

## 2 Theoretical basis

Until the end of the 1970s, the behavior of bounded rationality was believed to be chaotic and unsuited to modeling (Wakker, 2010). Until then, normative expected utility theory was seen to be the best describer of behavior (Friedman & Savage, 1948; Arrow, 1971). “Kahneman & Tversky’s (1979) prospect theory provided a major breakaway. It was the first descriptive theory that explicitly incorporated irrational behavior in an empirically realistic manner (Kahneman, 2003, p. 1456), while at the same time being systematic and tractable” (Wakker, 2010, p. 2).

Following rank-dependent or cumulative functional utility, first proposed by Quiggin (1982) for decision under risk, and second by Schmeidler (1989) for decision under uncertainty, Tversky and Kahneman provided an improved version of prospect theory in 1992. Within this improved version, they combined the empirical insights of Kahneman and Tversky (1979) with the theoretical insights of Schmeidler (1989). From 1992 on, there thus exists broad agreement in the research community that utility theory is not an adequate descriptive model of decision making.

The next several chapters present decision making models under uncertainty and risk to understand how individuals make decisions. This chapter starts with a brief summary of expected utility theory and some apparent anomalies. Then, it will be shown how the most widely accepted non-expected utility counterpart – prospect theory – integrates these anomalies in descriptive patterns into an alternative theory, which then will be presented briefly. This chapter concludes with the main limitation of prospect theory – the constitution and formation of reference points.

For more details on prospect theory, see Kahneman and Tversky (1979, 1984). For the extended theory of risk (known probabilities such as flipping a coin) to uncertainty and ambiguity (unknown probabilities such as filing a lawsuit), see Tversky and Kahneman (1991, 1992) for riskless choices. For experiments that illustrate these effects, please see Kahneman et al. (1990, 1991).

## 2.1 Expected utility theory

“It is no exaggeration to consider expected utility theory the major paradigm in decision making since the Second World War. It has been used prescriptively in management science (especially decision analysis), predictively in finance and economics, descriptively by psychologists, and has played a central role in theories of measurable utility” (Schoemaker, 1982, p. 529).

The mathematical form of expected utility theory goes back to Bernoulli (1738, 1954), who wanted to explain the St. Petersburg paradox.<sup>3</sup> He suggested that individuals do not maximize expected utility by the expectation of their monetary outcomes, but rather by the expected value of these outcomes (Kahneman & Tversky, 1984). Modern foundations of the theory are more than 60 years old and extend prior works with the development of an axiomatic choice-theoretic foundation. The two best-received variations of the theory are subjective expected utility theory in the case of uncertainty and the von Neumann–Morgenstern theory in the case of risk. Von Neumann and Morgenstern (1944, 1947) firstly formalized the theory and Savage (1954) further developed the theory for risk by including the notion of subjective<sup>4</sup> probabilities.

Expected utility theory suggests that individuals make risky or uncertain decisions by comparing their expected utility values. These expected utility values are weighted prospects (mental anticipations, expectations) yielded by multiplying the utility values of uncertain and risky outcomes by their respective preferences or probabilities. In subjective expected utility theory, the probabilities are personal or subjective parameters that characterize the decision maker (Wakker, 2010), according to the expectation of their utility values. Individuals maximize expected values based on those probabilities.

The fundamental expected utility theory of von Neumann and Morgenstern (1944) consists of four general and simple assumptions that define a rational

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<sup>3</sup> St. Petersburg issue: Consider the following well-known game, which involves flipping a fair coin until the first head occurs. The payoff of this experiment depends on the number of immediate heads. The first immediate head gives \$2, and after each tail the payoff will double. Basically, after 19 tails is possible to be a millionaire. Interestingly, individuals were willing to pay just something around \$5 to play this game (Wakker, 2010).

<sup>4</sup> For the contrast between objective and subjective probability, please see Fine (1973) and Fishburn (1986).

individual. It assumes that preference ordering over a given set of lotteries or gambles is complete, transitive, continuous, and independent.

*Completeness axiom:* This axiom assumes that decision makers (1) have well-defined preferences, (2) are able to choose between any two alternatives, and (3) are able to rank them all. Specifically, this means  $X$  is either preferred by an individual to  $Y$  ( $X \succ Y$ ), or is indifferent between them by  $X \sim Y$ , or  $Y$  to  $X$  is preferred ( $Y \succ X$ ).

*Transitivity axiom:* Transitivity assumes that decision makers also decide consistently. If an individual prefers forever  $X$ ,  $Y$ , and  $Z$  with  $X \succ Y$  and  $Y \succ Z$ , decision makers must prefer  $X \succ Z$ .

