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Arnold, Vladimir I. [Arnol′d, Vladimir Igorevich]

★**Real algebraic geometry.**

Translated from the 2009 Russian original by Gerald G. Gould and David Kramer.
Edited and with a foreword by Ilia Itenberg, Viatcheslav Kharlamov and Eugenii I. Shustin.
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This book is a translation of lecture notes from Russian with additional comments and notes by the editors. The book is aimed at advanced high school students, but works its way to the forefront of current research and unsolved problems. It introduces very advanced topics in a very relaxed and informal style. The book includes some exposition and definitions, some theorems and proofs and a lot of problems with hints or solutions. There are many illustrations to lead the reader to an intuitive understanding of the concepts being developed. The general theme of the book is, given an algebraic curve, what can be said of its topological structure in the plane? This is a special case of the first part of Hilbert's 16th problem, a topic taken up in detail in Chapter 4.

After a brief introduction, Chapter 2 takes up conic sections. It begins with standard results on foci and eccentricity, but continues to more advanced ideas including work in the complex plane and in 3 dimensions.

Chapter 3 does physics applications of conic sections and ellipsoids. It begins with an interesting (and historical) application to jet engines, then moves on to gravitational and magnetic fields.

Chapter 4 moves into much deeper mathematics. Projective geometry is introduced via perspective and its use in art. The chapter quickly moves to defining the real projective plane, conic sections therein, the Möbius strip and genus of the Riemann surface of a curve. Perhaps because Arnold has himself done considerable research in this area, the chapter is quite nontrivial. It has a discussion of an incorrect statement by Hilbert in his 16th problem and proceeds to discuss the correction and generalizations which have been proved, thanks to a very seminal paper by Arnold. This concerns the possible arrangements of ovals of an algebraic sixth degree curve in the real projective plane. There is then a detailed discussion of the number of topologically different polynomials of degree $n+1$ with n critical points. The discussion goes on to polynomials in more variables, primarily in the form of open problems which lend themselves to enumerative computational results.

Chapter 5 pushes further, considering algebraic curves in complex projective space. Remarkably, there is even a proof of the Riemann-Hurwitz Theorem on the genus of the Riemann surface of a smooth algebraic plane curve in the complex projective plane. The chapter ends with a quick discussion of elliptic functions and abelian integrals.

Chapter 6, which claims to be accessible to preschool children, leads the reader to compute the possible number of regions that can be obtained when the plane is cut by n lines. In the Appendix following this, the author's paper on this topic is reproduced.

Thomas C. Craven

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