Chapter 2
Robotic RMP Schemes and QP Formulations

Abstract Differing from the conventional pseudoinverse-type scheme, an optimization scheme (specifically, a minimization scheme) is presented and investigated in this chapter for online RMP of redundant robot manipulators. Such an RMP scheme, which takes into account the avoidance of joint physical limits (e.g., joint-angle limits and joint-velocity limits), aims at remedying the so-called joint-angle drift problem. Then, some other optimization schemes, which can be viewed as the extensions of the RMP scheme, are developed and investigated for the purpose of repetitive motion planning. These schemes are finally reformulated and unified as QP problems with different definitions of the same coefficients.

2.1 Introduction

A robot manipulator is redundant when more DOF are observed in relation to the minimum number of DOF required to perform a given end-effector primary task [1, 2]. One fundamental issue in operating such redundant robot systems is the online redundancy-resolution problem (or to say, the inverse kinematics problem) [1–6]. Note that there are multiple feasible solutions to the redundancy-resolution problem. By properly resolving the redundancy, the robots can avoid joint physical limits and environmental obstacles, apart from optimizing various secondary criteria [1–6]. To take full advantage of the redundancy, various computational schemes, including approaches based on QP, have thus been proposed, developed, and investigated. The conventional solution to such a redundancy-resolution problem mostly takes the pseudoinverse-type formulation (i.e., a minimum-norm particular solution plus a homogeneous solution) [2, 7–10]. Recent research [1, 3–6, 11–17] shows that the redundancy-resolution problem may be solved in a more favorable manner by using online optimization techniques (e.g., the QP formulation and approach).

In general, there are several extrinsic and inherent factors such as end-effector motion requirement, joint physical limits, and optimization of secondary criteria, which can greatly affect the motion planning of redundant robot manipulators, some allowing kinematic controls to become nonrepetitive [17]. If the redundancy-resolution schemes (e.g., the conventional pseudoinverse-type schemes) are unsuitable for a number of particular end-effector tasks, the final configuration of the robot manipulator may not coincide well with the initial configuration (i.e., the mo-

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tion planning of the robot manipulators is not repetitive). The so-called joint-angle drift phenomenon (also referred to as the nonrepetitive problem) implies that, when the robot end-effector tracks a closed path in its workspace, the joint variables may not return to their initial values after completing the end-effector task [11–17]. In other words, the trajectories obtained in the joint space may not be closed. Problems may arise from the robot manipulator's unpredictable behavior and even lead to less efficiency in readjusting the manipulator's configuration with self-motion at every cycle [18, 19]. Thus, it makes good sense if the repetitive motion planning (RMP) of redundant robot manipulators can be achieved by a scheme (e.g., the quadratic-form RMP scheme [12, 13, 15, 17]).

In this chapter, differing from the conventional pseudoinverse-type scheme, an RMP scheme (i.e., joint-angle drift remedy scheme) is presented and investigated for redundant robot manipulators. Note that joint physical limits are incorporated into such an RMP scheme which aims at making the kinematic control repeatable (also referred to as repetitive or cyclic). Then, being the extensions of the first RMP scheme, some other optimization schemes are developed and investigated for the purpose of repetitive motion planning. Finally, these schemes are reformulated and unified as QP problems with different definitions of the same coefficients, which can be solved readily by using recurrent neural networks [1, 3–6, 13, 15, 20] or numerical algorithms based on the linear variational inequality [21–24].

2.2 Physically Constrained RMP Scheme

The pseudoinverse-type solution of the redundancy resolution problem for the redundant robot manipulator, such as (1.3), may not be repetitive [7]. In other words, a closed path of the end-effector may not yield the closed trajectories in the joint space. Such a joint-angle drift is not acceptable for cyclic motion planning and control. The manipulator's configuration can be readjusted with manipulator self-motion processes (i.e., without changing the end-effector's position and orientation) [18, 19], but this would be less efficient. To make the inverse-kinematics solution itself repetitive, the minimization of the joint displacement between current and initial states could be exploited [12, 13, 15, 17]. In the formulation, the performance index used is

$$\frac{1}{2}(\dot{\theta}+\hat{c})^{\mathrm{T}}(\dot{\theta}+\hat{c}) \quad \text{with } \hat{c}=\lambda\big(\theta-\theta(0)\big), \tag{2.1}$$

