Chapter 2
Selected Topics in Revenue Management

This chapter introduces the theoretical basis of revenue management drawn upon in this study. It starts with a brief review on the historical development of revenue management, which is useful to understand the core ideas of the concept (see Sect. 2.2). Afterwards, the common characteristic of the industries having successful revenue management applications are discussed. Finally, a large part of this chapter is devoted to the capacity control approaches developed for single-leg and network revenue management problems faced by the airline industry.

2.1 Origins of Revenue Management

Revenue management (also called *yield management*) is a concept which emerged following the deregulation of the U.S. airline industry in the late 1970s. Prior to the deregulation, the services of the U.S. airline industry were regulated by the Civil Aeronautics Board (CAB). The fares of the airline products and the entrance into the new markets were under the strict control of the CAB. For example, a new airline was not allowed to enter the market or the existing airlines could not enter each others' networks, if it would harm the financial interests of an existing carrier (see Kole and Lehn 1999). With the intention of increasing the competition in the market, Congress passed the Airline Deregulation Act in 1978, which removed the governmental control over fares, routes and market entry. The deregulation has allowed the airlines to freely set the prices for their products and design their routes. Its further effects can be read in Morrison and Winston (1986), Bailey (1986) and U.S. GAO (1996).

A major consequence of the deregulation is the entrance of new low-cost airlines into the market, which were concentrated on simpler services with low operating costs (see Talluri and van Ryzin 2004b, Sect. 1.2.1). Hence, in comparison to the major airlines existing in the market, the low-cost carriers were able to charge lower fares for their products and better utilize the flight capacity. Phillips (2005),

D. Çetiner, *Fair Revenue Sharing Mechanisms for Strategic Passenger Airline Alliances*, Lecture Notes in Economics and Mathematical Systems 668,
DOI 10.1007/978-3-642-35822-7_2, © Springer-Verlag Berlin Heidelberg 2013

Sect. 6.1, states that People Express was one of the first low-cost airlines that entered the market. The substantial growth of People Express due to the low-cost strategy has posed a challenge for American Airlines, which was a major carrier in the market. To compete against the new successful entrant, American Airlines started to offer low fares, which stipulated some conditions like advance purchase requirements or a minimum length of stay. With this strategy, American Airlines was able to segment the market between leisure and business travelers and profit from the fact that these two segments have different willingness-to-pay. By offering low-fare tickets to leisure travelers, American Airlines was able to catch more customers while avoiding any loss in the revenue coming from the less price-sensitive business travelers having a higher willingness-to-pay.

Market segmentation is defined as "the process of subdividing a market into distinct subsets of customers that behave in the same way or have similar needs" (e.g., Zhang 2011). It is a key element in successful revenue management practices. Cleophas et al. (2011) present principles used for market segmentation in different industries. Zhang (2011) summarizes these in three categories: purchase pattern, product characteristics and customer characteristics. In the airline industry, customers are usually differentiated according to their time of purchase. The leisure customers tend to book further in advance of the booking horizon, whereas business travelers usually reserve later. Additionally, the product characteristics are varied to achieve segmentation. For instance, people who are obliged to change their schedules more frequently may not always be certain about their journeys. In such a case, passengers with a higher willingness-to-pay would prefer to purchase the ticket at a higher price for a refund opportunity allowing to cancel the reservation without any penalty. In the terminology of revenue management, such restrictions and conditions put on the products are described as fences whose function is to limit switching of the customers between different market segments (see Hanks et al. 1992; Kimes et al. 1998). For instance, by putting refund penalties or a minimum length of stay condition on low-fare tickets, airlines try to avoid that a customer who is ready to pay more prefers to buy the lower-fare ticket.

As it is given in Smith et al. (1992), through the implementation of a successful revenue management system, American Airlines has generated an estimated benefit of 1.4 billion dollars over a 3-year period starting 1988. Talluri and van Ryzin (2004b), p. 10, note that, pursuing revenue management strategies, airlines can create additional revenue constituting 4–5 % of their overall revenue.

The above mentioned advances at American Airlines establish the origin of the revenue management, which later attracted the attention of the other airlines all over the world as well as of other industries. According to Chiang et al. (2007) and Kimms and Klein (2005), beside the airline industry, revenue management policies are primarily practiced by hotel (see e.g., Bitran and Mondschein 1995; Bitran and Gilbert 1996; Badinelli 2000; Vinod 2004) and rental car (see e.g., Carroll and Grimes 1995; Geraghty and Johnson 1997; Steinhardt and Gönsch 2009) industries. Following these two, several other industries have adopted revenue management strategies including: railway (see e.g., Ciancimino et al. 1999), air cargo (see e.g., Kasilingam 1996; Amaruchkul 2007), manufacturing, (see e.g., Spengler et al. 2007;

Volling et al. 2012), media and broadcasting (see Kimms and Müller-Bungart 2007) and entertainment (see e.g., Drake 2008). The characteristics of these sectors that are relevant to revenue management are discussed in Talluri and van Ryzin (2004b), Chap. 10, and Cleophas et al. (2011). Additionally, Kimms and Klein (2005) provide mathematical models to solve revenue management problems encountered in several industries. Apart from the referred studies, the survey of Chiang et al. (2007) contains a detailed list of the industries practicing revenue management with relevant publications. Furthermore, Graf (2011) reviews the research on revenue management in the sectors, where strategic alliances are observed. However, there is no study considering the revenue management problem within alliances existing in any of these sectors except from airlines.

2.2 Revenue Management Core Ideas

Being familiar with the idea of increasing revenues through market segmentation, in this section, we define revenue management and introduce approaches developed for implementing it. As stated by Kimms and Klein (2005), there exist many definitions of revenue management in the literature, which are usually differentiated according to the area of application or the approach used. Kimms and Klein (2005) give some of these definitions and discuss their strengths and limitations in revealing the revenue management strategies of an industry. One more general definition of revenue management is given in Kimes (2000) as:

> Yield management is the application of information systems and pricing strategies to allocate the right capacity to the right customer at the right place at the right time.

In line with the above definition, airlines offer a flight to different customer segments at different prices. Each of the defined prices is referred to as a specific fare class of the flight. Talluri and van Ryzin (2004b), Sect. 1.6.1, differentiate between two approaches through which revenue management is realized: quantity-based and price-based. The former refers to the capacity control policies, whereas the latter includes the dynamic pricing strategies and auctions. In capacity control, the management of the capacity is the instrument used for maximizing the revenue. It is based on the principle of offering multiple products having different prices at the same time and changing the availability of the products for sale through capacity allocation decisions. In dynamic pricing, on the other hand, the price of the products is the control variable. Usually, a single product is offered and the price of it is modified during the selling horizon to control the demand and thereby the usage of the capacity. Auctions are regarded as a tool used for dynamic price changes and the literature on auctions within the context of revenue management is rare. We refer to Chiang et al. (2007) for several references and discuss briefly the relation between dynamic pricing and capacity control. In fact, these two approaches cannot be separated with clear boundaries and should be considered jointly (see e.g., Chiang et al. 2007). However, Talluri and van Ryzin (2004b), p. 176, specify some general characteristics of the market making the use of one approach more advantageous than another. Accordingly, dynamic pricing is preferred, if it is more flexible to adjust the prices, whereas capacity control is adopted in industries, where changing the supply is more flexible. However, there exist sectors enjoying the characteristics suitable for both of the approaches (see Müller-Bungart 2006). Considering the airline industry, for example, major airline companies usually commit prices in advance of the booking horizon, which limits the use of prices as a revenue management tool. Besides, the seats (belonging to the same cabin) reserved for different products are homogeneous in terms of the service quality such that they can easily be reallocated between different customer segments or flights. Therefore, most of the airlines use a capacity control policy. There are, however, some low-cost carriers using a dynamic pricing strategy (see Talluri and van Ryzin 2004b). In capacity control applications, airlines offer different products with predetermined prices and decide which products to make available for sale during the booking horizon in order to maximize the revenue. A product is closed to sale, if it is not profitable to allocate any seat capacity for it, in which case the customers who are willing to pay for that product are not allowed to buy tickets. On the other hand, Elmaghraby and Keskinocak (2003) highlight the growing interest for dynamic pricing applications in the retail industry. Due to the developments in facilitating price adjustments, the industry, concerned with seasonal products, practices revenue management through discounts and promotions that are typical forms of dynamic pricing (see also Talluri and van Ryzin 2004b).

This study handles the revenue sharing problem of passenger airline alliances, where capacity control instruments are used to maximize the revenue. Therefore, we do not discuss the dynamic pricing models. Extensive literature on these is provided in Bitran and Caldentey (2003) and Elmaghraby and Keskinocak (2003).

