Chapter 2
Philosophical Motivations

This introductory chapter will deal with what the main interpretational problems of canonical quantum theory are.

In particular, we will analyse how the mathematical formalism of quantum theory leads to a non-realist interpretation of the theory. The focus will lie on understanding and analysing the Kochen-Specker theorem (K-S theorem), which can be thought of as the main mathematical underlying reason why quantum theory is non realist. The interpretation which comes out is the well-known Copenhagen interpretation of quantum theory, which is an instrumentalist interpretation. However, such an interpretation leads to many conceptual problems.

Topos quantum theory is a way of overcoming such problems by re-defining quantum theory in the novel language of topos theory. The advantage of this language is that it renders the theory more realist, thus solving the above mentioned problems. In the process, however, one ends up with a multivalued/intuitionistic logic rather than a Boolean logic.

2.1 What Is a Theory of Physics and What Is It Trying to Achieve?

A theory of physics can be seen as a mathematical model which tries to answer three of the fundamental questions humanity has been and still is struggling to answer:

I What is a thing? (Heidegger)
II How are "things" related to one another?
III How do we get to know (1) and (2)?

The first two questions are related to ontological[1] issues while the third is of an epistemological[2] nature.

[1] Ontology comes from the Greek word meaning "being, that which is" and indicates the study of what things are in themselves and what can be said to exist.

[2] Epistemology comes from the Greek word meaning "knowledge" and it is the study concerning what is knowledge and how do we gain knowledge.

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The two main physics theories which presuppose to answer the above questions are

- Classical physics.
- Quantum theory.

The way in which these two theories have answered the above questions is by defining a mathematical model which is supposed to describe nature. The interpretation of this mathematical model then, in turn, gives rise to a philosophical view of the world which provides an answer to the question. In classical physics, the mathematical model developed is in accordance with our common believes about the world. In fact it is arguable whether our common believes modelled classical theory, but we will not delve into this issue. On the other hand, things are not so straightforward in quantum theory in which, as we will see is Sect. 2.3, the mathematical formalism of the theory seems, at time, to defy our common sense.

In any case, in order to fully understand how a philosophical picture of the world can be derived through the mathematical formulation of a theory of physics, we need to refine the questions 1–3 defined at the start of this section (see [21] for an in depth discussion). In particular, any theory of physics, worthy of that name, should address the following issues:

1. What is the system under investigation?
2. What is the ontological status of physical terms?
3. What is the epistemological status of physical terms?
4. How physical statements can be verified or falsified?
5. What is the relation between the mathematical model and the physical world?

As we will see, the different answers given to the above issues by classical theory and quantum theory, respectively, will highlight the radical differences between the two theories and the different interpretations which each of them gives to the "outside world".

Thus, summarising, the main idea exposed in this section is that the mathematical tools used to describe a physical system encode a philosophical position regarding the world.

In the following subsection, we will briefly analyse how questions 1–5 are dealt with in classical physics.

2.2 Philosophical Position of Classical Theory

The philosophical position of classical physics is that of a *realist theory*. Here, by *realist theory* we mean a theory in which

(i) properties can be ascribed to a system at any given time and do not depend on the act of measuring;

(ii) the underlying logic is Boolean (classical) logic[3] which is the same logic we employ in our language.

The realism of classical theory implies that a *thing* is defined[4] in terms of a bundle of properties, which are said to belong to the *thing* (system). The type of properties that we are dealing with are of two kinds:

1. *Internal properties* which belong exclusively to the system, for example the mass, charge etc.
2. *External properties* which define relations to other systems, for example position, velocity etc.

When defining what a thing is one usually considers internal properties. Thus, in a realist theory, the outside world (physical world) exists independently of us and physical terms represent[5] actual things existing 'out there '. This answers questions (1) and (2) of Sect. 2.1.

Regarding the epistemological question (3) and (4) of Sect. 2.1 one can say that in classical physics, knowledge is acquired through the process of measurement. In fact, *measurement* enables us to know the values of a given system. However, in this context, measurement is just another form of interaction, i.e. no special role is ascribed to measurement since both object and subject, as viewed from a classical perspective, exist out there independently of one another.

Finally it is worth mentioning that, generally, classical theory is thought of as being a deterministic theory,[6] i.e. given initial state of a system at a given time, it is possible to predict with certainty the state of the system at a subsequent time.

From the above discussion it emerges that classical theory provides a mathematical model which accurately describes the 'outside' world. Thus one says that there is a bijective correspondence between the 'outside' world (physical world) and the mathematical model put forward by classical theory.

We have thus answered all questions (1–5) of Sect. 2.1 for classical theory. We would now like to identify which mathematical constructs (and in particular the way in which they are defined) render classical theory a realist theory.

The answer to this question will be given in detail in subsequent Chapters, but for now we will restrict ourselves tn answering it in a very conceptual way, so as to give a general idea of the relation between mathematical constructs and induced philosophical ideas.

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[3]Boolean logic will be described later on in the book. For now we will simply say that a Boolean logic is the logic we use in our every-day thinking and in our language. Such a logic is characterised by the fact that (i) it is distributive, (ii) it only has two truth values {*true*, *false*} and (iii) the logical connectives are our linguistic logical connectives: "and", "exclusive or", "not", "if then".

[4]It is worth noting that our own language reflects a realist view of the world: "The tree *is* three meters tall".

[5]Care should be taken at this point since, quite often, physical terms only represent idealisations of real physical systems.

[6]In a stochastic approach the realist conditions (i) and (ii) at the beginning of the section still hold.

