

## Chapter 2

# Basic Properties of QCD

### 2.1 The QCD Lagrangian

In this and the following sections, the basic properties of QCD will be briefly discussed. A more detailed exposure of the many facets of QCD can be found in various textbooks, such as Yndurain (1983), Greiner et al. (1994), Ioffe et al. (2010).

QCD is a non-abelian  $SU(3)$  gauge theory with color charges as the generators of the gauge group. Its Lagrangian can be given as

$$\mathcal{L}_{QCD} = \sum_f \bar{q}_f (i \not{D} - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}, \quad (2.1)$$

where  $q_f$  are the quark fields with flavor  $f$ , which runs over all presently known six flavors  $u, d, s, c, b, t$  with the corresponding masses  $m_f$ . The quark fields in fact have two more indices, which are omitted above for simplicity. One is a spinor index running from 1 to 4, showing that the quark (and its anti-particle) are spin-1/2 particles, the other is a color index with values from 1 to 3, meaning that the quarks live in the fundamental representation of the  $SU(3)$  gauge group. The covariant derivative  $\not{D}$  contains the coupling between the quarks and gluon fields and is defined as

$$\not{D} = \gamma^\mu (\partial_\mu - ig A_\mu). \quad (2.2)$$

Here, the gluon field  $A_\mu$  is a  $3 \times 3$  matrix and lives in the adjoint representation of the  $SU(3)$  gauge group. Using the Gell-Mann matrices  $\lambda^a$ , it can be expanded as  $A_\mu = 1/2 \sum_a \lambda^a A_\mu^a$  ( $a = 1 \sim 8$ ). Furthermore,  $g$  stands for the gauge coupling constant. Finally, the last term of Eq.(2.1) represents the dynamics of the gluonic fields only. It can be expressed in terms of the field strength tensor  $G_{\mu\nu}^a$ , which is obtained from the gluon fields as

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c, \quad (2.3)$$

where  $f^{abc}$  are the structure constants of the  $SU(3)$  gauge group.

Note that in the simple description of this section, we have omitted ghost fields and possible gauge fixing terms, which are introduced during the quantization of the theory (Faddeev and Popov 1967).

## 2.2 Asymptotic Freedom

One important piece of evidence, suggesting that QCD is indeed the true theory to describe the strong interaction, was provided by the discovery of asymptotic freedom (Gross and Wilczek 1973; Politzer 1973). This property, which essentially means that the coupling constant  $g$  appearing in Eqs. (2.2) and (2.3) becomes small at large energies, can be derived through the renormalization procedure of QCD. As in any field theory, the perturbative quantum (loop) corrections in QCD contain ultra-violet divergences, which have to be renormalized for the theory to produce meaningful results. If (as it is the case for QCD) the theory is renormalizable, all these divergences can be absorbed into a redefinition of the bare coupling constant  $g$ , the bare masses  $m_f$  and the fields  $q$  and  $A$ . However, this redefinition will depend on the energy scale  $\mu$ , at which the renormalization is carried out, therefore introducing some dependence of the parameters of the theory on  $\mu$ . As  $\mu$  is an arbitrary parameter, which has been introduced by hand, the observables calculated from the theory should not depend on it. This requirement leads to several renormalization group equations (Callan 1970; Symanzik 1970), in which the part dealing with the coupling constant  $g$  is given as,

$$\mu \frac{\partial g}{\partial \mu} = \beta(g), \quad (2.4)$$

where  $\beta(g)$  is the  $\beta$ -function, which can be perturbatively calculated for small  $g$ . In QCD, this function has the following form (as an expansion in  $g$ ):

$$\beta(g) = -\beta_0 g^3 - \beta_1 g^5 + \dots, \quad (2.5)$$

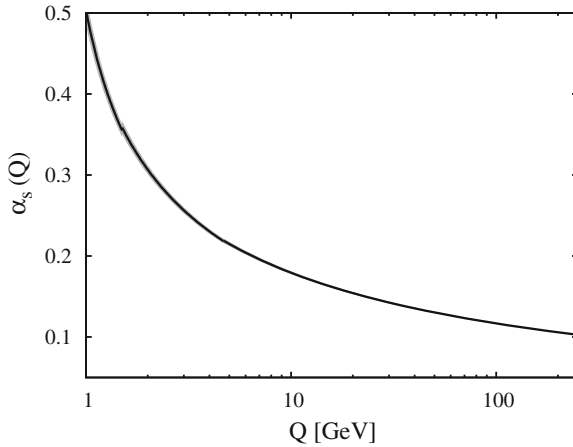
$$\beta_0 = \frac{1}{(4\pi)^2} \left( 11 - \frac{2}{3} N_f \right), \quad (2.6)$$

$$\beta_1 = \frac{1}{(4\pi)^4} \left( 102 - \frac{38}{3} N_f \right), \quad (2.7)$$

$N_f$  being the number of flavors. The fact that  $\beta_0$  is positive and that therefore  $\beta(g)$  is negative for sufficiently small values of  $g$  has been revealed in the papers of Gross and Wilczek (1973), Politzer (1973) and is equivalent to asymptotic freedom, as will be seen below.

