Acta Mechanica Solida Sinica **24**(1), 52–82 (2011)
5. Altenbach, H., Eremeyev, V.A.: On the shell theory on the nanoscale with surface stresses. Int. J. Eng. Sci. **49**(12), 1294–1301 (2011)
6. Altenbach, H., Eremeyev, V.A., Morozov, N.F.: Linear theory of shells taking into account surface stresses. Doklady Phys. **54**(12), 531–535 (2009)
7. Altenbach, H., Eremeyev, V.A., Morozov, N.F.: On equations of the linear theory of shells with surface stresses taken into account. Mech. Solids **45**(3), 331–342 (2010)
8. Dahmen, K., Lehwald, S., Ibach, H.: Bending of crystalline plates under the influence of surface stress—a finite element analysis. Surf. Sci. **446**(1–2), 161–173 (2000)
9. Eremeyev, V.A., Altenbach, H., Morozov, N.F.: The influence of surface tension on the effective stiffness of nanosize plates. Doklady Phys. **54**(2), 98–100 (2009)
10. Huang, D.W.: Size-dependent response of ultra-thin films with surface effects. Int. J. Solids Struct. **45**(2), 568–579 (2008)
11. Lu, P., He, L.H., Lee, H.P., Lu, C.: Thin plate theory including surface effects. Int. J. Solids Struct. **43**(16), 4631–4647 (2006)
12. Wang, Z.Q., Zhao, Y.P.: Self-instabilityand bending behaviors of nano plates. Acta Mechanica Solida Sinica **22**(6), 630–643 (2009)
13. Zhu, H.X., Wang, J.X., Karihaloo, B.: Effects of surface and initial stresses on the bending stiffness of trilayer plates and nanofilms. J. Mech. Mat. Struct. **4**(3), 589–604 (2009)
14. Seoánez, C., Guinea, F., Castro Neto, A.H.: Surface dissipation in nanoelectromechanical systems: unified description with the standard tunneling model and effects of metallic electrodes. Phys. Rev. B **77**(12), 125107 (2008)
15. Ru, C.Q.: Size effect of dissipative surface stress on quality factor of microbeams. Appl. Phys. Lett. **94**, 051905-1–051905-3 (2009)
16. Altenbach, H., Eremeyev, V.A., Morozov, N.F.: Surface viscoelasticity and effective properties of thin-walled structures at the nanoscale. Int. J. Eng. Sci. **59**, 83–89 (2012)
17. Povstenko, Y.Z.: Theoretical investigation of phenomena caused by heterogeneous surface tension in solids. J. Mech. Phys. Solids **41**(9), 1499–1514 (1993)
18. Lebedev, L.P., Cloud, M.J., Eremeyev, V.A.: Tensor Analysis with Applications in Mechanics. World Scientific, New Jersey (2010)
19. Christensen, R.M.: Theory of Viscoelasticity. An Introduction. Academic Press, New York (1971)
20. Tschoegl, N.W.: The Phenomenological Theory of Linear Viscoelastic Behavior. An Introduction. Springer, Berlin (1989)
21. Altenbach, H., Eremeyev, V.A.: On the bending of viscoelastic plates made of polymer foams. Acta Mechanica **204**(3–4), 137–154 (2009)

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