The main challenge of capacity control policies carried out by the airline industry is to determine the number of seats to be allocated to each product/segment dynamically during the reservation horizon so as to maximize the expected revenue. However, due to the characteristics of the industry like uncertainty in the demand behavior or diversity in the arrival patterns of various customer segments (see Sect. 2.3), obtaining an optimal dynamic control policy is not easy. On one hand, allocating seats to lower-valued products would improve the revenues by increasing the capacity utilization of the aircraft, but at the same time, potential higher-valued customers, who usually arrive later in the booking horizon, may be lost due to the lack of seats. The problem gets even more complicated, if additionally the network effects (Sect. 2.4.2) are considered. The revenue management problem faced by the airline industry is referred in the literature through several similar terms such as: seat allocation (see e.g., Curry 1990; Brumelle and McGill 1993; Lee and Hersh 1993), seat inventory control (see e.g., Belobaba 1989; Williamson 1992) and passenger-mix (see e.g., Glover et al. 1982) problem. In this study, we will use the term *seat allocation* and present several capacity control instruments developed for airlines.

2.3 Characteristics of Revenue Management Problems

Efficient implementation of revenue management instruments requires the fulfillment of some conditions. In other words, the industries, where revenue management can successfully be applied, share certain characteristics in common. Kimes (1989a) defines these characteristics as: relatively fixed capacity, ability to segment markets, perishable inventory, product sold in advance, fluctuating demand and low marginal sales costs/high marginal capacity change costs. They are also included in several subsequent works, e.g., Weatherford and Bodily (1992) and Talluri and van Ryzin (2004b), Sect. 1.3.3. Extensive discussions on the described features took place in Weatherford and Bodily (1992) and especially in Kimms and Klein (2005), who provide a refined classification covering some other relevant features. In the following parts of this section we present the characteristics classified by Kimms and Klein (2005) that create an appropriate environment for revenue management applications.

2.3.1 Integration of the External Factors

One of the most appealing characteristics of the sectors implementing revenue management is that the products/services cannot be stored to satisfy the demand from the stock. For this reason, the products/services are sold in advance of the production to integrate the external factor into the process. Müller-Bungart (2006) states that these external factors are usually the customers themselves or any information or physical goods supplied by the customers. Airline companies, for example, offer the products a long time before the departure. Since the seats on a flight cannot be stored, they are sold to the passengers prior to the departure time, which can be considered as the start of the production. The resulting drawback for the airlines is that they have to cope with some uncertainty, since they do not know beforehand the demand which arrives during the booking horizon. This stays in close relation to the property of heterogeneous customer behavior, which is presented next (see also Kimms and Klein 2005).

Generally, the integration of the external factor is identified with the service industries (see, e.g., Kimes 1989a; Fitzsimmons and Fitzsimmons 2006, Chap. 2). However, Müller-Bungart (2006) argues that it is also required in make-to-order manufacturing, since the information related to the customer order is needed to start the production. Underlining this common characteristic between the two industries, he emphasizes that revenue management is not restricted to the service sector. In their study, Kimms and Müller-Bungart (2003) provide a comparison of the revenue management applications in service and make-to-order manufacturing industries.

2.3.2 Heterogeneous Customer Behavior

An important characteristic inherent in successful revenue management applications is that the customers have different willingness-to-pay for the products. In such an environment, it is possible to segment the market into different types of customers and use price discrimination. In the passenger airline industry, the customers are differentiated as price sensitive and time sensitive, who generally have different times of purchase (see Weatherford and Bodily 1992). The time-sensitive business travelers usually make their reservations late in the booking horizon, whereas the more price-sensitive leisure travelers prefer to have less flexibility for a smaller price and book much earlier.

Making use of market segmentation, airlines offer discount-fare tickets with an attempt to catch more leisure travelers and in this way to increase the utilization of the capacity. The discount-fare tickets usually impose some restrictions like advance purchase requirements, cancellation fees, minimum length of stay requirements, etc. By imposing these restrictions, the business travelers, who would like to have flexibility, are prevented from buying the products for lower prices see (see Müller-Bungart 2006). According to Kimms and Klein (2005), by decreasing the price, the demand for a product can be increased and likewise, by setting higher prices, the demand can be directed to another product having more idle capacity. Heterogeneous customer behavior is essential for revenue management, since otherwise there would not arise the problem of accepting or rejecting an arriving request with an expectation to earn more revenue in the future. Instead, the arriving requests would be accepted with a first-come-first-served policy until there would not exist any capacity or demand (see Müller-Bungart 2006).

Another property related to the customer heterogeneity is the stochastic behavior of the demand, which is considered as a distinct characteristic in the work of Kimes (1989a). In the industries implementing revenue management policies, the demand for the products/services is usually uncertain and show variations over time. The uncertainty is inherent both in the total demand arriving and in the distribution of this demand over the booking horizon. Kimms and Klein (2005) point out that the stochastic behavior of the demand is not a necessary condition for implementing revenue management, but it has a great influence on the applied revenue management policy. Since the demands are unknown, forecasts about the demand for each product are required to develop a capacity control mechanism, which maximizes the expected revenue.

2.3.3 Restricted Operational Flexibility of Capacity

Revenue management is concerned with the efficient allocation of the available resources to the demand. Therefore, a major characteristics common to the firms or industries implementing revenue management techniques is the limited flexibility of the capacity. This concept comprises the two features given by Kimes (1989a): relatively fixed capacity and perishable inventory. In fact, both of these features are related to the potential use of the resources to satisfy the incoming demand. According to Kimes (1989a), the capacity is relatively fixed, because the amount of the resources cannot be changed to satisfy the incoming demand. Weatherford and Bodily (1992) and Kimms and Klein (2005) discuss this definition by examining the sectors, which are well known to apply revenue management instruments. According to Kimms and Klein (2005), airlines can respond to the demand by using aircrafts with different capacities (see Kimms and Klein 2005). However, making short-term capacity adjustments is generally very costly. For example, to satisfy a little excess demand, an aircraft with 100 seats has to be included as resource, which is economically not reasonable. Therefore, capacity adjustments are long-term decisions that are made considering the long-term state of the operations (see Müller-Bungart 2006). In the industries practicing revenue management, the capacity cannot be adjusted in the short-term, because it is impossible due to some technical reasons or does not payoff due to the high adjustment costs and long adjustments times (see Müller-Bungart 2006).

Another key feature of the resources associated to the limited operational flexibility is that the capacity cannot be used to satisfy future demand. As it is mentioned above, this characteristic is remarked as the perishability of the inventory in the literature (see Kimes 1989a). Nahmias (2011) defines perishable items as products that have an expiration date at which their utility goes to zero. In other words, the good cannot be stored and therefore provides no gain if it is not used up to its time limit. Considering the airlines, an unsold seat on an aircraft has no value after the departure, since it cannot be used to fulfill the demand for a further flight. Underlining this distinguishing feature, some publications refer to revenue or yield management as Perishable-Asset Revenue Management (PARM) (see Weatherford and Bodily 1992).

2.3.4 Standardized Products

A standardized product range includes products or services offered for a long time with given, well defined characteristics. In the airline industry, a product refers to a flight between an origin-destination (OD) pair with a specific fare class. At the time when the products are announced, information about the arrival, departure times, price and purchasing conditions is provided (see Müller-Bungart 2006). According to Kimms and Klein (2005), the revenue management approaches price differentiation and capacity control rely on the standardized product range. In the presence of uncertain demand, in order to develop an efficient control policy, one needs information about the future demand for each product, which is only obtainable when the products are standardized and offered for a long time. Furthermore, the behavior of the demand and the related parameters like the expected demand, can only be correctly predicted when the products are well defined with fixed attributes (see Kimms and Klein 2005).

In the next sections, where the capacity control approaches are presented, the role of the standardized product range in modeling and solving revenue management problems become more apparent.

2.4 Capacity Control Approaches for Airline Revenue Management Problems

The research on the capacity control approaches of the airlines includes models and algorithms to determine the seat allocations for each product in order to maximize the expected revenue obtained at the end of the reservation horizon and subsequently translate them into effective booking control policies. Primarily, there are two major types of policies proposed for capacity control: booking limit control policy and the bid-price control policy. We present the use of these policies both for single-resource and network capacity control problems. However, it is worth noting that this study deals with airline alliance networks and for this reason concentrates more on network capacity control approaches.

The modeling approaches developed for single-resource and network capacity control problems have been classified into static and dynamic models in the literature. Static models assume that the lower-fare customers arrive before the higher fare class customers. In other words, the passengers belonging to different fare classes arrive in non-overlapping intervals. This behavior of the passengers is often called as low-before-high arrival pattern. Dynamic models, on the other hand, do not require this assumption and allow an arbitrary order of customer arrivals (see Talluri and van Ryzin 2004b, Sects. 2.2 and 2.5).

In the following sections, we examine the capacity control approaches to be applied in single-leg and network problems. We give models to find the values for the used control variables and describe how they are transformed into booking limit and bid-price control policies.

Within the context of passenger airlines, single-resource capacity control describes the control of the capacity of a single flight leg. Therefore, it is applicable to the capacity control problems arising in single-leg flights and is also used to provide approximate solutions for network problems where the control of each flight leg in the network is made separately (see Talluri and van Ryzin 2004b, Sect. 2.1). Since we consider in this study the capacity control problem faced by passenger airlines, we refer to single-resource capacity control also as single-leg problems.

