Preface

This book presents a geometrical introduction to special relativity. By *geometrical*, it is meant that the adopted point of view is four dimensional from the very beginning. The mathematical framework is indeed, from the first chapter, that of *Minkowski spacetime*, and the basic objects are the vectors in this space (often called *4-vectors*). Physical laws are translated in terms of geometrical operations (scalar product, orthogonal projection, etc.) on objects of Minkowski spacetime (4-vectors, worldlines, etc.).

Many relativity textbooks start rather by a three-dimensional approach, using space + time decompositions based on inertial observers. Only in the second stage they introduce 4-vectors and Minkowski spacetime. In this respect, they are faithful to the historical development of relativity. A more axiomatic approach is adopted here, setting from the very beginning the full mathematical framework as one of the postulates of the theory. From this point of view, the chosen approach is similar to that adopted in classical mechanics or quantum mechanics, where usually the exposition does not follow the history of the theory. The history of relativity is undoubtedly rich and fascinating, but the objective of this book is the learning of special relativity within a consistent and operational setting, from the bases up to advanced topics. The text is, however, enriched with historical notes, which include references to the original works and to the studies by historians of science.

Usually, the geometric approach is reserved for *general relativity*, i.e. for the incorporation of the gravitational field in relativity theory.[1] We employ it here for special relativity, taking into account a geometric structure much simpler than that of general relativity: while the latter is based on the concept of *differentiable manifold*, special relativity relies entirely on the concept of *affine space*, which can be identified with the space $\mathbb{R}^4$. Consequently, the mathematical prerequisites are relatively limited; they are mostly linear algebra at the level of the first two years of university. The mathematics used here is actually the same as those of a course

[1]Two notable exceptions are the monographs by Costa de Beauregard (1949) and Synge (1956).

of classical mechanics, provided one is ready to take into account two things: (i) vectors do not belong to a linear space of dimension three, but four, and (ii) the scalar product of two vectors is not the standard scalar product in Euclidean space but is given by a privileged symmetric bilinear form, the so-called *metric tensor*. Once this is accepted, physical results are obtained faster than by means of the "classic" three-dimensional formulation, and a more profound understanding of relativity is acquired. Moreover, learning general relativity is made much easier, starting from such an approach.

In connection with the four-dimensional approach, another characteristic of this monograph is to lay the discussion of physically measurable effects on the most general type of observer, i.e. allowing for accelerated and rotating frames. On the opposite, most of (all?) special relativity treatises are based on a privileged class of observers: the *inertial* ones. Although it is true that for these observers the perception of physical phenomena is the simplest one (for instance, for an inertial observer, light in vacuum moves along a straight line and at a constant speed), the real world is made of accelerated and rotating observers. Therefore, it seems conceptually clearer to discuss first the measures performed by a generic observer and to treat afterwards the particular case of inertial observers. Conversely, if one restricts first to inertial observers, it becomes cumbersome to extend the discussion to general observers. As a matter of fact, this is to a great extent the source of the various "paradoxes" that appeared in the course of the development of relativity. As mentioned above, the three-dimensional approach to relativity is based on inertial observers, since one may associate with each observer of this kind a global decomposition of spacetime in a "time" part and a "space" part.

One of the consequences of the "general observer" approach adopted here is the least weight attributed to the famous *Lorentz transformation* between the frames of two inertial observers. This transformation, which is usually introduced in the first chapter of a relativity course, appears here only in Chap. 6. In particular, the physical effects of time dilation or aberration of light are derived (geometrically) in Chaps. 2 and 4, without appealing explicitly to the Lorentz transformation. Similarly, the *principle of relativity*, on which special relativity has been founded at the beginning of the twentieth century (hence its name!), is mentioned here only in Chap. 9, at the occasion of a historical note.

The plan of the book is as follows. The full mathematical framework (Minkowski spacetime) is set in Chap. 1. The concepts of worldline and proper time are then introduced (Chap. 2) and are illustrated by a detailed exposition of the famous "twin paradox". Chapter 3 is entirely devoted to the definition of an observer and his (local) rest space. This is done in the most general way, taking into account acceleration as well as rotation. The notion of observer being settled, we are in position to address kinematics. This is performed in two steps: (i) by fixing the observer in Chap. 4 (introduction of the Lorentz factor, as well as relative velocity and relative acceleration) and (ii) by discussing all the effects induced by a change of observer in Chap. 5 (laws of velocity composition and acceleration composition, Doppler effect, aberration, image formation, "superluminal" motions in astrophysics). The two chapters that follow are entirely devoted to the Lorentz group, exploring its algebraic structure (Chap. 6), with the introduction of boosts and Thomas rotation, and its Lie group structure (Chap. 7). Chapter 8 focuses on the privileged class of inertial observers, with the introduction of the Poincaré group and its Lie algebra. The dynamics starts in Chap. 9, where the notion of 4-momentum is presented, as well as the principle of its conservation for any isolated system. On its side, Chap. 10 is devoted to the conservation of angular momentum and to the concepts of centre of inertia and spin. Relativistic dynamics is subsequently reformulated in Chap. 11 by means of a principle of least action. The conservation laws appear then as consequences of Noether theorem. A Hamiltonian formulation of the dynamics of relativistic particles is also presented in this chapter. Chapter 12 focuses on accelerated observers, discussing kinematical aspects (Rindler horizon, clock synchronization, Thomas precession) as well as dynamical ones (spectral shift, motion of free particles). A second type of non-inertial observers is studied in Chap. 13: the rotating ones. This chapter ends with an extensive discussion of the Sagnac effect and its application to laser gyrometers in inertial guidance systems on board airplanes.

