Contents

1 Runge–Kutta (–Nyström) Methods for Oscillatory Differential Equations 1
1.1 RK Methods, Rooted Trees, B-Series and Order Conditions 1
1.2 RKN Methods, Nyström Trees and Order Conditions 8
1.2.1 Formulation of the Scheme 8
1.2.2 Nyström Trees and Order Conditions 9
1.2.3 The Special Case in Absence of the Derivative 18
1.3 Dispersion and Dissipation of RK(N) Methods 19
1.3.1 RK Methods 20
1.3.2 RKN Methods 21
1.4 Symplectic Methods for Hamiltonian Systems 22
1.5 Comments on Structure-Preserving Algorithms for Oscillatory Problems 23
References 24

2 ARKN Methods 27
2.1 Traditional ARKN Methods 27
2.1.1 Formulation of the Scheme 28
2.1.2 Order Conditions 29
2.2 Symplectic ARKN Methods 32
2.2.1 Symplecticity Conditions for ARKN Integrators 33
2.2.2 Existence of Symplectic ARKN Integrators 37
2.2.3 Phase and Stability Properties of Method SARKN1s2 . . . 41
2.2.4 Nonexistence of Symmetric ARKN Methods 43
2.2.5 Numerical Experiments 44
2.3 Multidimensional ARKN Methods 48
2.3.1 Formulation of the Scheme 48
2.3.2 Order Conditions 52
2.3.3 Practical Multidimensional ARKN Methods 57
References 60

3 ERKN Methods . . . 63
3.1 ERKN Methods . . . 63
3.1.1 Formulation of Multidimensional ERKN Methods . . . 64
3.1.2 Special Extended Nyström Tree Theory . . . 65
3.1.3 Order Conditions . . . 73
3.2 EFRKN Methods and ERKN Methods . . . 76
3.2.1 One-Dimensional Case . . . 77
3.2.2 Multidimensional Case . . . 80
3.3 ERKN Methods for Second-Order Systems with Variable Principal Frequency Matrix . . . 82
3.3.1 Analysis Through an Equivalent System . . . 82
3.3.2 Towards ERKN Methods . . . 83
3.3.3 Numerical Illustrations . . . 86
References . . . 88

4 Symplectic and Symmetric Multidimensional ERKN Methods . . . 91
4.1 Symplecticity and Symmetry Conditions for Multidimensional ERKN Integrators . . . 91
4.1.1 Symmetry Conditions . . . 92
4.1.2 Symplecticity Conditions . . . 94
4.2 Construction of Explicit SSMERKN Integrators . . . 99
4.2.1 Two Two-Stage SSMERKN Integrators of Order Two . . . 99
4.2.2 A Three-Stage SSMERKN Integrator of Order Four . . . 101
4.2.3 Stability and Phase Properties of SSMERKN Integrators . . . 103
4.3 Numerical Experiments . . . 105
4.4 ERKN Methods for Long-Term Integration of Orbital Problems . . . 111
4.5 Symplectic ERKN Methods for Time-Dependent Second-Order Systems . . . 111
4.5.1 Equivalent Extended Autonomous Systems for Non-autonomous Systems . . . 112
4.5.2 Symplectic ERKN Methods for Time-Dependent Hamiltonian Systems . . . 113
4.6 Concluding Remarks . . . 115
References . . . 117

5 Two-Step Multidimensional ERKN Methods . . . 121
5.1 The Scheifele Two-Step Methods . . . 121
5.2 Formulation of TSERKN Methods . . . 124
5.3 Order Conditions . . . 127
5.3.1 B-Series on SENT . . . 127
5.3.2 One-Step Formulation . . . 132
5.3.3 Order Conditions . . . 133
5.4 Construction of Explicit TSERKN Methods . . . 136
5.4.1 A Method with Two Function Evaluations per Step . . . 136
5.4.2 Methods with Three Function Evaluations per Step . . . 138
5.5 Stability and Phase Properties of the TSERKN Methods . . . 142

5.6 Numerical Experiments . . . 145
References . . . 148

6 Adapted Falkner-Type Methods . . . 151
6.1 Falkner's Methods . . . 151
6.2 Formulation of the Adapted Falkner-Type Methods . . . 152
6.3 Error Analysis . . . 156
6.4 Stability . . . 161
6.5 Numerical Experiments . . . 166
Appendix A Derivation of Generating Functions (6.14) and (6.15) . . . 169
Appendix B Proof of (6.24) . . . 170
References . . . 171

7 Energy-Preserving ERKN Methods . . . 173
7.1 The Average-Vector-Field Method . . . 173
7.2 Energy-Preserving ERKN Methods . . . 175
7.2.1 Formulation of the AAVF methods . . . 175
7.2.2 A Highly Accurate Energy-Preserving Integrator . . . 176
7.2.3 Two Properties of the Integrator AAVF-GL . . . 178
7.3 Numerical Experiment on the Fermi–Pasta–Ulam Problem . . . 179
References . . . 183

8 Effective Methods for Highly Oscillatory Second-Order Nonlinear Differential Equations . . . 185
8.1 Numerical Consideration of Highly Oscillatory Second-Order Differential Equations . . . 185
8.2 The Asymptotic Method for Linear Systems . . . 187
8.3 Waveform Relaxation (WR) Methods for Nonlinear Systems . . . 190
References . . . 195

9 Extended Leap-Frog Methods for Hamiltonian Wave Equations . . . 197
9.1 Conservation Laws and Multi-Symplectic Structures of Wave Equations . . . 197
9.1.1 Multi-Symplectic Conservation Laws . . . 197
9.1.2 Conservation Laws for Wave Equations . . . 199
9.2 ERKN Discretization of Wave Equations . . . 200
9.2.1 Multi-Symplectic Integrators . . . 200
9.2.2 Multi-Symplectic Extended RKN Discretization . . . 200
9.3 Explicit Extended Leap-Frog Methods . . . 208
9.3.1 Eleap-Frog I: An Explicit Multi-Symplectic ERKN Scheme . . . 208
9.3.2 Eleap-Frog II: An Explicit Multi-Symplectic ERKN-PRK Scheme . . . 215
9.3.3 Analysis of Linear Stability . . . 215
9.4 Numerical Experiments . . . 217
9.4.1 The Conservation Laws and the Solution . . . 217
9.4.2 Dispersion Analysis . . . 224
References . . . 229

Appendix First and Second Symposiums on Structure-Preserving Algorithms for Differential Equations, August 2011, June 2012, Nanjing 231

Index 233

http://www.springer.com/978-3-642-35337-6

Structure-Preserving Algorithms for Oscillatory Differential Equations

Wu, X.; You, X.; Wang, B.

2013, XII, 236 p. 40 illus., 2 illus. in color., Hardcover
ISBN: 978-3-642-35337-6