2.4.1 Single-Leg Capacity Control

Single-leg capacity control approaches generally deal with the problem of allocating the capacity of a single flight consisting of only one flight leg to different fare classes so as to maximize the expected revenue. Hence, a product is defined as a specific fare class of the flight. Therefore, we use in this section the terms *product* and *fare class* interchangeably. Later, in Sect. 2.4.2, we will consider network problems, where there are multiple flights each having several fare classes. As noted by Talluri and van Ryzin (2004b), Sect. 2.1, in practice, the capacity control problems for flights using multiple resources may also be solved by dividing them into several single-leg problems.

2.4.1.1 Booking Limit Control Policy

In order to implement a booking limit control policy, *booking limits* have to be determined, which restrict the number of seats that can be sold to a specific product. There exist two ways for using booking limits in capacity control policies: partitioned or nested. The distinction between them will be clarified in the following parts of this section.

Partitioned Booking Limits

In the partitioned booking limit policy, the total capacity of the flight is divided into separate blocks, where each block defines the capacity allocated to a specific product or fare class. Therefore, a *partitioned booking limit* of a product refers to the maximum number of seats that can be sold solely to that product. At this point it is worth to define the *protection level* of a product, which describes the number of seats to be reserved for a particular class or classes. Consider a single-leg flight having a capacity of C, which is allocated to different fare classes denoted by $k = 1, \ldots, p$. It is assumed that $k = 1$ shows the highest fare class and $k = p$ the lowest. Let BL_k and x_k denote the booking limit and the protection level of the product k, respectively. By definition, within a partitioned allocation policy, the protection levels and the booking limits are equivalent. Furthermore, because each booking limit defines a separate block of the seats, the booking limits of all the products sum up to C, which is represented in Fig. 2.1.

As it is stated in Lee and Hersh (1993) and Talluri and van Ryzin (2004b), Sect. 2.1.1.1, using partitioned booking limits is not an effective approach, because the demands for the products are random. Consider the case where the demand for a fare class k exceeds its booking limit. According to a partitioned booking control, all the requests beyond the booking limit BL_k will be denied, even if there is still capacity available in the blocks $BL_{k+1}, \ldots, BL_p$. As a result, the airlines may realize losses in revenue, which can be avoided using a nested booking control policy.

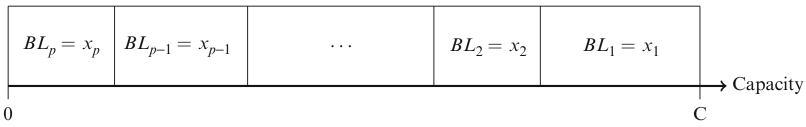

Fig. 2.1 Illustration of partitioned booking limits (see Lee and Hersh 1993)

Nested Booking Limits

A nested booking control policy allows more valuable products to access to the capacity reserved for less valuable products. Before the implementation of the policy, a so-called nesting order should be constructed, which ranks the products according to their values to the airline. For single-leg problems, the determination of the nesting order is trivial. Obviously, a high-fare class customer is more valuable than a lower-fare class customer indicating that ranking should be based on the fare class. Contrary to single-leg problems, ranking of the products in an airline network is not straightforward. Since the products do not use the same resources, it is difficult to identify which product is more valuable to the network. The lower fare class of a longer flight may contribute to the network revenue more than a higher fare class of a shorter flight, or vice versa. Furthermore, since each resource has a different capacity it is not easy to define a unique order, which is valid across all the resources (see Talluri and van Ryzin 2004b, Sect. 3.1.2.2).

In this section we demonstrate the implementation of nested booking limits in single-leg capacity control problems. In Sect. 2.4.2.2 we will consider nesting of the products in airline networks and present several approaches. For both types of problems, there are several studies demonstrating that using nested booking limits results in higher expected revenues than using partitioned booking limits (see e.g., Williamson 1992).

The illustration of the nested booking control on a single-leg flight is given in Fig. 2.2. The term x_k, refers to the *nested protection level* of the product k, which represents the number of seats protected for fare classes $k, k-1, \dots, 1$ against the lower fare classes. The *nested booking limits* describe the number of seats that can be sold to a product and its lower-ranked products. Therefore, the booking limit of a product k is equal to the capacity remaining after protecting necessary seats for the higher fare classes $k-1, k-2, \dots, 1$. The highest fare class $k = 1$ has access to all the available seats. Mathematically, it is formulated as:

$$BL_k = \begin{cases} C, & \text{for k} = 1, \\ C - x_{k-1}, & \text{for k} = 2, \dots, \text{p}. \end{cases}$$

As it can be seen from the Fig. 2.2, the capacity available to a fare class k, is also available to the fare classes $k-1, \dots, 1$. There is a relation between nested and

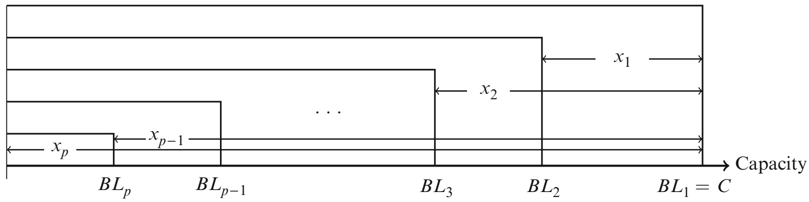

Fig. 2.2 Illustration of nested booking limits (see Lee and Hersh 1993)

partitioned booking limits: the nested booking limit of a product is the sum of the partitioned booking limits of itself and all the lower-ranked products.

During the booking process, equivalent capacity control policies are applied by using booking limits or protection levels. Talluri and van Ryzin (2004b), Sect. 2.1.1.3 describe capacity control with nested protection levels. Accordingly, a request for a fare class k is accepted, if the remaining capacity of the flight exceeds the number of seats to be protected for the higher-ranked products, which is denoted by x_{k-1}. In Sect. 6.7, we implement a nested capacity control approach, which makes use of the protection levels for accept/reject decisions. There are two alternatives to implement a nested capacity control policy: *standard nesting* and *theft nesting*. In the former approach, the protection levels are updated during the booking horizon following the acceptance of a reservation, whereas they remain constant in the latter one. Suppose that a request for a fare class k is accepted. If standard nesting is applied, it is assumed that there is no need to protect an additional seat for the fare classes $k, k-1, \ldots, 1$ from the lower fare classes. As a result, the protection levels $x_k, x_{k+1}, \ldots, x_{p-1}$ are reduced by one. If theft nesting is applied, on the other hand, the same number of seats is kept to be protected for the fare classes $k, k-1, \ldots, 1$, even though a request for fare class k is accepted.

The work of Littlewood (1972) is the first to address the problem of determining the optimal booking limits for single-leg flights. He considers a problem with two fare classes and proposes a static model suggesting that a customer request should be accepted as long as its fare exceeds the future expected revenue of the seat. This result, which is called "Littlewood's rule" in the literature, is the basis of subsequent revenue management research. Consider a single-leg flight with two fare classes denoted by f_1 and f_2, where $f_1 > f_2$. Suppose that a customer belonging to the lower fare class arrives, which should either be accepted or rejected. Let c be the remaining capacity on the flight when the request arrives. Let the random variable D_i denote the demand for fare class i. According to Littlewood's rule, the request should be accepted if and only if

$$f_2 \geq f_1 Pr(D_1 \geq c), \tag{2.1}$$

The above equation can be explained as follows: If the arriving request is rejected, the seat can be sold to a higher fare class customer and a revenue of f_1 would be earned. However, this can happen only when $D_1 \geq c$. Hence, the term

$f_1 Pr(D_1 \geq c)$ corresponds to the expected marginal revenue of the seat and the customer is accepted if f_2 is at least as much as this expected revenue. Hence, the optimal policy implies that the lower-fare class customers should be rejected if the remaining capacity does not exceed x_1^*. In other words, x_1^* refers to the optimal protection level for the fare class one. Suppose that D_i follows a discrete distribution, then x_1^* satisfies:

$$f_2 < f_1 Pr(D_1 \geq x_1^*) \text{ and } f_2 \geq f_1 Pr(D_1 \geq x_1^* + 1). \tag{2.2}$$

In case of a continuous distribution for demand, x_1^* is found as follows:

$$f_2 = f_1 Pr(D_1 > x_1^*), \text{ which means } x_1^* = F_1^{-1}(1 - \frac{f_2}{f_1}), \tag{2.3}$$

where $F_i(.)$ refers to the cumulative distribution function of the demand for fare class i. The policies given in (2.2) and (2.3) can be translated into a nested booking limit control policy such that $BL_1 = C$ and $BL_2 = C - x_1^*$, assuming C is the initial capacity of the flight. More detailed descriptions of Littlewood's rule are found in Talluri and van Ryzin (2004b), Sect. 2.2.1 and Klein and Steinhardt (2008), Sect. 3.2.1. The note provided by Netessine and Shumsky (2002) describes the single-leg revenue management problem with two fare classes as a newsvendor problem, which is an interesting result. Similar to the newsvendor problem it can be thought that, if too many seats are protected, an overage cost of f_2 incurs, because the seat cannot be sold. Otherwise, if too few seats are protected, then an underage cost of $f_1 - f_2$ incurs, because the extra revenue which would come from the higher fare customers cannot be earned.