The second part of the book opens in Chap. 14, where the physical object under focus is no longer a particle but a field. This part starts by three purely mathematical chapters to introduce the notions of tensor (Chap. 14), tensor field (Chap. 15) and integration over a subdomain of spacetime (Chap. 16). Among other things, these chapters present the p-forms and exterior calculus, which are very useful not only for electromagnetism but also for hydrodynamics. We felt necessary to devote an entire chapter to integration in order to introduce with enough details and examples the notions of submanifold of Minkowski spacetime, area and volume element; integral of a scalar or vector field; and flux integral. The chapter ends by the famous Stokes' theorem and its applications. Equipped with these mathematical tools, we proceed to electromagnetism in Chap. 17. Here again, the emphasis is put on the four-dimensional aspect: the electromagnetic field tensor $\boldsymbol{F}$ is introduced first, and the electric and magnetic field vectors $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ appear in a second stage. The motion of charged particles and the various types of particle accelerators are discussed in this chapter. Chapter 18 presents Maxwell equations, here also in a four-dimensional form, which is intrinsically simpler than the classical set of three-dimensional equations involving $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$. The Liénard–Wiechert potentials are derived in this chapter, leading to the electromagnetic field generated by a charged particle in arbitrary motion. Chapter 19 introduces the concept of energy–momentum tensor, a fundamental tool for the dynamics of continuous media in relativity. The principles of conservation of energy–momentum and angular momentum are notably presented in a "continuous" version, as opposed to the "discrete" version considered in Chaps. 9 and 10. The energy–momentum of the electromagnetic field can then be discussed in depth in Chap. 20. In that chapter, the energy and momentum radiated away by a moving charge are computed. A particular case is constituted by synchrotron radiation, whose applications in astrophysics and in synchrotron facilities are discussed. Chapter 21 introduces relativistic hydrodynamics, first in a standard form and next making use of the exterior calculus presented in Chaps. 14–16. The latter approach facilitates greatly the derivation of relativistic generalizations of the classical theorems of fluid mechanics. Two particularly important and contemporary applications are explored in this chapter: relativistic jets in astrophysics and the quark-gluon plasma produced in heavy ion colliders. At last, the book ends by the problem of gravitation (Chap. 22): after some discussion about the unsuccessful attempts to incorporate gravitation in special relativity, the theory of general relativity is briefly introduced. Let us point out that the study of accelerated observers performed in Chap. 12 allows one, via the equivalence principle, to treat easily some relativistic effects of gravitation, such as the gravitational redshift or the bending of light rays.

The book contains six purely mathematical chapters (Chaps. 1, 6, 7, 14, 15 and 16). The aim is to introduce in a consistent and gradual way all the tools required for special relativity, up to rather advanced topics. As a monograph devoted to a theory whose foundations are more than a hundred years old, the book does not contain any truly original result. One may, however, note the general expression of the 4-acceleration of a particle in terms of its acceleration and velocity both relative to a generic observer (i.e. accelerated or rotating) [Eq. (4.60)]; the composition law of relative accelerations resulting from a change of observer and providing the relativistic generalization of centripetal and Coriolis accelerations [Eq. (5.56)]; the complete classification of restricted Lorentz transformations from a null eigenvector (Sect. 6.4); the elementary and relatively short derivation of Thomas rotation in the most general case (Sect. 6.7.2); the expressions of energy and momentum relative to an observer, taking into account the acceleration and rotation of that observer [Eqs. (9.12) and (9.13)]; the computation of the discrepancy between the rest space of an observer and his simultaneity hypersurface (Sect. 12.3); the expression of the 4-acceleration of an observer in terms of physically measurable quantities [Eq. (12.73)]; the equation of motion of a free particle in Rindler coordinates [Eqs. (12.75) and (12.82)]; and the demonstration that the nonrelativistic limit of the canonical equation of fluid dynamics is the Crocco equation (Sect. 21.5.4).

One of the book's limitations is the classical domain: no topic related to quantum mechanics is treated. In particular, spinors and representations of the Poincaré group are not discussed (see, e.g., Cartan (1966), Naber (2012), Penrose and Rindler (1984), Naïmark (1962)). Although these notions are not quantum by themselves, they are mostly used in relativistic quantum theory, notably to write Dirac equation—which we do not address here.

Notes

Notations: In order to facilitate the reading, mathematical notations and symbols introduced in the course of the text are collected in the notation index (p. 761). Throughout the text, the abbreviation *iff* stands for *if, and only if.*

Web page: The page http://relativite.obspm.fr/sperel is devoted to the book. It contains the errata, the clickable list of bibliographic references, all the links listed in Appendix B, as well as various complements. The reader is invited to use this page to report any error that he/she may find in the text.

This book has been first published in French language by EDP Sciences & CNRS Editions in 2010 (Gourgoulhon 2010). The differences with respect to that version are rather minor: they regard some improvements in the presentation and in the figures, as well as some updates in the bibliography.

Meudon, France Éric Gourgoulhon

Springer

http://www.springer.com/978-3-642-37275-9

Special Relativity in General Frames
From Particles to Astrophysics
Gourgoulhon, E.
2013, XXX, 784 p. 178 illus., 19 illus. in color.,
Hardcover
ISBN: 978-3-642-37275-9