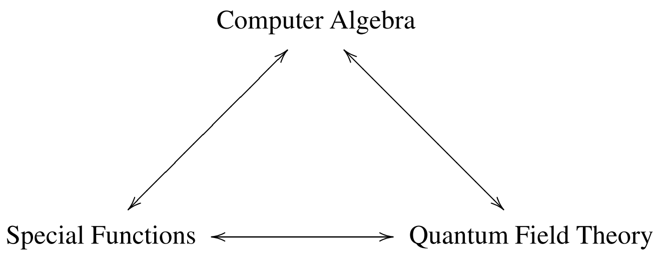
Preface

The research topics of computer algebra, special functions and quantum field theory have been deriving outstanding achievements from computational, algorithmic and theoretical point of view. As it turns out, there is a strong overlap of common interests concerning mathematical, physical and computer science aspects, and in the last years, the topics started a vital and promising interaction in the field of the automated computation of multi-loop and multi-leg Feynman diagrams in precision calculations. This observation has led, e.g. to an intensive cooperation between RISC (Research Institute for Symbolic Computation) of the Johannes Kepler University Linz and DESY (Deutsches Elektronen-Synchrotron). In order to push forward the interaction of the three research fields, the summer school and conference "Integration, Summation and Special Functions in Quantum Field Theory" organized by the European Network LHCPhenonet in cooperation with RISC and DESY was held at Hagenberg/Austria. Here central topics have been introduced with the special emphasis to present the current developments and to point out further possible connections.

This book collects the presented work in form of survey articles for a general readership. It aims at pushing forward the interdisciplinary ties between the very active research areas of computer algebra, special functions and quantum field theory. The driving questions of this book can be summarized as follows:

- How do special functions, such as generalized hypergeometric series, Appell functions, nested harmonic sums, nested multiple polylogarithms and multiple zeta values, emerge in quantum field theories?
- What properties do these functions and constants have and how are they related to each other?
- How can one extract information from such functions or how can one simplify voluminous expressions in terms of such functions with computer algebra, in particular with the help of symbolic summation and symbolic integration?
- What is the irreducible analytic and algebraic structure of multi-loop and multi-leg Feynman integrals?

This book tries to throw light to the underlying problems and to work out possible future cooperations between the different fields:



We emphasize that the interdisciplinary aspects are also reflected in the spirit of the articles. The authors have different backgrounds concerning mathematics, computer science and theoretical physics, and their different approaches bring in new aspects that shall push forward the presented topics of this book.

In this regard, we highlight the following rising aspects:

In Harmonic Sums, Polylogarithms, Special Numbers, and their Generalizations (J. Ablinger, J. Blümlein), *special functions* such as nested sums, associated iterated integrals and special constants which hierarchically appear in the evaluation of massless and massive Feynman diagrams at higher loops are discussed. In particular, the properties of harmonic sums and their generalizations of cyclotomic sums, generalized harmonic sums and sums containing binomial and inverse-binomial weights are worked out that give rise to the simplification of such sums by means of *computer algebra*.

In Multiple Zeta Values and Modular Forms in Quantum Field Theory (D. Broadhurst), properties of *special functions* like multiple zeta values and alternating Euler sums are worked out, and it is indicated where they arise in *quantum field theory*. In particular, the article deals with massive Feynman diagrams whose evaluations yield polylogarithms of the sixth root of unity, products of elliptic integrals and L-functions of modular forms inside their critical strips.

In Computer-Assisted Proofs of Some Identities for Bessel Functions of Fractional Order (S. Gerhold, M. Kauers, C. Koutschan, P. Paule, C. Schneider, B. Zimmermann), big parts of the *computer algebra software* of the combinatorics group of RISC are used to prove a collection of identities involving Bessel functions and other *special functions*. These identities appear in the famous Handbook of Mathematical Functions by Abramowitz and Stegun, as well as in its successor, the DLMF, but their proofs were lost. Here generating functions and symbolic summation techniques are utilized to produce new proofs for them.

In Conformal Methods for Massless Feynman Integrals and Large N_f Methods (J. A. Gracey), the large N method based on *conformal integration methods* is presented that calculates high-order information on the renormalization group functions in a *quantum field theory*. The possible future directions for the large N methods are discussed in light of the development of more recent techniques such as the Laporta algorithm.

In The Holonomic Toolkit (M. Kauers), an overview over standard techniques for *holonomic functions* is given covering, e.g. big parts of Feynman integrals coming from *quantum field theory*. It gives a collection of standard examples and states several fundamental properties of holonomic objects. Two techniques which are most useful in applications are explained in some more detail: *closure properties*, which can be used to prove identities among holonomic functions, and *guessing*, which can be used to generate plausible conjectures for equations satisfied by a given function.

In Orthogonal Polynomials (T. H. Koornwinder), an introduction to *orthogonal polynomials* is presented. It works out the general theory and properties of such special functions, and it is concerned with constructive aspects on how certain formulas can be derived. Special classes, such as *Jacobi polynomials*, *Laguerre polynomials* and *Hermite polynomials*, are discussed in details. It ends with some remarks about the usage of *computer algebra* for this theory.