where $\theta(0)$ is the initial state of the joint-angle vector. The design parameter, $\lambda>0$, is used to scale the magnitude of the manipulator response to the joint displacement. The design parameter λ should be set as large as the robot system would permit or selected appropriately for simulative and/or experimental purposes. Note that theoretical analysis for the effectiveness of such a performance index (2.1) on RMP of robots will be presented in the next chapter via two different approaches, namely, the gradient-descent method and Zhang et al.'s neural-dynamic method. Now, based on the repetitive performance index (2.1), we have the following basic problem formulation for RMP of redundant robot manipulators:

$$\text{minimize} \quad \frac{1}{2}(\dot{\theta} + \hat{c})^{\mathrm{T}}(\dot{\theta} + \hat{c}) \quad \text{with } \hat{c} = \lambda\big(\theta - \theta(0)\big),$$
$$\text{subject to} \quad J(\theta)\dot{\theta} = \dot{r}.$$

For the above basic RMP scheme, the performance index can be reformulated as $(\dot{\theta} + \hat{c})^{\mathrm{T}}(\dot{\theta} + \hat{c}) = \dot{\theta}^{\mathrm{T}}\dot{\theta} + 2\hat{c}^{\mathrm{T}}\dot{\theta} + \hat{c}^{\mathrm{T}}\hat{c}$. Since such an RMP scheme is resolved at the joint-velocity level and the decision variable vector is the joint velocity $\dot{\theta}$, the joint angle θ can be regarded as a constant in the minimization of the performance index [i.e., "minimize $(\dot{\theta} + \hat{c})^{\mathrm{T}}(\dot{\theta} + \hat{c})/2$"]. In addition, $\hat{c}^{\mathrm{T}}\hat{c}$ is positive and viewed as a constant (with respect to $\dot{\theta}$), which is thus set aside from "minimize $(\dot{\theta} + \hat{c})^{\mathrm{T}}(\dot{\theta} + \hat{c})/2$". Therefore, the minimization of $(\dot{\theta} + \hat{c})^{\mathrm{T}}(\dot{\theta} + \hat{c})/2$ is equivalent to the minimization of $\dot{\theta}^{\mathrm{T}}\dot{\theta}/2 + \hat{c}^{\mathrm{T}}\dot{\theta}$. Besides, in view of the fact that almost all robot manipulators are physically constrained by their joint-angle limits and joint-velocity limits, we have the following more realistic and useful RMP scheme for redundant robot manipulators [12, 13, 15, 17]:

$$\text{minimize} \quad \frac{1}{2}\dot{\theta}^{\mathrm{T}}\dot{\theta} + \hat{c}^{\mathrm{T}}\dot{\theta} \quad \text{with } \hat{c} = \lambda\big(\theta - \theta(0)\big), \tag{2.2}$$
$$\text{subject to} \quad J(\theta)\dot{\theta} = \dot{r}, \tag{2.3}$$
$$\theta^{-} \leq \theta \leq \theta^{+}, \tag{2.4}$$
$$\dot{\theta}^{-} \leq \dot{\theta} \leq \dot{\theta}^{+}, \tag{2.5}$$

where (2.3) is exactly (1.2), which describes the end-effector primary task, i.e., the path. More importantly, the avoidance of joint-angle limits $\theta^{\pm}$ and joint-velocity limits $\dot{\theta}^{\pm}$ are considered, with superscripts $^{+}$ and $^{-}$ denoting the upper and lower limits of a joint variable vector (e.g., θ or $\dot{\theta}$), respectively. Since the performance index (2.2) is quadratic, the above optimization scheme (2.2)–(2.5) is hereafter termed quadratic-form physically-constrained RMP scheme for presentation convenience, or termed physically-constrained RMP scheme for simplicity in this book. Lastly, the comparison between the quadratic-form physically constrained RMP scheme and the pseudoinverse-based techniques can be presented as follows [17].

Remark 2.1 Unlike the conventional pseudoinverse-based techniques [2, 7, 8], the physically constrained RMP scheme plans repetitive motion of redundant manipulators in an explicit inverse-free manner, in order to avoid expensive $O(n^3)$ computation. Moreover, physical constraints [i.e., in the form of bound constraints (2.4) and (2.5)] can be incorporated into the presented physically constrained RMP scheme (2.2)–(2.5), whereas pseudoinverse-based techniques do not consider physical constraints; and this is also true for algorithmic singularities [8].