Based on the study of Littlewood (1972), Belobaba (1987a), and Belobaba (1989) propose the Expected Marginal Seat Revenue (EMSR)-a heuristic for single-leg problems with a number of fare classes $p \geq 2$. The heuristic provides nested protection levels for each fare class. Although the EMSR-a heuristic results in a quite different booking control policy than the optimal one, the resulting expected revenues are very close to optimal (see Curry 1990; Wollmer 1992; Brumelle and McGill 1993). A slightly modified version of the EMSR-a is the so-called EMSR-b method (see e.g., Belobaba and Weatherford 1996). In the following, we explain the computation of the protection levels with EMSR-a and EMSR-b heuristics.

EMSR-a heuristic solves the multi-class problem by performing the Littlewood‘s Rule given in (2.2) and (2.3) for pairs of fare classes. Suppose that we need to determine the booking limit for the fare class $k + 1$. Hence, we have to find the protection level x_k, which corresponds to the number of seats that should be protected for the products $k, k-1, \ldots, 1$. Consider a pair of fare classes $k + 1 (2 \leq k + 1 \leq p)$ and $j (1 \leq j \leq k)$. By means of the Littlewood‘s Rule, the EMSR-a heuristic computes, for each pair, the protection level x_j^{k+1} that represents the number of seats to be protected for the product j from the access of the product $k + 1$. Then, x_k is found as the sum of these protection levels over $j = 1, \ldots, k$

such that

$$x_k = \sum_{j=1}^{k} x_j^{k+1}. \tag{2.4}$$

EMSR-b reduces the problem with p fare classes to two-class problems by aggregating the demand of the products. Consider again the fare class $k + 1$ for which we try to find the value of x_k. Recall that x_k represents the number of seats to be protected for the fare classes $k, k - 1, \ldots, 1$. Let $\hat{D}_j$ be the aggregate future demand for these fare classes such that

$$\hat{D}_k = \sum_{j=1}^{k} D_j. \tag{2.5}$$

Moreover, let $\overline{f}_k$ be the weighted-average revenue of the fare classes $k, k-1, \ldots, 1$, which is computed by:

$$\overline{f}_k = \frac{\sum_{j=1}^{k} f_j E[D_j]}{\sum_{j=1}^{k} E[D_j]}. \tag{2.6}$$

Then, the protection level x_k is found through the Littlewood's Rule considering the fare class $k + 1$ with a fare of f_{k+1} and the aggregated fare class with a demand of $\hat{D}_k$ and a fare of $\overline{f}_k$ as a pair.

One important advantage of the EMSR heuristics is that they are easy to implement. Hence, they are widely used by the airline companies (see Belobaba and Weatherford 1996). The study of Graf (2011) considers the EMSR heuristics within the context of airline alliances. Both versions of the heuristic are implemented to the revenue management problem of a single-leg airline alliance with two partners.

Several other static models to determine the optimal booking limits for different fare classes in a single-leg problem are developed by Curry (1990), Wollmer (1992), and Brumelle and McGill (1993). The difference between these three studies is the assumption about the distribution of the demand for the fare classes. For an overview on the seat allocation problem and the early works on the subject, we refer to the study of Belobaba (1987b), Kimes (1989b) and Weatherford and Bodily (1992).

The assumption about the low-before-high arrival pattern has been relaxed by Robinson (1995), but the study still assumes that customers of different fare classes arrive in non-overlapping intervals. A major contribution to the single-resource capacity control problems is due to Lee and Hersh (1993), who provide a discrete time dynamic model. It is shown that for each fare class there exists an optimal booking limit or an optimal decision period, after which the requests of the corresponding fare classes should not be accepted. Subramanian et al. (1999) extended the pioneer study of Lee and Hersh (1993) for single-resource capacity allocation problems to include overbooking, cancellations and no-shows. In contrast to the case without overbooking, they found out that the booking limits may not be monotone in time remaining until departure. Furthermore, at a certain time in the booking horizon it may be optimal to accept a lower fare class customer while rejecting a higher fare class. Brumelle and Walczak (2003) enhanced the model of Subramanian et al. (1999) by including batch bookings, where a customer can request multiple seats at a time. Lee and Hersh (1993) also consider batch bookings in which case the optimal policy is characterized by optimal decision periods. Lautenbacher and Stidham (1999) review the static and dynamic control policies developed for single-leg problems and propose a Markov decision process model formulation which encompasses all these models as special cases.

Zhao and Zheng (2001) incorporate customer choice behavior in a single-leg seat allocation problem with two fare classes. Another study considering customer choice behavior in single-leg revenue management problems is due to Talluri and van Ryzin (2004a), who propose an approach based on the dynamic model given by Lee and Hersh (1993).

2.4.1.2 Bid-Price Control Policy

Another policy applicable to single-leg problems is the bid-price control policy, which is introduced by Simpson (1989) and further studied by Williamson (1992). In fact, the Littlewood's Rule presented in the previous section is equivalent to a bid-price control strategy for a problem with two fare classes. Here, we outline the policy for general single-leg problems. Bid-price policy can also be applied to network capacity control problems. In this section, we briefly introduce the policy and in the next section, we examine the policy within the context of network problems and review the relevant literature.

In general, *bid-prices* are threshold values defined for the resources, which are used to approximate the opportunity cost of using one unit capacity on the resource (see Talluri and van Ryzin 2004b, Sect. 3.1.2.3). In the airline revenue management context, the bid-prices are determined for each flight leg and show the opportunity cost of reserving a seat on the associated flight leg. In single-leg problems, the bid-price describes the minimum acceptable value that the airline will earn from selling a ticket (see Klein 2007). The policy implies that an incoming request should be accepted, if and only if there is available capacity and the corresponding fare is at least as much as the bid-price. To implement a bid-price control policy one needs only to store a single bid-price value, which may seem as an advantage over a booking limit control policy, where a booking limit value is stored for each fare class (see Williamson 1992). However, a major drawback of working with bid-prices is that once the bookings are opened to a fare class, there is no limit on the number of seats that can be sold to that fare class. This is not an efficient policy, because as the remaining capacity decreases, it may be more profitable to protect seats for customers with a higher-willingness-to-pay. The problem is handled by updating the bid-prices during the booking horizon taking the available capacity into account (see de Boer et al. 2002). Clearly, the amount of data to be stored will expand, because in the presence of updating there will exist one threshold value for each state variable (e.g., time, capacity). However, bid-prices are effective only if they are updated (see Talluri and van Ryzin 2004b, Sect. 2.1.1.4).

A popular way to compute bid-prices is introduced by Williamson (1992), which makes use of the linear programming models developed for network revenue management problems. The mentioned models and their use in calculation of the bid-prices are presented in the next section. Since the models can be applied also to single-leg problems, the bid-prices for single-leg problems are calculated in the same manner. Several procedures for updating the bid-prices are presented in Klein (2007).

2.4.2 Network Capacity Control

If the products of an airline are solely composed of flights including a single flight leg, the total expected revenue can be maximized through maximizing the revenue of each flight separately (see Williamson 1992, p. 36). However, airlines do not offer direct flights between every origin-destination pair. They usually operate on hub-and-spoke networks including many flights composed of more than a single flight leg. There are, however, some low-cost carriers, which are operating on networks consisting of only direct flights (see Phillips 2005, for examples).

Figure 2.3 shows a *hub-and-spoke* network of an airline, which consists of two hubs and nine spokes. The gray nodes represent the spokes which are connected to the hubs A and B. Direct flights are offered between any hub-spoke pair and between two hubs. However, a flight between two spokes consists of at least two flight legs.

The sale of a multi-leg flight is restricted by the capacity limits of the used flight legs. Therefore, an allocation decision made for a flight affects the available capacity of the connecting flights. Hence, optimizing each flight leg locally cannot make sure that the network revenue is maximized. Revenue management strategies should be applied, which take the network effects into account.

The following example, which is adapted from Klein and Steinhardt (2008), Sect. 3.3.1, shows how leg-based control policies fail to find the correct capacity allocation decisions. Consider the network given in Fig. 2.4 including two flight legs. The airline offers the following single-leg flights: Düsseldorf (D) – Munich (M) and Munich – New York (N). Additionally, it offers a flight between Düsseldorf and New York with an intermediate stop in Munich. Thus, we have flights between the following OD pairs: DM, MN, DN.

Let us assume that there exist two fare classes for each of these OD pairs, where $f_{1OD} \geq f_{2OD}$. Furthermore, the airline expects one request for the flight DM with a fare of $f_{1DM} = 400$ and one request for the flight DN with a fare of $f_{2DN} = 600$. It is assumed that there is one seat available on the flight leg Düsseldorf – Munich, whereas the flight leg Munich – New York has ample capacity. Suppose that the request for DM arrives. In a leg-based booking control policy, the request will be accepted, since it belongs to the highest fare class. However, rejecting it and

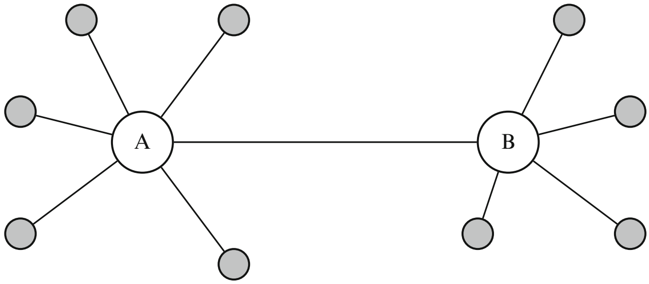

Fig. 2.3 A hub-and-spoke network of an airline

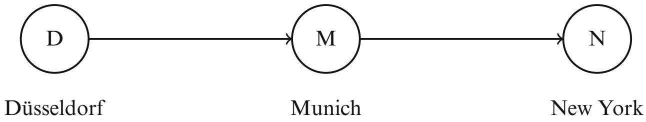

Fig. 2.4 A simple network with two flight legs

reserving the request for the flight DN would generate a revenue of 600 instead of 400.