The elements/concepts whose mathematical description render classical theory a realist theory are the following:

1. *State space*. In classical physics, the state space S is defined to be the collection of all states $s_i \in S$ of the system such that each s_i at a given time t_i, embodies all the properties of the system at that time.
2. *Definition of physical quantities*. A physical quantity in classical physics is identified with the collection of values it can have for a given system. This is a consequence of the fact that in classical physics quantities do have values and are characterised by these values.
3. *Definition of propositions*. The type of propositions dealt with in classical physics are of the form "$A \in \Delta$", meaning "the physical quantity A takes its values in the interval $\Delta \subseteq \mathbb{R}$". In classical physics such a proposition is identified with the collection of states $s_i \in S$ for which the quantity A does have values which lie in the interval Δ. Hence "$A \in \Delta$" is identified with a subset of the state space, precisely that subset for which the proposition is true.
4. *Boolean logic*. The logic governing classical propositions is Boolean logic which is a distributive logic and admits only two truth values: $\{true, false\}$. Verification of such truth values is done through the measurement interaction.
5. *Probabilities*. Epistemic interpretation of probabilities which, in a sense, represent what we know of the system. In this view the discrepancy between our knowledge and the actual state of the system can be made arbitrarily small by refining either our measurements or the description of the system.

Later in the book we will describe, in details, how the above classical concepts are mathematically represented but, for now, it suffices to say that it is precisely the way in which the above elements of the theory are mathematically expressed which renders classical theory a realist theory. In fact, when considering quantum theory we will see how the same elements are mathematically described in a very different way. This will induce a different conceptual understanding of such elements which, in turn, will imply a different philosophical interpretation of the theory.

2.3 Philosophy Behind Quantum Theory

If we analyse the mathematical formalism of quantum theory, we immediately realise that the theory is non-realist (with the definition of realist given above). In fact the above conditions [(i), (ii)] at the start of Sect. 2.2 do not strictly hold[7] in quantum theory. In fact the formalism of quantum theory implies a clear distinction between measuring apparatus and measuring system, such that the act of measuring gets ascribed a special status. In this setting, measurement becomes a means for assigning a probabilistic spread of outcomes rather than a means to determine properties of the system. Thus the very concept of properties ceases to have its common sense

[7] We will clarify this later on in the book.

meaning, since its definition is now intertwined with the act of measurement. It is as if properties acquire the status of latent attributes which are brought into existence by the act of measurement, but which can not be said to exist independently of such measurement. Therefore it becomes meaningless to talk about a physical system as possessing properties. The interpretation that results is the so called *instrumentalist interpretation* of quantum theory which is a non-realist interpretation [21]. This interpretation of quantum theory makes it very difficult to give answers to questions (1–5) of Sect. 2.1. The reason being that in quantum theory (as it has been expressed so far) there is no distinction between ontological and epistemological status of physical terms. In fact, very roughly, one can say that quantum theory is a mathematical model which, in a way, gives rise to "things" through the measuring process. Thus a "thing" becomes simply a result of a measurement which, as such, only describes what we assume exists 'out there'. In turn, physical statements seem only to represent our knowledge of events rather than events themselves. Thus the philosophical position of quantum theory can be summarised by the following statements:

1. Properties can not be said to be possessed by a system a priori. All that can be said is that after a measurement is performed, the system "acquires" the "latent" properties (state-vector reduction).
2. Any statement regarding 'states of affairs' about a system can only be made a posteriori after measurement. However such statements can not be regarded as describing properties of the system, on the contrary, they describe probabilities of possible measurement outcomes.
3. Measurement becomes a very special type of interaction.
4. Clear distinction between observer and observed system. By this we mean a qualitative distinction, not merely a quantitate one. In fact it is true that in any branch of physics which requires experiments, there is *ipso fatto* a distinction between the physics and the system they are studying. However in a classical regime this distinction is temporary and quantitative, since classical physics is governing both observer and observed system. However in the quantum case the distinction is qualitative since the two systems undergo different *types* of physics.
5. States are not seen as bearers of physical properties but are simply the most efficient tools to enable one to determine/compute predictions for possible measurements, i.e. predictions of probabilities of outcomes, not outcomes themselves.
6. Quantum theory is deterministic *but* what evolves are now predicted probabilities of measurement results, *not* actual measurements.
7. Relative frequency interpretation of probabilities. In this interpretation certain uncertainties can not be eliminated by refinements of our measurement.

The above features of quantum theory, which directly derive from the mathematical representation of the theory, imply a non realist interpretation of quantum theory.

Such an interpretation, although it works for some situations, causes various conceptual problems in the context of quantum gravity and quantum cosmology.

2.4 Conceptual Problems of Quantum Theory

The canonical mathematical formulation of quantum theory leads to an interpretation which has many conceptual obstacles for a fully coherent theory. These problems can be summarised as follows:

- Notions of 'measurement' and 'external observer' pose problems when dealing with cosmology. In fact, in this case there can be no external observer since we are dealing with a closed system. But this then implies that the concept of 'measurement' plays no fundamental role which, in turn, implies that the standard definition of probabilities, in terms of relative frequency of measurements, breaks down.
- The existence of the Planck scale suggests that there is no *a priori* justification for the adoption of the notion of a continuum in the quantum theory used in formulating quantum gravity.
- Standard quantum theory employs, in its formulation, the use of a fixed spatio-temporal structure (fixed background). This is needed to make sense of its instrumentalist interpretation, i.e. it needs a space-time in which to make a measurement. This fixed background seems to cause problems in quantum gravity where one is trying to make measurements of space-time properties. In fact, if the action of making a measurement requires a space time background, what does it mean to measure space time properties?
- Given the concept of superposition present in quantum theory, by applying such a concept to quantum gravity we would have to account for the occurrence of quantum superpositions of eigenstate properties of space, time and space-time.

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