*Continuity axiom:* The continuity assumption is needed to ensure that for any gamble some probabilities exist by which decision makers are indifferent between the best and worst alternatives. If a decision maker prefers  $X$  to  $Y$  and  $Y$  to  $Z$  ( $X \succ Y \succ Z$ ), then there exists a possible combination of  $X$  and  $Z$  for which a decision maker is indifferent between  $X$  and  $Z$  and the gamble  $Y$ .

*Independence axiom:* The independence axiom implies that replacing a prospect in a mixture ( $X$  &  $Y$ ) by another similar prospect  $Z$  does not affect the preference value of the mix (Wakker, 2010). Imagine that gamble  $X$  is weakly preferred to gamble  $Y$  if a compound gamble that yields  $X$  with probability  $q$  and the third gamble  $Z$  with the probability  $1-q$  is weakly preferred to a compound gamble that yields  $Y$  with probability  $q$  and  $Z$  with probability  $1-q$ , for any gamble  $Z$ . Therefore, the choice between  $X$  and  $Y$  is independent of the third one (Holt, 1986).

In sum, “the key characteristics of this general maximization model are: (1) a holistic evaluation of alternatives, (2) separable transformations on probabilities and outcomes, and (3) an expectation-type operation that combines probabilities and outcomes multiplicatively (after certain transformations)” (Schoemaker, 1982, p. 530). Thus, mathematical properties are additive and if the independence axiom is also satisfied utility is linear (Andreoni & Sprenger, 2011). In other words, if all four axioms are given, the decision maker is assumed to be rational and the preferences are able to be represented by a utility function.

Several variants of the expected utility model exist (e.g., certainty equivalence theory (DeFinetti, 1937; Schneeweiss, 1974; Handa, 1977), subjective expected utility (Ramsey, 1931; Savage, 1954; Edwards, 1955; Quiggin, 1982), subjectively weighted utility (Karmarkar, 1978)). Schoemaker (1982) provided an overview of these major expected utility variants. The alternatives depend on how utility is measured, what types of probability transformations are allowed, and how the outcomes are measured (Schoemaker, 1982). For further explanations and empirical examples, please see also, for instance, Wakker (2010).

## **2.2 Violations of expected utility theory and the need for an alternative theory**

In a perfect world, expected utility theory would be ideal, as it provides exact measurements of utility and gives perfect predictions (Wakker, 2010). However, usually there exists the behavior of bounded rationality and biases, and thus it is not a reliable model for depicting human behavior. In particular, the non-linearity of preferences (Allais, 1953; Camerer & Ho, 1994), loss aversion behavior (Kahneman & Tversky, 1984), reference dependency (Kahneman & Tversky, 1979), framing effects (Tversky & Kahneman, 1986), and mental accounting (Thaler, 1999) cannot be sufficiently explained. The following chapter provides several behavioral phenomena of bounded rationality that violate the assumptions of expected utility theory.

### *Non-linear decision weights*

Expected utility theory assumes that the decision maker's preferences over prospects are represented by linear responses to variations in probability and utility (Wakker, 2010). However, studies have shown non-linear preferences in choices (Allais, 1953; Machina, 1987; Camerer & Ho, 1991), and that the ideas of transitivity ( $X > Y$  and  $Y > Z$ , then  $X > Z$ ) could be questioned (Tversky & Kahneman, 1986). For instance, the famous Allais paradox (Allais, 1953) shows that some behavior is better explained by utility functions that are non-linear. Imagine the following situation. Although the probabilities of winning a lottery are minuscule (.002; .001), most participants choose the prospect that offers the larger gain (3000; 6000), while in another lottery with substantial probabilities of winning

(.90; .45), people choose the prospect where winning is probable (3000; 6000).<sup>5</sup> Moreover, the following phenomena support the non-linearity of preferences.

### Certainty effect

According expected utility theory, individuals should weight prospects by respective probabilities. However, individuals overweight outcomes that are considered to be certain relative to outcomes that are merely probable (Kahneman & Tversky, 1979). For instance, the Allais paradox (1953) for uncertainty shows that the difference between the probabilities of .99 and 1.00 has more impact on preferences than the difference between .10 and .11 (Tversky & Kahneman, 1992).

### Source dependency

Besides the degree of uncertainty, a decision maker's preferences also depend on the source of uncertainty. "Ellsberg (1961) observed that people prefer to bet on an urn containing equal numbers of red and green balls, rather than on an urn that contains red and green balls in unknown proportions" (Tversky & Kahneman, 1992, p. 298). Moreover, individuals more often prefer a bet on an event in their fields of competence to a bet on a matched chance event even though the previous probability is ambiguous and the latter is clear (Heath & Tversky, 1991; Tversky & Kahneman, 1992).

