Preface to Books 2–3

Books 2–3 correspond to Chapters V–IX of the first edition. They study schemes and complex manifolds, two notions that generalise in different directions the varieties in projective space studied in Book 1. Introducing them leads also to new results in the theory of projective varieties. For example, it is within the framework of the theory of schemes and abstract varieties that we find the natural proof of the adjunction formula for the genus of a curve, which we have already stated and applied in Section 2.3, Chapter 4. The theory of complex analytic manifolds leads to the study of the topology of projective varieties over the field of complex numbers. For some questions it is only here that the natural and historical logic of the subject can be reasserted; for example, differential forms were constructed in order to be integrated, a process which only makes sense for varieties over the (real or) complex fields.

Changes from the First Edition

As in the Book 1, there are a number of additions to the text, of which the following two are the most important. The first of these is a discussion of the notion of the algebraic variety classifying algebraic or geometric objects of some type. As an example we work out the theory of the Hilbert polynomial and the Hilbert scheme. I am very grateful to V.I. Danilov for a series of recommendations on this subject. In particular the proof of Theorem 6.7 is due to him. The second addition is the definition and basic properties of a Kähler metric, and a description (without proof) of Hodge's theorem.

Prerequisites

Varieties in projective space will provide us with the main supply of examples, and the theoretical apparatus of Book 1 will be used, but by no means all of it. Different sections use different parts, and there is no point in giving exact indications. References to the Appendix are to the Algebraic Appendix at the end of Book 1.

Prerequisites for the reader of Books 2–3 are as follows: for Book 2, the same as for Book 1; for Book 3, the definition of differentiable manifold, the basic theory of analytic functions of a complex variable, and a knowledge of homology, cohomology and differential forms (knowledge of the proofs is not essential); for Chapter 9, familiarity with the notion of fundamental group and the universal cover. References for these topics are given in the text.

Recommendations for Further Reading

For the reader wishing to go further in the study of algebraic geometry, we can recommend the following references.

For the cohomology of algebraic coherent sheaves and their applications: see Hartshorne [37].

An elementary proof of the Riemann–Roch theorem for curves is given in W. Fulton, Algebraic curves. An introduction to algebraic geometry, W.A. Benjamin, Inc., New York–Amsterdam, 1969. This book is available as a free download from http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf.

For the general case of Riemann–Roch, see A. Borel and J.-P. Serre, Le théorème de Riemann–Roch, Bull. Soc. Math. France **86** (1958) 97–136,

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