The above example illustrates the challenge in multi-leg revenue management problems for a very small network. Obviously, the determination of the optimal policy becomes much more complicated, if networks of more realistic size are considered. In the following section, we present the network revenue management approaches developed for making the capacity management decisions in airline networks.

2.4.2.1 Optimal Dynamic Network Control Policy

Talluri and van Ryzin (1998) examine the network revenue management problem and proposes a dynamic programming formulation for the optimal control policy. Their results are also provided in Talluri and van Ryzin (2004b), Sect. 3.2.

Consider an airline network having m flight legs denoted by $l = 1, \dots, m$. By combining these flight legs, the airline offers a number p of ODFs labeled through $k = 1, \dots, p$. In the network setting, a product k defines a flight between an origin destination (OD) pair belonging to the fare class F. Throughout this study, the abbreviation *ODF* is used to refer to a product. Let a_{lk} be the resource coefficient of the ODF k on flight leg l such that

$$a_{lk} = \begin{cases} 1 & \text{if flight leg } l \text{ is used by ODF } k, \\ 0 & \text{otherwise.} \end{cases}$$

Let us also define the *resource matrix* A, which includes the elements a_{lk}, and A_k indicating the kth column of the matrix A. Assume that c_l is the available capacity on the flight leg l at any time during the booking horizon. To define the state of the network, we introduce the vector c which is determined through the *remaining capacity* on the flight legs. That is, $c = (c_1, \ldots, c_m)^\mathsf{T}$. If a request for ODF k is accepted, the new state vector will be $c - A_k$. Otherwise, it remains the same.

The booking horizon consists of time periods $t = 1, \ldots, T$, and the flight departs at time period $T + 1$. Furthermore, the time intervals are chosen sufficiently small such that the probability of more than one arrival is negligible. Let the vector $f = (f_1, \ldots, f_p)$ show the fares of the ODFs. Note that throughout this study, we use the notations $f_k = f(ODF) = f(OD, F)$ to refer to the fare of a flight between a specific OD pair belonging to the fare class F. The demand for the ODFs is modeled through the random vector $F(t)$, where $F(t) = (F_1(t), \ldots, F_p(t))$. A positive value of $F_k(t)$ (i.e., $F_k(t) = f_k \geq 0$) shows that a request for the ODF k arrives in period t with a corresponding fare of f_k. An $F_k(t)$ value equal to zero, on the other hand, means that no request for the ODF k arrives in period t. The vector $F(t)$ has a joint distribution in each period t, which is assumed to be known. Moreover, the distribution of $F(t)$ is independent across time periods. Note that, since the number of arrivals in a period cannot exceed one, at most one component of $F(t)$ can be positive.

Suppose that a request arrives at the time period t, which is going to be either accepted or rejected. Let the vector $u(t) = (u_1(t), \ldots, u_p(t))^\mathsf{T}$ denote this decision, where $u_k(t)$ is equal to 1, if the request for the ODF k is accepted in period t, and 0 otherwise. Since the decision depends on the state of the network and the fare of the ODF, $u_k(t)$ is modeled as a function of c and f_k such that $u_k(t) = u_k(t, c, f_k)$. In any time period, given a capacity vector c, the vectors $u(t)$ are defined through the set $\mathcal{U}(\mathrm{c})$, where $\mathcal{U}(\mathrm{c}) = \{u \in \{0, 1\}^p : Au \leq c\}$

We define $V_t(c)$, which represents the maximum expected revenue to attain in the periods $t, t + 1, \ldots, T$ given that the remaining capacity is c in period t. The value of $V_t(c)$ can be computed through the following Bellman equation:

$$V_t(c) = E\left[\max_{u \in U(c)} \{F(t)u(t, c, f) + V_{t+1}(c - Au)\}\right]$$

with the conditions $V_{T+1}(c) = 0$ for all c and $V_t(0) = 0$ for all t.

Talluri and van Ryzin (1998) prove that $V_t(c)$ is finite for all finite c and the optimal control policy defined through u_k^* is such that

$$u_k^*(t, c, f_k) = = \begin{cases} 1 & \text{if } f_k \geq V_{t+1}(c) - V_{t+1}(c - A_k) \text{ and } A_k \leq c, \\ 0 & \text{otherwise.} \end{cases} \tag{2.7}$$

The term $V_{t+1}(c) - V_{t+1}(c - A_k)$ is referred to as the opportunity cost of accepting the request due to the decrease in the available capacity. The optimal policy described in (2.7) implies that a request should be accepted if its fare exceeds its opportunity cost and there is enough capacity available.

However, as it is pointed out in Talluri and van Ryzin (2004b), p. 83, determining an optimal policy is almost impossible, when the size of real airline networks is considered. In other words, computing the value function $V_t(c)$ is intractable due to the curse of dimensionality of the state space. Therefore, approximation methods are used for network problems. Talluri and van Ryzin (2004b) differentiate between two approaches. (1) approximating the value function by using mathematical models and (2) decomposing the high dimensional dynamic program developed for the whole network into one-dimensional single-leg problems which are to be solved independently. The following section presents the mathematical models developed for solving network revenue management problems. For network decomposition approaches, we refer to Talluri and van Ryzin (2004b), Sect. 3.4. An extension of this approach to customer choice behavior is found in Liu and van Ryzin (2008).

2.4.2.2 Mathematical Programming Based Approximations

The mathematical programming based approaches include approximating the expected revenue by means of deterministic and probabilistic mathematical models. The aim of these models is to allocate the capacity of the resources to the products so as to maximize the expected revenue. They are regarded as static approximations to the expected network revenue, since they provide one time allocation of the available capacity (see Talluri and van Ryzin 1998). To improve the quality of the approximation, the models are usually resolved and the allocations are reoptimized a number of times during the booking horizon by taking the realized demand and the remaining capacity into account (see Talluri and van Ryzin 2004b, Sect. 3.3).

Before introducing the different mathematical models developed for airline network revenue management problem, it is necessary to give the assumptions common to all these models. The mathematical models maximize the expected network revenue over all the products of the airline. It is assumed that the demands of the ODFs are independent such that switching of the customers between the fare classes is not allowed. Furthermore, the distribution of the demand is known for each ODF separately. With the objective of maximizing the expected revenue, the models determine the number of seats to be allocated to each ODF.

Deterministic Modeling Approach

The most popular model used to approximate the expected revenue in airline networks is the Deterministic Linear Programming (DLP) Model presented in e.g., Williamson (1992) and de Boer et al. (2002). The model is deterministic since it maximizes the expected revenue by assuming that the demand for an ODF is always equal to its expected demand. Hence, the stochastic nature of the demand is not taken into account, which appears to be a major deficiency of the model at first glance. However, the simulation studies of Williamson (1992) and de Boer et al. (2002) have shown supporting results to prefer DLP over other more complicated models which will be presented in the following parts of this section. In fact, the DLP model takes its roots from the network formulation proposed by Glover et al. (1982).

The formulation of the DLP model to find the optimal partitioned seat allocations for a network of an airline is as follows:

$$DLP : \; Maximize \sum_{k=1}^{p} f_k Y_k \tag{2.8}$$

$$subject\ to$$

$$\sum_{k=1}^{p} a_{lk} Y_k \le C_l \qquad l = 1, \ldots, m, \tag{2.9}$$

$$Y_k \le E[D_k] \qquad k = 1, \ldots, p, \tag{2.10}$$

$$Y_k \ge 0 \qquad k = 1, \ldots, p. \tag{2.11}$$

The objective function (2.8) maximizes the expected revenue of the airline over all the products. The decision variable Y_k refers to the number of seats allocated to the ODF k. Constraint set (2.9) ensures that the capacity limits of the flight legs are not violated. Thus, the total number of seats allocated on a flight leg is limited by C_l, which is the initial capacity of the flight leg l. Note that if the model is solved many times during the booking horizon, the actual remaining capacity vector is used. Constraint set (2.10) implies that the number of seats allocated to an ODF cannot exceed its expected demand, which is denoted by $E[D_k]$. Finally, (2.11) is the non-negativity constraint. It is known that for linear or single-hub networks, DLP model gives integer allocations, provided that the capacities of the flight legs and the demand expectations for the ODFs in the constraints (2.9) and (2.10) are integer (see de Boer et al. 2002).