In Creative Telescoping for Holonomic Functions (C. Koutschan), a broad overview of the available *summation and integration algorithms* for *holonomic functions* is presented. In particular, it is worked out how the underlying algorithms can be executed within the Mathematica package HolonomicFunctions. Special emphasis is put on concrete examples that are of particular relevance for problems coming, e.g. from special functions and *physics*.

In Renormalization and Mellin transforms (D. Kreimer and E. Panzer), the Hopf algebraic framework is utilized to study renormalization in a kinetic scheme. Here a direct *combinatorial description* of renormalized amplitudes in terms of *Mellin transform* coefficients is given using the universal property of rooted trees. The application to scalar *quantum field theory* reveals the scaling behaviour of individual Feynman graphs.

In Relativistic Coulomb Integrals and Zeilberger's Holonomic Systems Approach I (P. Paule, S. K. Suslov), *symbolic summation algorithms* such as Zeilberger's extension of Gosper's algorithm and a parameterized variant are utilized to calculate *recurrence relations* and transformation formulas for generalized hypergeometric series. More precisely, the basic facts within the theory of relativistic Coulomb integrals are presented, and the presented summation technology is used to tackle open problems there.

In Hypergeometric Functions in Mathematica® (O. Pavlyk), a short introduction to the constructive theory of *generalized hypergeometric functions* is given dealing, e.g. with differential equations, Mellin transforms and Meijer's G-functions. Special emphasis is put on concrete examples and notes on the implementation in the *computer algebra system Mathematica*.

In Solving Linear Recurrence Equations with Polynomial Coefficients (M. Petkovšek, H. Zakrajšek), *computer algebra algorithms* for finding polynomial, rational, hypergeometric, d'Alembertian and Liouvillian *solutions of linear recurrences* with polynomial coefficients are described. In particular, an alternative proof of a recent result of Reutenauer's is given that Liouvillian sequences are precisely the interlacing of d'Alembertian ones. In addition, algorithms for factoring linear recurrence operators and finding the minimal annihilator of a given holonomic sequence are presented.

In Generalization of Risch's Algorithm to Special Functions (C. G. Raab), *indefinite integration algorithms* in differential fields are presented. In particular, the basic ideas of Risch's algorithm for *elementary functions* and generalizations thereof are introduced. These algorithms give rise to more general algorithms dealing also with *definite integration*, i.e. calculating linear recurrences and differential equations integrals involving extra parameters.

In Multiple Hypergeometric Series – Appell Series and Beyond (M. J. Schlosser), a collection of basic material on *multiple hypergeometric series* of Appell type is presented covering *contiguous relations*, *recurrences*, *partial differential equations*, *integral representations* and *transformations*. More general series and related types such as Horn functions, Kampé de Fériet series and Lauricella series are introduced.

In Simplifying Multiple Sums in Difference Fields (C. Schneider), difference field algorithms for *symbolic summation* are presented. This includes the *simplification of indefinite nested sums*, *computing recurrence relations* of definite sums and *solving recurrence relations*. Special emphasis is put on new aspects in how the summation problems are rephrased in terms of difference fields, how the problems are solved there and how the derived results can be reinterpreted as solutions of the input problem. In this way, large-scale summation problems for the evaluation of Feynman diagrams in *quantum field theories* can be solved completely automatically.

In Potential of FORM 4.0 (J. A. M. Vermaseren), the *computer algebra system* FORM is presented that is heavily used in *quantum field theory* for large-scale calculations. Special emphasis is put on the main new features concerning factorization algorithms, polynomial arithmetic, special functions and code simplification.

Finally, in Feynman Graphs (S. Weinzierl), *Feynman graphs* and the associated *Feynman integrals* are discussed. It presents four different definitions from the mathematical and physical point of view. In particular, the most prominent class of *special functions*, the multiple polylogarithms, with their algebraic properties are worked out, which appear in the evaluation of Feynman integrals. The final part is devoted to Feynman integrals, which cannot be expressed in terms of multiple polylogarithms. Methods from algebraic geometry provide tools to tackle these integrals.

In addition, we want to emphasize the following fascinating presentations that are not part of this book, but which contributed substantially to our summer school and conference "Integration, Summation and Special Functions in Quantum Field Theory": the key note lecture Mate is Meta by Bruno Buchberger, Hypergeometric Functions and Loop Integrals by Nigel Glover, Polynomial GCDs and Factorization by Jürgen Gerhard and Holonomic Summation and Integration by Frédéric Chyzak.

The present project has been supported in part by the EU Network LHCPhenonet PITN-GA-2010-264564; the Austrian Science Fund (FWF) grants (P20347-N18, DK W1214); the Research Institute for Symbolic Computation, RISC; and Deutsches Elektronen-Synchrotron, DESY, which are kindly acknowledged.

We would like to thank I. Brandner-Foissner, T. Guttenberger, R. Oehme-Pöchinger (RISC) and M. Mende (DESY) for their help in organizing this meeting.

Zeuthen, Germany
Hagenberg, Austria
May, 2013

Johannes Blümlein
Carsten Schneider

http://www.springer.com/978-3-7091-1615-9

Computer Algebra in Quantum Field Theory
Integration, Summation and Special Functions
Schneider, C.; Bluemlein, J. (Eds.)
2013, XIV, 411 p. 41 illus., Hardcover
ISBN: 978-3-7091-1615-9