### *Reference dependence and loss aversion*

Utility theory considers the assumption that the outcomes of risky prospects are evaluated as final asset positions. However, there are overwhelming observations that suggest that decision makers are often more sensitive to how an outcome differs from some reference level than to the absolute level of the decision outcome itself (Rabin, 1997). In particular, several studies have provided evidence that decision outcomes are described as gains and losses to some subjective ref-

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<sup>5</sup> The "Allais paradox, can be interpreted as saying that when probability of a worst outcome are tiny (...), a person demands a huge increase in the probability of the best outcome to compensate for increasing the probability of the worst outcome by a little (...); but when the probability of the worst outcome is large (...), a person is more willing to accept an increase in the likelihood of the worst outcome to increase the probability of the best outcome (...)." (Rabin, 1997, p. 10).

erence level or reference point (e.g., Kahneman & Tversky, 1979; Camerer, 1998). According to the psychological analysis of value, reference levels play a role in determining preferences (Tversky & Kahneman, 1991). The following phenomena stress the salience of changes from reference points as the basics of human behavior:

### Reflection effect

Individuals tend to be risk-averse over prospects involving gains, which is accompanied by risk-seeking behavior over prospects involving losses (Kahneman & Tversky, 1979). This implies that individuals prefer risk-averse behavior in the domain of gains, but risk-seeking behavior in the domain of losses in order to compensate for possible deficits. Of primary importance for the reflection effect is that outcomes are coded relative to a reference point (Schneider & Lopes, 1986). Moreover, experiments have shown that choices between prospects are not determined solely by the probability of final states but rather by perceived gains and losses compared with this reference point (Kahneman & Tversky, 2000).

### Risk-seeking

Individuals tend to prefer small probabilities of winning a large prize to the expected value of that prospect (Tversky & Kahneman, 1992). In addition, risk-seeking occurs when individuals must choose between a secure loss and a substantial probability of a large loss (Tversky & Kahneman, 1992). Therefore, individuals simultaneously tend to (unknowingly) take unfair gambles to avoid risk (e.g., life insurance) and unfair gambles that increase risk (e.g., playing roulette).

### Loss aversion

Loss aversion refers to the fact that losses loom larger than gains (Kahneman & Tversky, 1984). According to Kahneman and Tversky (1979, p. 279), “the aggravation that one experiences in losing a sum of money appears to be greater than the pleasure associated with gaining the same amount.” Therefore, an individual’s displeasure from monetary loss is greater than the pleasure from the same amount of gain (Rabin, 2002). This observed asymmetry between gains

and losses is too extreme to be explained by transaction cost or income effects (Tversky & Kahneman, 1992) as expected utility theory is.

The difference between loss aversion and risk aversion refers to the behavior that decision makers are risk-averse for even small amounts of money. Tversky and Kahneman (1991) showed that decision makers evaluate moderate losses roughly twice as much as equal sized gains.

#### Endowment effect

Loss aversion is related to the endowment effect observed by Thaler (1980). Some studies have shown that purchasing and selling prices for the same good are often diverse (Kahneman et al., 1990). Once a decision maker owns a good, he/she immediately values it more than before he/she owned it (Rabin, 1997). Research has provided evidence that the selling price is generally much larger than purchase price (e.g., Kahneman et al., 1990). This is because decision makers treat the newly owned good as a part of their reference level and consider not having the good to be a loss, whereas decision makers without that good consider not having that good as remaining at their reference level (Rabin, 1997).

#### Disposition effect

Shefrin and Statman (1985) showed that as decision makers dislike losses more than they enjoy gains, and as they are more willing to gamble in the domain of losses (Montier, 2002), investors hold onto stocks that have lost value relative to their purchase prices too long and are eager to sell stocks that have increased in value (see also Weber & Camerer, 1998). However, the purchase price of a stock should actually not affect whether you decide to sell it or not.

This effect has also been observed in other settings such as the real estate market or in the context of stock options (Odean, 1998; Grinblatt & Keloharju, 2001; Genesove & Mayer, 2001). Odean (1998) found out that “when an investor in his sample sells shares, he has a greater propensity to sell shares of a stock that has risen in value since purchase rather than of one that has fallen in value” (see also Barberis & Xiong, 2009, p. 751).

### Status quo bias

Samuelson and Zeckhauser (1988) referred to status quo bias as exaggerated preferences for the status quo. Thus, individuals tend to stick to their status quo situations. Decision makers seem to prefer the status quo to changes that involve losses, even when losses are coupled with gains in other dimensions (Rabin, 2002). Knetsch (1989) demonstrated status quo bias by giving one group of students candy bars and another group mugs. After a while, the students had the opportunity to trade their gifts – a mug for a candy bar or vice versa. Interestingly, 90 percent of both students groups chose not to trade.