Chen et al. (1998) have shown that the objective function value of the DLP constitutes an upper bound for the network revenue. Approximation approaches giving tighter upper bounds are proposed by Adelman (2007) and Topaloglu (2009). A comparison of these modeling approaches is given in Talluri (2009) together with improved versions leading to tighter bounds. Talluri (2009) states that obtaining tighter bounds is essential, since the booking policies implied by the models giving tighter upper bounds result in higher expected revenues.

Probabilistic Modeling Approaches

A general mathematical model for approximating the value function which incorporates the probabilistic nature of the demand is provided in Talluri and van Ryzin

(1998). The formulation of the so-called *probabilistic non-linear programming* (PNLP) model is as follows:

$$PNLP: \ Maximize \sum_{k=1}^{p} f_k E[min\{D_k, Y_k\}] \tag{2.12}$$

subject to

$$\sum_{k=1}^{p} a_{lk} Y_k \leq C_l \qquad l = 1, \ldots, m, \tag{2.13}$$

$$Y_k \geq 0 \qquad k = 1, \ldots, p. \tag{2.14}$$

The distinction between PNLP and DLP is that the PNLP works with the random demands of the ODFs, which are denoted by D_k, instead of the expected values. Note that the actual number of seats sold will be equal to this allocation, if the demand for the ODF is at least as many as Y_k. Otherwise, if the demand turns out to be less than Y_k, some of the seats will remain empty. Hence, the term $E[min\{D_k, Y_k\}]$ in (2.12) corresponds to the expected sales of the ODF that leads to a non-linear objective function.

Like the deterministic model, PNLP also finds out partitioned booking limits for the ODFs so as to maximize the expected revenue. Since the implied allocations are certainly feasible for the airline network, the solution of the PNLP provides a lower bound for the optimal expected revenue (see also Chen et al. 1998).

An equivalent linear programming model is presented in Williamson (1992) and de Boer et al. (2002), which can be used when the demand distributions are discrete. The formulation of the so-called expected marginal revenue (EMR) model is as follows:

$$EMR: \ Maximize \sum_{k=1}^{p} \sum_{z=1}^{\hat{B}_k} f_k Pr(D_k \geq z) Y_k(z)$$

subject to

$$\sum_{k \in \mathcal{E}_l} \sum_{z=1}^{\hat{B}_k} Y_k(z) \leq C_l \text{ for } l = 1, \ldots, m, \tag{2.15}$$

$$0 \leq Y_k(z) \leq 1 \text{ for } k = 1, \ldots, p \text{ and } z = 1, \ldots, \hat{B}_k. \tag{2.16}$$

The decision variable $Y_k(z)$ shows whether z or more seats are allocated to the ODF k. It is not defined as a binary variable, because the linear programming formulation above has turned to be tight (see Williamson 1992; de Boer et al. 2002). Then, the total number of seats allocated to an ODF is equal to the sum of $Y_k(z)$ over all possible z values. The constraint set (2.15) makes sure that the capacity of the flight legs are not exceeded, where the set $\mathcal{E}_l$ includes the ODFs traveling through the flight leg l. The domain of z for an ODF k is given in constraint (2.16). Due to the capacity limitations, the value of z cannot exceed the maximum number of seats that can be allocated to the ODF k. Therefore, $\hat{B}_k = min_{l \in \mathcal{T}_k}\{C_l\}$, where $\mathcal{T}_k$ denotes the set of flight legs traveled by ODF k.

Like the DLP and PNLP models, the solution of EMR defines non-nested, partitioned seat allocations for the ODFs. The fact that $E[min\{D_k, Y_k\}] = \sum_{z=1}^{Y_k} Pr(D_k \geq z)$ for discrete distributions and the decreasing property of $Pr(D_k \geq z)$ in z leads to the equivalence of the PNLP and EMR models.

A major difficulty of working with the EMR model is the large number of decision variables of type $Y_k(z)$, which grows very fast with the increase in the number of ODFs and the capacity of the flight legs. Although PNLP has a non-linear objective function, solving it is often more efficient than solving the EMR model (see Williamson 1992; Talluri and van Ryzin 2004b, p. 96).

Pak and Piersma (2002) present the capacity control approaches to single-leg and network seat allocation problems. The study includes a review of the above mentioned mathematical models.

2.4.2.3 Implementing a Nesting Booking Limit Control Policy

Extensive comparisons of the DLP and EMR models are provided in Williamson (1992) and de Boer et al. (2002), which demonstrate results supporting the use of the deterministic model. They first have found the seat allocations by using the deterministic and probabilistic linear programming models. Then, nesting the ODFs according to their contributions to the network revenue, they have simulated the booking process using the seat allocations implied by each model. The expected revenues obtained by the simulation have turned out to be usually higher with a booking control policy based on the deterministic model than with the one based on the probabilistic model. The mentioned studies investigate also the performance of the models when incorporated into a bid-price control policy. It is observed that the bid-prices obtained using the deterministic model result in higher revenues than the ones implied by the probabilistic model.

As it was stated previously, determining a nesting order for network problems is not straightforward. Williamson (1992) considers three strategies for a nesting order of an airline network: nesting by fare class, nesting by fares and nesting by shadow prices. The drawback of the first strategy arises when a lower-fare customer of a long flight may contribute more than a high-fare customer of a shorter flight. In the second approach, the ODFs are nested only considering their fare values. Thus, the multi-leg flights have priority over single-leg flights, since they generally have higher fares. Consider a multi-leg ODF, to which no seats are allocated in the mathematical model. If the ODFs are nested according to their fares, these multi-leg ODF will have access to the capacity reserved for the single-leg ODFs, which have smaller fare values. As a result, although the partitioned booking limit of the multi-leg ODF is zero, a significant number of seats can be sold to this ODF during the booking process. These seats, however, could be sold to single-leg flight ODFs, which are more valuable to the network (see Williamson 1992, Sect. 4.2.2)

The third strategy is developed with an attempt to better reflect the contribution of the ODFs to the network revenue. It is primarily based on the shadow prices of the constraints in the mathematical models. In nesting the deterministic seat allocations, Williamson (1992) uses the dual prices of the demand constraints (2.10) in the DLP model. The shadow price refers, all else being held constant, to the increment in the revenue if the expected demand $E[D_k]$ of an ODF is increased by one. Hence, the ODFs with higher shadow prices are ranked higher in the nesting order. In the EMR model, however, there does not exist any demand constraint. Moreover, an increase in the mean demand indicates a change in the function $Pr(D_k \geq z)$. Therefore, we need to resolve the EMR model to obtain the change in the revenue if the expected demand of an ODF is increased by one. Obviously, this is a cumbersome task if the number of products in an airline network is imagined.

To handle this difficulty, de Boer et al. (2002) propose a nesting heuristic, which uses the dual prices of the capacity constraints (2.9) and (2.15) in deterministic and probabilistic cases, respectively. They approximate the opportunity cost of a flight by adding the dual prices of the flight legs. Then, the net contribution of the ODF to the network revenue is estimated by subtracting this amount from its fare. Let ω_l be the shadow price of the capacity constraint for flight leg l. Then, the net contribution of the ODF k, denoted by NC_k, is computed as follows:

$$NC_k = f_k - \sum_{l \in \mathcal{T}_k} \omega_l . \tag{2.17}$$

We adapt this heuristic in Sect. 6.7, where we conduct a simulation study to assess the validity of the nucleolus concept for the revenue sharing problem of airline alliances.

2.4.2.4 Bid-Price Network Control Policy

Bid-price policy is a popular instrument for controlling the capacity in airline networks. In the previous section, we have introduced the policy and its use in single-leg problems. In this section, we discuss the application of the bid-price policy to network problems. Considering a network revenue management problem, the bid-price policy implies that a request for a flight should be accepted if its fare is equal to or exceeds the sum of the bid-prices of the traveled flight legs. This method is usually referred to as the additive bid-price approach, since the value of a flight is approximated by the sum of the values of the traveled flight legs (see Bertsimas and Popescu 2003; Müller-Bungart 2006).

Williamson (1992) describes the bid-price as the marginal value of the last seat on a flight leg. As mentioned before, the bid-prices can be calculated using the linear programs developed for the network capacity control problem. Consider the DLP model presented at the beginning of this section. The bid-price of a flight leg is equal to the shadow price of the corresponding capacity constraint in (2.9). Williamson

(1992) investigates the performance of the bid-prices based on DLP and PNLP model. However, the simulation studies of Williamson (1992) and de Boer et al. (2002) show that DLP based bid-prices generate higher expected revenues than the ones based on the PNLP model.