Remark 2.2 Based on the extended Jacobian technique [9, 10], if Jacobian matrix J is a full-rank matrix of dimension $m \times n$, then any matrix P for which $JP = I$

can be obtained by adding appropriately selected $n-m$ constraint-rows to J such that the $n \times n$ extended Jacobian matrix obtained becomes nonsingular. However, such $n-m$ constraints may not take full advantage of redundancy (e.g., they may not permit inequality-based obstacle avoidance [5]) and may artificially limit the end-effector workspace. In addition, the extended Jacobian technique also limits the total number of the physical constraints (i.e., $n-m$), which may have less practical significance in the field of engineering. In contrast, by using the presented physically constrained RMP scheme (2.2)–(2.5) that incorporates the avoidance of joint physical limits [i.e., (2.4) and (2.5)] as a subtask of redundancy-resolution, the proposed framework can also handle other subtasks, such as obstacle avoidance [5], formulated in terms of constraints and/or performance indices.

2.3 Extensions of Physically Constrained RMP Scheme

As mentioned above, the minimization of the joint displacement between current and initial states should be exploited to achieve the RMP purpose. Based on such an idea, some extensions of the above physically constrained RMP scheme (2.2)–(2.5) are discussed, developed, and investigated in this section to remedy the joint-angle drift phenomenon of redundant robot manipulators.

2.3.1 Extension I

In this subsection, the extension-I optimization scheme [being the first extension of the RMP scheme (2.2)–(2.5)] is developed and investigated for RMP of redundant robot manipulators.

The performance index for the extension-I optimization scheme is formulated as

$$\frac{1}{2}\dot{\theta}^{\mathrm{T}} W \dot{\theta} + \hat{c}^{\mathrm{T}} \dot{\theta}, \tag{2.6}$$

where $W := \|\theta - \theta(0)\|_2^2 I$ (with $\|\cdot\|_2$ denoting the two-norm (or termed, Euclidean norm) of a vector, and I denoting the identity matrix), and $\hat{c}$ is defined as before. Note that such a performance index is obtained by defining the scalar-valued error function $\hat{\varepsilon} = \ln(\|\theta - \theta(0)\|_2^2)$ and exploiting the gradient-descent method [16]. Please refer to Appendix A for proof details. Thus, by incorporating the joint physical limits into the scheme formulation, we have the following RMP scheme for physically constrained redundant robot manipulators [16]:

$$\text{minimize} \quad \frac{1}{2}\dot{\theta}^{\mathrm{T}} W \dot{\theta} + \hat{c}^{\mathrm{T}} \dot{\theta} \quad \text{with } \hat{c} = \lambda\big(\theta - \theta(0)\big), \tag{2.7}$$

$$\text{subject to} \quad J(\theta)\dot{\theta} = \dot{r}, \tag{2.8}$$

$$\theta^{-} \le \theta \le \theta^{+}, \tag{2.9}$$

$$\dot{\theta}^- \leq \dot{\theta} \leq \dot{\theta}^+. \tag{2.10}$$

Hereafter, for presentation convenience, the above scheme (2.7)–(2.10) is termed in this book the extension-I optimization scheme (or simply, extension-I scheme) for repetitive motion planning of redundant robot manipulators. The efficacy of the extension-I optimization scheme (2.7)–(2.10) is shown in Appendix B.

2.3.2 Extension II

Facing the success of the aforementioned RMP schemes [i.e., scheme (2.2)–(2.5) and scheme (2.7)–(2.10)] [12, 13, 15–17], we may think of a new performance index for such drift-free purposes, which is formulated as follows [11]:

$$\frac{1}{2}\dot{\theta}^{\mathrm{T}} W \dot{\theta}, \tag{2.11}$$

where W is defined as before, i.e., $W = \|\theta - \theta(0)\|_2^2 I$. Evidently, by minimizing such a performance index (2.11) we can minimize "clearly" the squared norm of joint displacement between current and initial states [in the sense that "when θ equals $\theta(0)$, (2.11) is minimized to be zero; otherwise, it is positive"]. Therefore, with the performance index (2.11) minimized and the joint physical limits considered, the extension-II optimization scheme for the RMP purposes of redundant robot manipulators is formulated as below [11]:

$$\text{minimize} \quad \frac{1}{2}\dot{\theta}^{\mathrm{T}} W \dot{\theta}, \tag{2.12}$$

$$\text{subject to} \quad J(\theta)\dot{\theta} = \dot{r}, \tag{2.13}$$

$$\theta^- \leq \theta \leq \theta^+, \tag{2.14}$$

$$\dot{\theta}^- \leq \dot{\theta} \leq \dot{\theta}^+, \tag{2.15}$$

which is the second extension of physically constrained RMP scheme (2.2)–(2.5). It is worth pointing out here that the extension-II optimization scheme (2.12)–(2.15) can be derived from the extension-I optimization scheme (2.7)–(2.10) with design parameter λ being zero. In this sense, the extension-II optimization scheme can be viewed as a special case of the extension-I optimization scheme. However, the inability of extension-II optimization scheme (2.12)–(2.15) is shown in Appendix C.