### *Framing effect and mental accounting*

A further challenge to the general model of rational choice consists of the dependence of choices on the description and interpretation of decision problems. Specially, this includes:

#### Framing

The rational theory of choice assumes that equivalent formulations of a choice problem should result in the same preference order (Arrow, 1982). However, there exists empirical evidence that variation in the framing of options results in systematically different preferences (Tversky & Kahneman, 1986). Hence, the way a decision problem is presented influences the choices, even given the same data (see, for instance, the phenomenon discovered by Allais, 1953).

#### Heuristics of choice

Individuals use a variety of heuristic procedures within complex decision situations in order to simplify the evaluation of choices. For example, heuristic procedures include computational shortcuts, editing operations, such as the elimination of common components, and imitating non-essential differences (Tversky, 1969). Moreover, the heuristics of choice depend on the formulation of the problem, the method of elicitation, and the context of choice (Tversky & Kahneman, 1992). For instance, Wang's (2008) findings highlight the importance of interpersonal affect-based heuristics in the context of predicting public choices.

### Mental accounting: integration and segregation

Thaler's (1985) idea of mental accounting described the procedure that individuals organize decisions and outcomes by integrating some and segregating others, often violating standard economic theory (Kahneman & Tversky, 2000). This implies that individuals may control their own rewards by choosing to close or to keep mental accounts open. Hence, mental accounting refers to the cognitive process of how individuals keep track of where their money is going and keep spending under control (Thaler, 1999). An additional effect that considers closing and keeping an account is the sunk cost effect. Although following expected utility theory the historical cost should not affect the rational decision maker's best choice, experiments provide evidence that costs that have already been sunk are indeed integrated into decision choices.

### *Conclusion*

Based on these behaviors of bounded rationality, there is consensus in the research community that utility theory as commonly interpreted and applied is not an adequate descriptive model (i.e., the prediction and explanation of actual behavior – what individuals actually do). It is rather a normative model (i.e., what individuals should do) of an idealized decision maker and not a description of the behavior of real people (Tversky & Kahneman, 1986).

Many alternative descriptive models of choices have been proposed in response to this challenge (see, for instance, Machina, 1987; Camerer, 1989). However, there is general agreement that prospect theory is the most promising alternative. Prospect and cumulative prospect theory<sup>6</sup> (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992), motivated by the failures of the rational model, explains loss aversion, risk-seeking, and non-linear preferences, incorporates framing processes, and accommodates source preferences (Tversky & Kahneman, 1992). This could help better understand the violations mentioned above and other phenomena.

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<sup>6</sup> In 1992, Tversky and Kahneman (1992) presented an updated version of prospect theory called cumulative prospect theory. The new version still incorporates a value function and a probability weighting function. However, they applied the idea of Quiggin (1982) by considering non-linear rank-dependent weighting probability assessments as opposed to individual probabilities in order to include non-linear preferences.

## 2.3 Prospect theory

The most cited paper in *Econometrica* is on Kahneman and Tversky's prospect theory, the most important revision of subjective expected utility theory. The theory was founded by the two psychologists in 1979. It is an extension of the classical decision-theoretical perspective by predicting individual choices, even in cases in which expected utility theory is violated.

Prospect theory has two main features compared with expected utility theory. First, decision choices are based on gains and losses rather than a reference point in contrast to expected utility theory, in which decision choices are based on the probabilities of the final asset (Benartzi & Thaler, 1995). The key observation is that decision making begins by identifying a reference point (e.g., current wealth state, aspiration level), from which people tend to be risk-averse for gains and risk loving for losses. Prospect theory replaces the traditional utility function for wealth with an S-shaped function that shows changes in wealth rather than levels of wealth. Changes in wealth are defined by positive and negative changes to a reference point. Therefore, whether an individual perceives an outcome as a gain or as a loss depends on how the reference point is coded.

Second, the value and weighting functions are non-linear in the probabilities, while the utility function assumes linear probabilities. Prospect theory assumes choice consequences and that probabilities (which are replaced by decision weights) influence the evaluation of choices, and therefore the decision process. In prospect theory, decision weights are defined as a reflection of the impact of events on the overall attractiveness of a choice. Hence, they are monotonic (function that preserves the given order), but not necessarily linear (Schoemaker, 1982). Furthermore, prospect theory provides specific assumptions on how these consequences and decision weights will be transformed into subjective amounts. For instance, it allows different decision weighting for gains and losses. Hence, risk attitudes can be different for losses from those for gains in every respect (Wakker, 2010).

Although prospect theory was developed with the help of hypothetical gambles (for an overview of the experiments, see Levin et al., 1998), their predictions are not limited to behavior under laboratory conditions. The theory is supported in the domains of stock markets, labor economics, consumer goods and

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