Talluri and van Ryzin (1999) point out that working with bid-prices based on the DLP model may provide poor results. This occurs due to the fundamental assumption of the DLP model that the demand for an ODF is always equal to its expected demand. Suppose that the expected demand traveling through a flight leg is less than the capacity. In that case, the DLP will result in zero bid-prices. However, considering the high variation in the demand, the value of the flight leg might be larger than zero. In order to incorporate uncertainty, Talluri and van Ryzin (1999) propose a randomized approach, which is called RLP (Randomized Linear Programming), to compute bid-prices in an airline network. They simulate n independent sample demand realizations for each ODF, which are randomly generated from the given probability distribution. Let the vector $D^1, \ldots, D^n$ show these samples such that $D^i = D^i_1, \ldots, D^i_p$ denote the realized demand of the ODF $k = 1, \ldots, p$ in the ith sample. Then the parameter $E[D_k]$ in the constraint set (2.10) is replaced by D^i_k and the model is solved. The bid-price of a flight leg is set to the average of the shadow prices obtained over the n runs. It is observed by Talluri and van Ryzin (1999) that RLP provides small improvements compared to the DLP, which, however, are significant especially when the expected demand values are high. In this study we adapt this randomized approach to the alliance revenue management problem and present it in Sect. 6.4. Bertsimas and Popescu (2003) discuss the drawbacks of using an additive bid-price approach and indicate that the bid-prices are not well defined, if the underlying model has multiple dual solutions. They provide a non-additive bid-price method and show that it performs better than the traditional additive approach. The method estimates the opportunity cost of a flight using the objective function value of the DLP model, which does not depend on the value of the dual solution. The study of Klein (2007) defines bid-prices depending on some easily observable parameters, which facilitates the updating process.

Methods for calculating bid-prices, which include the dynamic nature of the customer arrivals have been developed by Adelman (2007) and Topaloglu (2009). Adelman (2007) proposes time dependent bid-prices determined using a linear programming approximation to the dynamic programming formulation of the network revenue management problem. The method presented by Topaloglu (2009) gives time and capacity dependent bid-prices. It is based on the decomposition of the network problem into single-leg problems by relaxing constraints that link different flight legs. By considering only one leg a time Topaloglu (2009) was able to compute time and capacity dependent bid-prices. His numerical studies show that the bid-prices result in higher expected revenues than the DLP and RLP based ones. For a discussion of different bid-price control approaches, we refer to Müller-Bungart (2006) and Klein (2007).

2.4.3 Overbooking

Overbooking is the oldest revenue management instrument, which has also been used before the deregulation (see McGill and van Ryzin 1999). It is a strategy initiated by the airline industry with an attempt to increase the capacity utilization of the flights against the negative effect of the so-called cancellations and no-shows. For the airlines, it is not uncommon that the passengers cancel their reservations at any time during the booking horizon. Moreover, no-shows are likely to happen, which describe the passengers with reservations who fail to show up at the flight time without notice. Hence, the airlines face the risk of departing with a high number of empty seats, which could be sold to other passengers. To compensate this revenue loss due to cancellations and no-shows, the airlines follow an overbooking policy. Roughly speaking, overbooking is concerned with selling more seats for a flight than the physical capacity of the aircraft to decrease the number of seats that remain empty at the time of departure. American Airlines reported that about 15 % of the seats in 1990 would remain empty if they limited the bookings by the aircraft capacity (see Smith et al. 1992). However, the extra revenue that would be realized through overbooking a flight must be traded off against the costs, which incur due to denied boardings. Denied boarding refers to the rejection of a passenger with a valid ticket due to insufficient capacity. It arises when the flight is oversold as a result of overbooking, i.e., when more passengers show up at the departure time than the available seat capacity of the flight. Denied boarding costs include the compensation provided to the rejected passengers and the loss of goodwill that arises due to the customer dissatisfaction (see Chatwin 1998). Taking this tradeoff into account, Dunleavy (1995) describes the objective of overbooking as the determination of the number of bookings that will be accepted beyond the physical capacity so that the flight will depart with a minimum number of empty seats and a minimum number of denied boardings.

The research on airline overbooking started with the work of Beckmann (1958), who proposes a static cost-based model to compute the overbooking limit for a single-leg one-fare class problem. Static models do not consider the dynamics of the customer arrivals and cancellations during the booking horizon. The overbooking limits are determined for the time the model is implemented and recomputed periodically to be aligned with the changes in the state of the system. Among other static models, the study of Thompson (1961) provides overbooking limits which restrict the probability of overselling to a specified level. It was further enhanced by Taylor (1962) and Rothstein and Stone (1967). The pioneer work of Littlewood (1972) applies the marginal revenue theory to determine overbooking limits for a single-leg flight having two fare classes. Shlifer and Vardi (1975) describe static models for single-leg flights with two fare classes and a two-leg flight with a single type of customers. A survey of the early research on the airline overbooking problem is provided in Rothstein (1985).

In the dynamic models, the overbooking limits are adapted whenever a request arrives by taking the possible new arrivals and cancellations in the subsequent periods as well as the no-shows into account (see Klein 2007, Sect. 4.2.1). The first dynamic overbooking model has been developed by Rothstein (1971) for a single-leg problem with a single fare class. For the same type of problem, Chatwin (1998) examines the conditions under which a booking limit policy is optimal. In fact, overbooking and seat allocation decisions are closely interrelated. The major input influencing the seat allocation decisions is the capacity of the flight legs, which depends on the overbooking level. The applied capacity control policy, on the other hand, affects the overbooking level through influencing the opportunity cost of the capacity (see Karaesmen 2001). Despite this, by determining the seat allocation decisions, it is generally assumed that the overbooking limits are calculated a priori. One exception is the work of Subramanian et al. (1999), which proposes a discrete time Markov decision process formulation to determine the overbooking limits for single-leg flights having multiple fare classes. The model requires a multidimensional state variable, unless the cancellation and no-show probabilities are independent of the fare class. As a result, obtaining exact solutions in the presence of overbooking is challenging even for single-leg cases. Recently, Aydin et al. (2010) have proposed static and dynamic models for single-leg overbooking problems, which provide advantages in terms of computational effort and give results close to the optimal expected value.

There are only a few publications considering overbooking decisions in airline networks. Bertsimas and Popescu (2003) and Karaesmen (2001) propose solutions for joint capacity allocation and overbooking problem by incorporating denied boarding costs into the DLP model. A dynamic model has been developed by Erdelyi and Topaloglu (2010), where the problem is solved through decomposing it into a sequence of single leg problems. The resulting expected revenues turned out to be higher than the ones obtained using approximations based on the deterministic model.

Apart from the airlines, the application of overbooking becomes also appealing in the hotel and rental car sectors, where cancellations and no-shows are highly common. As it is mentioned in Phillips (2005), another factor leading to the rise of overbooking in these two sectors is the uncertainty in the available capacity due to the so-called overstays and understays. In hotel sector, for example, there is the possibility that the customers leave the hotel earlier than they promise on the reservation. Therefore, even if there is no cancellation or no-show, the rooms stay empty in the absence of an overbooking strategy. The studies on hotel overbooking include the works of Rothstein (1974), Ladany (1976) and Liberman and Yechiali (1978). A list of publications concerning the overbooking problem in airline and hotel industries is provided in McGill and van Ryzin (1999) and Chiang et al. (2007).

References

Adelman D (2007) Dynamic bid prices in revenue management. Oper Res 55:647–661

Amaruchkul K (2007) Single-leg air-cargo revenue management. Transp Sci 41:457–469

Aydin N, Birbil SI, Frenk JBG, Noyan N (2010) Single-leg airline revenue management with overbooking. http://www.optimization-online.org/DB_FILE/2010/04/2603.pdf. Accessed 16 Nov 2011

Badinelli RD (2000) An optimal, dynamic policy for hotel yield management. Eur J Oper Res 121:476–503
Bailey EE (1986) Price and productivity change following deregulation: the US experience. Econ J 96:1–17
Beckmann MJ (1958) Decision and team problems in airline reservations. Econometrica 26: 134–145
Belobaba PP (1987a) Air travel demand and airline seat inventory management. Ph.D. thesis, Massachusetts Institute of Technology
Belobaba PP (1987b) Airline yield management: an overview of seat inventory control. Transp Sci 21:63–73
Belobaba P (1989) Application of a probabilistic decision model to airline seat inventory control. Oper Res 37:183–197
Belobaba P, Weatherford LR (1996) Comparing decision rules that incorporate customer diversion in perishable asset revenue management situations. Decis Sci 27:343–363
Bertsimas D, Popescu I (2003) Revenue management in a dynamic network environment. Transp Sci 37:257–277
Bitran G, Caldentey R (2003) An overview of pricing models for revenue management. Manuf Serv Oper Manage 5:203–230
Bitran GR, Gilbert SM (1996) Managing hotel reservations with uncertain arrivals. Oper Res 44:35–49
Bitran GR, Mondschein SV (1995) An application of yield management to the hotel industry considering multiple day stays. Oper Res 43:427–443
Brumelle SL, McGill JI (1993) Airline seat allocation with multiple nested fare classes. Oper Res 41:127–137
Brumelle S, Walczak D (2003) Dynamic airline revenue management with multiple semi-Markov demand. Oper Res 51:137–148
Carroll WJ, Grimes RC (1995) Evolutionary change in product management: experiences in the car rental industry. Interfaces 25:84–104
Chatwin RE (1998) Multiperiod airline overbooking with a single fare class. Oper Res 46:805–819
Chen VCP, Gunther D, Johnson EL (1998) A Markov decision based approach to the airline YM problem. Working Paper, Georgia Institute of Technology
Chiang W-C, Chen JC, Xu X (2007) An overview of research on revenue management: current issues and future research. Int J Revenue Manage 1:97–128
Ciancimino A, Inzerillo G, Lucidi S, Palagi L (1999) A mathematical programming approach for the solution of the railway yield management problem. Transp Sci 33:168–181
Cleophas C, Yeoman I, McMahon-Beattie U, Veral E (2011) The applications of revenue management and pricing. In: Yeoman I, McMahon-Beattie U (eds) Revenue management: a practical pricing perspective. Palgrave Macmillan, Houndmills/Basingstoke/Hampshire/ New York, pp 9–16
Curry RE (1990) Optimal airline seat allocation with fare classes nested by origins and destinations. Transp Sci 24:193–204
de Boer SV, Freling R, Piersma N (2002) Mathematical programming for network revenue management revisited. Eur J Oper Res 137:72–92
Drake MJ (2008) Optimal timing of switches between product sales for sports and entertainment tickets. Nav Res Logist 55:59–75
Dunleavy HN (1995) Airline passenger overbooking. In: Jenkins D (ed) Handbook of airline economics. McGraw-Hill, New York, pp 469–476
Elmaghraby W, Keskinocak P (2003) Dynamic pricing in the presence of inventory considerations: research overview, current practices and future directions. Manage Sci 49:1287–1309
Erdelyi A, Topaloglu H (2010) A dynamic programming decomposition method for making overbooking decisions over an airline network. INFORMS J Comput 22:443–456
Fitzsimmons JA, Fitzsimmons MJ (2006) Service management: operations, strategy, information technology. McGraw-Hill/Irwin, Boston