2.3.3 Extension III

Based on lots of simulative results for the physically constrained RMP scheme [12, 13, 15, 17], we find that the linear part of the performance index (2.2) (i.e., the so-called drift-free criterion $\hat{c}^{\mathrm{T}}\dot{\theta}$) plays an important role in the success of the RMP scheme (2.2)–(2.5) on redundant robot manipulators. Thus, in this subsection, the following linear performance index is exploited in order to achieve the repetitive motion planning of redundant robot manipulators:

$$\hat{c}^{\mathrm{T}}\dot{\theta}, \tag{2.16}$$

where $\hat{c}$ is defined as before, i.e., $\hat{c} = \lambda(\theta - \theta(0))$. Then, similar to the previous subsections, by minimizing the performance index (2.16) and considering the joint physical limits, we have the following redundancy-resolution scheme for the RMP purpose [14]:

$$\text{minimize} \quad \hat{c}^{\mathrm{T}}\dot{\theta} \quad \text{with } \hat{c} = \lambda\big(\theta - \theta(0)\big), \tag{2.17}$$

$$\text{subject to} \quad J(\theta)\dot{\theta} = \dot{r}, \tag{2.18}$$

$$\theta^{-} \le \theta \le \theta^{+}, \tag{2.19}$$

$$\dot{\theta}^{-} \le \dot{\theta} \le \dot{\theta}^{+}, \tag{2.20}$$

which is called the extension-III optimization scheme in this book for presentation convenience. Note that such a performance index is also the linear part of the performance index (2.7), which has great effect on the success of the extension-I optimization scheme (2.7)–(2.10) (please refer to the simulation part in Appendix B for details) [16]. Evidently, the extension-I optimization scheme can be obtained by combining the extension-II optimization scheme (viewed as the quadratic part) and the extension-III optimization scheme (viewed as the linear part). The interesting results about the extension-III optimization scheme (2.17)–(2.20) are shown in Appendix D.

In summary, being extensions of physically constrained RMP scheme (2.2)–(2.5), these optimization schemes have been developed and discussed in this section for achieving the RMP purpose of redundant robot manipulators.

2.4 QP Reformulation and Unification

In the previous sections, the presented physically constrained RMP scheme (2.2)–(2.5) and its three extensions [i.e., extension-I optimization scheme (2.7)–(2.10), extension-II optimization scheme (2.12)–(2.15), and extension-III optimization scheme (2.17)–(2.20)] have been developed and investigated for RMP of redundant robot manipulators. In this section, these optimization schemes are reformulated and unified as QP problems. Note that the QP unification of these schemes brings more insights into the wealth of existing solutions, as well as a better understanding of future researches (e.g., the unification of various redundancy-resolution schemes via the QP technique).

Since the redundancy-resolution problem is solved at the joint-velocity level, the limited joint-angle range $[\theta^{-}, \theta^{+}]$ (in the presented scheme formulations) has to be

Table 2.1 Definitions of the coefficients Q and $\hat{q}$ for the presented optimization schemes

Scheme	Definition of Q	Definition of $\hat{q}$
Physically constrained RMP scheme (2.2)–(2.5)	$Q := I \in R^{n\times n}$	$\hat{q} := \hat{c} \in R^n$
Extension-I optimization scheme (2.7)–(2.10)	$Q := W \in R^{n\times n}$	$\hat{q} := \hat{c} \in R^n$
Extension-II optimization scheme (2.12)–(2.15)	$Q := W \in R^{n\times n}$	$\hat{q} := 0 \in R^n$
Extension-III optimization scheme (2.17)–(2.20)	$Q := 0 \in R^{n\times n}$	$\hat{q} := \hat{c} \in R^n$

converted into an expression based on the joint velocity $\dot{\theta}$ [12, 13, 15, 17]. The new bound constraint can thus be written as

