

Contents

Part I Basic Techniques and Models

1	Notions of Mathematical Finance	3
1.1	Financial Modelling	3
1.2	Stochastic Processes	5
1.3	Further Reading	8
2	Elements of Numerical Methods for PDEs	11
2.1	Function Spaces	11
2.2	Partial Differential Equations	12
2.3	Numerical Methods for the Heat Equation	15
2.3.1	Finite Difference Method	15
2.3.2	Convergence of the Finite Difference Method	17
2.3.3	Finite Element Method	20
2.4	Further Reading	25
3	Finite Element Methods for Parabolic Problems	27
3.1	Sobolev Spaces	27
3.2	Variational Parabolic Framework	31
3.3	Discretization	33
3.4	Implementation of the Matrix Form	34
3.4.1	Elemental Forms and Assembly	35
3.4.2	Initial Data	38
3.5	Stability of the θ -Scheme	39
3.6	Error Estimates	41
3.6.1	Finite Element Interpolation	41
3.6.2	Convergence of the Finite Element Method	43
3.7	Further Reading	45
4	European Options in BS Markets	47
4.1	Black–Scholes Equation	47
4.2	Variational Formulation	51

4.3	Localization	52
4.4	Discretization	54
4.4.1	Finite Difference Discretization	54
4.4.2	Finite Element Discretization	54
4.4.3	Non-smooth Initial Data	55
4.5	Extensions of the Black–Scholes Model	58
4.5.1	CEV Model	58
4.5.2	Local Volatility Models	62
4.6	Further Reading	64
5	American Options	65
5.1	Optimal Stopping Problem	65
5.2	Variational Formulation	67
5.3	Discretization	68
5.3.1	Finite Difference Discretization	68
5.3.2	Finite Element Discretization	69
5.4	Numerical Solution of Linear Complementarity Problems	70
5.4.1	Projected Successive Overrelaxation Method	71
5.4.2	Primal–Dual Active Set Algorithm	72
5.5	Further Reading	74
6	Exotic Options	75
6.1	Barrier Options	75
6.2	Asian Options	77
6.3	Compound Options	79
6.4	Swing Options	82
6.5	Further Reading	84
7	Interest Rate Models	85
7.1	Pricing Equation	85
7.2	Interest Rate Derivatives	87
7.3	Further Reading	90
8	Multi-asset Options	91
8.1	Pricing Equation	91
8.2	Variational Formulation	93
8.3	Localization	95
8.4	Discretization	96
8.4.1	Finite Difference Discretization	96
8.4.2	Finite Element Discretization	98
8.5	Further Reading	102
9	Stochastic Volatility Models	105
9.1	Market Models	105
9.1.1	Heston Model	106
9.1.2	Multi-scale Model	106
9.2	Pricing Equation	108
9.3	Variational Formulation	110

9.4	Localization	113
9.5	Discretization	114
9.5.1	Finite Difference Discretization	115
9.5.2	Finite Element Discretization	116
9.6	American Options	119
9.7	Further Reading	122
10	Lévy Models	123
10.1	Lévy Processes	123
10.2	Lévy Models	126
10.2.1	Jump–Diffusion Models	126
10.2.2	Pure Jump Models	127
10.2.3	Admissible Market Models	128
10.3	Pricing Equation	128
10.4	Variational Formulation	131
10.5	Localization	134
10.6	Discretization	135
10.6.1	Finite Difference Discretization	136
10.6.2	Finite Element Discretization	137
10.7	American Options Under Exponential Lévy Models	140
10.8	Further Reading	143
11	Sensitivities and Greeks	145
11.1	Option Pricing	145
11.2	Sensitivity Analysis	147
11.2.1	Sensitivity with Respect to Model Parameters	147
11.2.2	Sensitivity with Respect to Solution Arguments	151
11.3	Numerical Examples	152
11.3.1	One-Dimensional Models	153
11.3.2	Multivariate Models	154
11.4	Further Reading	155
Part II Advanced Techniques and Models		
12	Wavelet Methods	159
12.1	Spline Wavelets	160
12.1.1	Wavelet Transformation	161
12.1.2	Norm Equivalences	162
12.2	Wavelet Discretization	163
12.2.1	Space Discretization	164
12.2.2	Matrix Compression	165
12.2.3	Multilevel Preconditioning	167
12.3	Discontinuous Galerkin Time Discretization	168
12.3.1	Derivation of the Linear Systems	171
12.3.2	Solution Algorithm	172
12.4	Further Reading	175

13	Multidimensional Diffusion Models	177
13.1	Sparse Tensor Product Finite Element Spaces	178
13.2	Sparse Wavelet Discretization	181
13.3	Fully Discrete Scheme	184
13.4	Diffusion Models	185
13.4.1	Aggregated Black–Scholes Models	186
13.4.2	Stochastic Volatility Models	189
13.5	Numerical Examples	191
13.5.1	Full-Rank d -Dimensional Black–Scholes Model	191
13.5.2	Low-Rank \tilde{d} -Dimensional Black–Scholes	192
13.6	Further Reading	195
14	Multidimensional Lévy Models	197
14.1	Lévy Processes	197
14.2	Lévy Copulas	198
14.3	Lévy Models	205
14.3.1	Subordinated Brownian Motion	206
14.3.2	Lévy Copula Models	208
14.3.3	Admissible Models	209
14.4	Pricing Equation	211
14.5	Variational Formulation	212
14.6	Wavelet Discretization	213
14.6.1	Wavelet Compression	215
14.6.2	Fully Discrete Scheme	217
14.7	Application: Impact of Approximations of Small Jumps	218
14.7.1	Gaussian Approximation	218
14.7.2	Basket Options	222
14.7.3	Barrier Options	226
14.8	Further Reading	228
15	Stochastic Volatility Models with Jumps	229
15.1	Market Models	229
15.1.1	Bates Models	230
15.1.2	BNS Model	231
15.2	Pricing Equations	231
15.3	Variational Formulation	234
15.4	Wavelet Discretization	238
15.5	Further Reading	244
16	Multidimensional Feller Processes	247
16.1	Pseudodifferential Operators	247
16.2	Variable Order Sobolev Spaces	250
16.3	Subordination	253
16.4	Admissible Market Models	256
16.5	Variational Formulation	259
16.5.1	Sector Condition	259
16.5.2	Well-Posedness	260

16.6 Numerical Examples	262
16.7 Further Reading	267
Appendix A Elliptic Variational Inequalities	269
A.1 Hilbert Spaces	269
A.2 Dual of a Hilbert Space	271
A.3 Theorems of Stampacchia and Lax–Milgram	273
Appendix B Parabolic Variational Inequalities	275
B.1 Weak Formulation of PVI's	275
B.2 Existence	277
B.3 Proof of the Existence Result	278
Index	297

Computational Methods for Quantitative Finance

Finite Element Methods for Derivative Pricing

Hilber, N.; Reichmann, O.; Schwab, C.; Winter, C.

2013, XIII, 299 p. 56 illus., 47 illus. in color., Hardcover

ISBN: 978-3-642-35400-7