Geraghty MK, Johnson E (1997) Revenue management saves national car rental. Interfaces 27:107–127
Glover F, Glover R, Lorenzo J, McMillan C (1982) The passenger-mix problem in the scheduled airlines. Interfaces 12:73–79
Graf M (2011) Revenue management for strategic alliances with applications to the airline industry. Ph.D. thesis, University of Duisburg-Essen
Hanks RD, Cross RG, Noland RP (1992) Discounting in the hotel industry: a new approach. Cornell Hotel Restaur Adm Q 33:15–23
Karaesmen I (2001) Three essays on revenue management. Ph.D. thesis, Columbia University, New York
Kasilingam RG (1996) Air cargo revenue management: characteristics and complexities. Eur J Oper Res 96:36–44
Kimes SE (1989a) The basics of yield management. Cornell Hotel Restaur Adm Q 30:14–19
Kimes SE (1989b) Yield management: a tool for capacity-constrained service firms. J Oper Manage 8:348–363
Kimes SE (2000) A strategic approach to yiel management. In: Ingold A, McMahon-Beattie U, Yeoman I (eds) Yield management: strategies for the service industries. Continuum, London/New York, pp 3–14
Kimes SE, Chase RB, Choi S, Lee PY, Ngonzi EN (1998) Restaurant revenue management: applying yield management to the restaurant industry. Cornell Hosp Q 39:32–39
Kimms A, Klein R (2005) Revenue Management im Branchenvergleich. Zeitschrift für Betriebswirtschaft, Ergänzungsheft 1 "Revenue Management", Gabler, Wiesbaden, pp 1–30
Kimms A, Müller-Bungart M (2003) Revenue Management beim Verkauf auftragsorientierter Sachleistungen. Working Paper, Technical University Bergakademie Freiberg
Kimms A, Müller-Bungart M (2007) Revenue management for broadcasting commercials: the channel's problem of selecting and scheduling the advertisements to be aired. Int J Revenue Manage 1:28–44
Klein R (2007) Network capacity control using self-adjusting bid-prices. OR Spectr 29:39–60
Klein R, Steinhardt C (2008) Revenue Management: Grundlagen und Mathematische Methoden, Springer, Berlin/Heidelberg
Kole SR, Lehn KM (1999) Deregulation and the adaptation of governance structure: the case of the U.S. airline industry. J Financ Econ 52:79–117
Ladany SP (1976) Dynamic operating rules for motel reservations. Dec Sci 7:829–841
Lautenbacher CJ, Stidham S (1999) The underlying Markov decision process in the single-leg airline yield-management problem. Transp Sci 33:136–146
Lee TC, Hersh M (1993) A model for dynamic airline seat allocation control with multiple seat bookings. Transp Sci 27:252–265
Liberman V, Yechiali U (1978) On the hotel overbooking problem – an inventory system with stochastic cancellation. Manage Sci 24:1117–1126
Littlewood K (1972) Forecasting and control of passenger bookings. In: AGIFORS proceedings, Nathanya, vol 12, pp 95–117
Liu Q, van Ryzin GJ (2008) On the choice-based linear programming model for network revenue management. Manuf Serv Oper Manage 10:288–310
McGill JI, van Ryzin GJ (1999) Revenue management: research overview and prospects. Transp Sci 33:233–256
Müller-Bungart M (2006) Revenue management with flexible products – models and methods for the broadcasting industry. Springer, Berlin/Heidelberg/New York
Morrison S, Winston C (1986) The economic effects of airline deregulation. The Brookings Institution, Washington DC
Nahmias S (2011) Perishable inventory systems. Springer, New York
Netessine S, Shumsky R (2002) Introduction to the theory and practice of yield management. INFORMS Trans Educ 3:34–44
Pak K, Piersma N (2002) Overview of OR techniques for airline revenue management. Stat Neerl 56:480–496
Phillips RL (2005) Pricing and revenue optimization. Stanford University Press, Stanford

Robinson LW (1995) Optimal and approximate control policies for airline booking with sequential nonmonotonic fare classes. Oper Res 43:252–263
Rothstein M (1971) An airline overbooking model. Transp Sci 5:180–192
Rothstein M (1974) Hotel overbooking as a Markovian sequential decision process. Dec Sci 5: 389–404
Rothstein M (1985) OR and the airline overbooking problem. Oper Res 33:237–248
Rothstein M, Stone W (1967) Passenger booking levels. In: AGIFORS proceedings, Noordwijk, vol 7, pp 392–435
Shlifer E, Vardi Y (1975) An airline overbooking policy. Transp Sci 9:101–114
Simpson RW (1989) Using network flow techniques to find shadow prices for market and seat inventory control. Memorandum M89-1, MIT Flight Transportation Laboratory
Smith BC, Leimkuhler JF, Darrow RM (1992) Yield management at American airlines. Interfaces 22:8–31
Spengler T, Rehkopf S, Volling T (2007) Revenue management in make-to-order manufacturing–an application to the iron and steel industry. OR Spectr 29:157–171
Steinhardt C, Gönsch J (2009) Dynamic programming decomposition approaches for revenue management with planned upgrades. Working Paper, Department of Analytics & Optimization, University of Augsburg
Subramanian J, Stidham S, Lautenbacher CJ (1999) Airline yield management with overbooking, cancellations, and no-shows. Transp Sci 33:147–167
Talluri K (2009) On bounds for network revenue management. Working Paper, Pompeu Fabra University
Talluri KT, van Ryzin GJ (1998) An analysis of bid-price controls for network revenue management. Manage Sci 44:1577–1593
Talluri KT, van Ryzin GJ (1999) A randomized linear programming method for computing network bid prices. Transp Sci 33:207–216
Talluri KT, van Ryzin GJ (2004a) Revenue management under a general discrete choice model of consumer behavior. Manage Sci 50:15–33
Talluri KT, van Ryzin GJ (2004b) The theory and practice of revenue management. Springer, New York
Taylor CJ (1962) The determination of passenger booking levels. In: AGIFORS proceedings, Fregene, vol 2, pp 93–116
Thompson HR (1961) Statistical problems in airline reservation control. J Oper Res Soc 12: 167–185
Topaloglu H (2009) Using Lagrangian relaxation to compute capacity-dependent bid prices in network revenue management. Oper Res 57:637–649
U.S. GAO (1996) Airline deregulation: changes in airfares, service, and safety at small, medium-sized, and large communities, report to the Chairman, committee on commerce, science, and transportation, U.S. Senate
Vinod B (2004) Unlocking the value of revenue management in the hotel industry. J Revenue Pricing Manage 3:178–190
Volling T, Akyol DE, Wittek K, Spengler T (2012) A two-stage bid-price control for make-to-order revenue management. Comput Oper Res 39:1021–1032
Weatherford LR, Bodily SE (1992) A taxonomy and research overview of Perishable-Asset revenue management: yield management, overbooking, and pricing. Oper Res 40:831–844
Williamson EL (1992) Airline network seat control: methodologies and revenue impacts. Ph.D. thesis, Massachusetts Institute of Technology
Wollmer RD (1992) An airline seat management model for a single leg route when lower fare classes book first. Oper Res 40:26–37
Zhang M (2011) Fencing in the practice of revenue management. In: Yeoman I, McMahon-Beattie U (eds) Revenue management: a practical pricing perspective. Palgrave Macmillan, Houndmills/Basingstoke/Hampshire/New York, pp 136–149
Zhao W, Zheng Y-S (2001) A dynamic model for airline seat allocation with passenger diversion and no-shows. Transp Sci 35:80–98

http://www.springer.com/978-3-642-35821-0

Fair Revenue Sharing Mechanisms for Strategic Passenger Airline Alliances
Çetiner, D.
2013, XV, 168 p. 27 illus., Softcover
ISBN: 978-3-642-35821-0