

Preface

This book is aimed at providing a unified eigenvalue density formulation of matrix models and discussing the corresponding phase transitions and critical phenomena. The purpose of this book is to systematically classify the transition models based on the potential functions that define the consequent integrable systems, string equations, eigenvalue densities, free energies and critical points considered in the transitions.

The unification of different couplings or interactions is one of the important research topics in quantum chromodynamics (QCD) theory that can help to study the dominance of confinement or asymptotic freedom in low or high energy scale. Hilbert space theory and Fourier analysis are the necessary mathematical tools to work on the physical problems. As a foundation of the Hilbert space theory, orthogonal polynomials can provide a new mathematical background for further investigating the fundamental concepts such as position and momentum described in the Heisenberg uncertainty principle that is tightly related to matrix models which are important in QCD.

In quantum physics the probability amplitude for a particle to travel from one point to another in a given time is characterized by propagator. By the Fourier transform, the propagator becomes a singular function in the momentum space, where the singularity is related to the uncertainty. Correlation function in the matrix models can avoid such singularity in large- N asymptotics that leads the eigenvalue density to represent the momentum so that free energy function can be properly formulated. String equation is a tool in the momentum aspect to construct the eigenvalue densities and give parameter relations in the models for finding the phase transitions. The associated integrable systems imply a unified formula for the different densities in the matrix models by using the Lax pair structures obtained from the corresponding orthogonal polynomials.

Analytic results derived from the integrable systems will reduce the mathematical complexity in discussing the phase transition problems of the matrix models. The different density phases can be generated from the scalings of the string equation and associated discrete differential equations in the Lax pair with proper periodic reductions implemented by using an index folding technique in large- N scaling.

The different scalings can have a common case which is the critical point separating two phases with different conditions. The string equation properly establishes the nonlinear relations between the parameters in the model such that behaviors of the free energy around the critical point can be easily obtained, either analytically or by the expansions according to the parameter relations. The first-, second- and third-order transition models can be created by using the string equation, Toda lattice and corresponding integrable systems. Expansions for the coupling parameters in association with the double scalings present a new strategy to find the divergence transitions or critical phenomena with a fractional power-law.

Phase transition models discussed by using the string equations differ from the transitions in traditional models such as the Ising models, but there is a similarity between the periodic reduction that reorganizes wave functions in the momentum aspect using the index folding and the idea of renormalization in statistical mechanics that reorganizes the particles in the position aspect. Typically, the critical point in the bifurcation transition is when the center or radius parameter is bifurcated, while the critical point in the renormalization method is a fixed point in an iteration process. The power-law at the critical point in the momentum aspect is derived from the algebraic equation reduced from the string equation that is the parameter condition(s) different from the renormalization groups considered in the Ising models. In addition, eigenvalue density on multiple disjoint intervals and corresponding free energy can be referred to study Seiberg-Witten differential and prepotential in the Seiberg-Witten theory which is developed to solve the mass gap problem in quantum Yang-Mills theory.

The organization of this book is as follows. Chapter 1 is about the physical background of the matrix models. The unified model proposed in this book is fundamental for the phase transitions. Chapter 2 is for the reduction of eigenvalue densities from the integrable systems. In Chaps. 3 and 4, various transitions will be discussed for the Hermitian matrix models. In Chaps. 5 and 6, we will talk about the transitions and critical phenomena in the unitary matrix models, including the Gross-Witten third-order phase transition. Chapter 7 deals with the Marcenko-Pastur distribution, McKay's law and their generalizations in association with the Laguerre and Jacobi polynomials.

This book is about how to find the phase transitions, which can be used as a reference book for researchers and students in the fields of phase transitions and critical phenomena in quantum physics. I thank Professor J. Bryce McLeod for his directions and help on both the related works in this book and previous works. I thank Craig A. Tracy for introducing the random matrix theory, and thank Gunduz Caginlalp, Xinfu Chen, Palle Jorgensen, Juan Manfredi and William Troy for the useful discussions or suggestions. And I thank Xing-biao Hu, Zaijiu Shang and Lianwen Zhang for the helpful discussions and encouragement.

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