$$\mu\left(\theta^- - \theta\right) \le \dot{\theta} \le \mu\left(\theta^+ - \theta\right), \tag{2.21}$$

where large values of parameter $\mu > 0$ may cause quick joint deceleration as the robot approaches its limits [13, 15, 17]. Normally, μ is determined in order for the converted feasible region of $\dot{\theta}$ (i.e., by the joint-angle limits $[\theta^-, \theta^+]$) to be not smaller than the original region made by the joint-velocity limits $[\dot{\theta}^-, \dot{\theta}^+]$. In mathematics, μ should be more than or equal to $2\max_{1\le i\le n}\{\dot{\theta}_i^+/(\theta_i^+ - \theta_i^-), -\dot{\theta}_i^-/(\theta_i^+ - \theta_i^-)\}$ [13, 15, 17].

Combining the bound-constraints (2.21) and $\dot{\theta}^- \le \dot{\theta} \le \dot{\theta}^+$ yields a unified dynamic bound-constraint, $\xi^- \le \dot{\theta} \le \xi^+$, where the ith elements of ξ^- and ξ^+ are defined respectively as [13, 15, 17]:

$$\xi_i^- = \max\left\{\dot{\theta}_i^-, \mu\left(\theta_i^- - \theta_i\right)\right\}, \qquad \xi_i^+ = \min\left\{\dot{\theta}_i^+, \mu\left(\theta_i^+ - \theta_i\right)\right\}.$$

Based on the above handling of joint physical limits (i.e., the combination of joint-angle and joint-velocity limits), the presented scheme for RMP purpose can be reformulated and unified as the following quadratic program:

$$\text{minimize} \quad \frac{1}{2}\hat{x}^{\mathrm{T}}Q\hat{x} + \hat{q}^{\mathrm{T}}\hat{x}, \tag{2.22}$$

$$\text{subject to} \quad J\hat{x} = \hat{d}, \tag{2.23}$$

$$\xi^- \le \hat{x} \le \xi^+, \tag{2.24}$$

where the decision variable vector $\hat{x} := \dot{\theta}$ and $\hat{d} = \dot{r}$. Note that the definitions of coefficients $Q \in R^{n\times n}$ and $\hat{q} \in R^n$ for the presented optimization schemes are different from those for each other, which are shown in Table 2.1. So, the quadratic-form or linear-form schemes in the table all can be termed QP-based schemes as they can be reformulated, unified and solved as QPs; and the linear-form extension-III optimization scheme (2.17)–(2.20) can also be termed an LP-based scheme as it can be reformulated and solved as LP (note that LP is a special case of QP and that the linear form is a special case of the quadratic form).

Remark 2.3 As seen from Table 2.1, $Q = I$ and $\hat{q} = \hat{c}$ are set in the QP reformulation for the presented RMP scheme (2.2)–(2.5). Therefore, the coefficient matrix

Q is positive definite, which implies that the objective function in (2.22) is strictly convex. In view of the feasible region made by linear constraints (2.23) and (2.24) being a closed convex set, it follows from [25] that the constrained optimal-solution to QP problem (2.22)–(2.24) [corresponding to the most preferred physically constrained RMP scheme (2.2)–(2.5)] exists uniquely. In light of the uniqueness property, the continuity of the QP solution can thus be guaranteed. It is worth pointing out that the presented unified QP problem (2.22)–(2.24) can be solved by using the recurrent neural networks (e.g., the LVI-based primal–dual neural network) [1, 3–6, 13, 15, 20] and LVI-based numerical algorithms [21–24], which are detailed in Chaps. 4 through 7 of Part II of this book.

2.5 Chapter Conclusions

In this chapter, the physically constrained RMP scheme (2.2)–(2.5) has been presented and investigated to remedy the joint-angle drift phenomena in motion planing of redundant robot manipulators (i.e., to make the kinematic control repeatable). In addition, being the extensions of (2.2)–(2.5), three optimization schemes [i.e., extension-I optimization scheme (2.7)–(2.10), extension-II optimization scheme (2.12)–(2.15), and extension-III optimization scheme (2.17)–(2.20)] have been developed and investigated to achieve the purpose of RMP for redundant robot manipulators. More importantly, these optimization schemes have been finally reformulated and unified as QP problems depicted in (2.22)–(2.24) with different definitions of the same coefficients [i.e., with different Q and $\hat{q}$ in (2.22)].

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