Solving Eq. (2.4) and keeping for simplicity only the leading  $\beta_0$  term, the following result can be obtained:

$$\alpha_s(\mu) = \frac{1}{4\pi \ln(\mu^2/\Lambda_{\text{QCD}}^2)}. \quad (2.8)$$



**Fig. 2.1** The value of  $\alpha_s$  as a function of the energy  $Q$ , obtained from the  $\beta$ -function including corrections up to 4-loops (Bethke 2009). The shaded region shows its corresponding numerical uncertainty. The discontinuity seen in the plot at around 1.5 GeV stems from the matching between the 3- and 4-flavor  $\beta$ -function, which has to be implemented in this energy region

Here,  $\alpha_s$  stands for  $g^2/(4\pi)$  and the integration constant  $\Lambda_{\text{QCD}}$  is known as the QCD scale parameter. Its actual value is about 200–300 MeV, depending on how many flavors one considers to be active. From this equation, one sees that the value  $\alpha_s$  decreases with larger energy  $\mu$ . Nowadays, the  $\beta$ -function is known up to 4-loops ( $g^9$ ) (Bethke 2009), giving values of  $\alpha_s$  as shown in Fig. 2.1. As can be observed from this figure, as long as the energy scale  $Q$  is much larger than  $\sim 1$  GeV,  $\alpha_s$  is small and a perturbative treatment is meaningful, while for energy scales around or below  $\sim 1$  GeV,  $\alpha_s$  becomes so large that a perturbative expansion will eventually break down. It is therefore clear that non-perturbative methods are necessary for studying low-energy QCD processes.

## 2.3 Symmetries of QCD

### 2.3.1 Gauge Symmetry

Gauge symmetry is in a certain sense the most important symmetry of QCD, as the QCD Lagrangian was indeed constructed on the basis of gauge invariance. It demands that the theory should be invariant under the following gauge transformation. The fermionic fields change as

$$q'(x) = U(x)q(x), \quad (2.9)$$

where  $U(x)$  is a  $3 \times 3$  unitary matrix in color space, which generally depends on the space-time point  $x$ . Because of this dependence, Eq. (2.9) is a local transformation and the corresponding gauge symmetry a local symmetry. At the same time, the gauge fields  $A_\mu(x)$  are transformed as

$$A'_\mu(x) = U(x)A_\mu(x)U^\dagger(x) + \frac{i}{g}U(x)\partial_\mu U^\dagger(x), \quad (2.10)$$

with the same  $U(x)$  as in Eq. (2.9). It is not difficult to show that these transformations act on the covariant derivative of Eq. (2.2) in the following way:

$$D'_\mu(x) = U(x)D_\mu(x)U^\dagger(x). \quad (2.11)$$

This then immediately shows that the first term of the Lagrangian in Eq. (2.1), which describes the motion of the quark fields, is gauge invariant. For the second term, involving only gluonic fields and their mutual interactions, it is convenient to note that the field strength tensor of Eq. (2.3) (contracted with  $\frac{1}{2}\lambda^a$ ) can be expressed as

$$G_{\mu\nu}(x) = \frac{i}{g}[D_\mu, D_\nu], \quad (2.12)$$

from which, together with Eq. (2.11), it follows that this object transforms as

$$G'_{\mu\nu}(x) = U(x)G_{\mu\nu}(x)U^\dagger(x). \quad (2.13)$$

As the last term of Eq. (2.1), can be written down as the trace of two  $G_{\mu\nu}(x)$ 's with contracted Lorentz indices, one can see from Eq. (2.13) that it is also gauge invariant, as it should be.

In actual calculations, one often makes use of the freedom of choosing a gauge to simplify the algebraic manipulations. For calculations of the operator product expansion in QCD sum rules, for instance, the so-called Fock-Schwinger gauge is a convenient choice, as we will discuss in Chapter 3 and Appendix B. On the other hand, any physical result obtained from QCD should not depend on the gauge, in which it was calculated. Therefore, to verify the gauge independence of some result can serve as a useful check of the calculation. Furthermore, for formulating QCD on a space-time lattice in order to carry out Monte-Carlo simulations, gauge invariance also has provided essential guidance (Wilson 1974).

### 2.3.2 Chiral Symmetry

As will be explained below, chiral symmetry is not an exact symmetry of the QCD Lagrangian of Eq. (2.1), but is only valid in the limit of small quark masses. However, because the masses of the  $u$ - and  $d$ -quarks are much smaller than  $\Lambda_{\text{QCD}}$ , the typical

scale of QCD, these can be treated as small perturbations. Therefore chiral symmetry becomes a useful concept, at least for the  $u$ - and  $d$ -quark sector, which plays the most prominent role in almost all low-energy hadronic processes.

To discuss chiral symmetry, one first has to introduce left-handed and right-handed quarks, which are defined as follows

$$q_L = P_L q, \quad q_R = P_R q, \quad (2.14)$$

with

$$P_L = \frac{1}{2}(1 - \gamma_5), \quad P_R = \frac{1}{2}(1 + \gamma_5). \quad (2.15)$$

Here, it is clear that the projection operators  $P_{L,R}$  satisfy the necessary conditions  $P_{L,R}^2 = P_{L,R}$ ,  $P_L P_R = 0$ ,  $P_L + P_R = 1$ . Rewriting the QCD Lagrangian of Eq. (2.1) with the help of the left- and right-handed quark fields of Eq. (2.14), we obtain

$$\mathcal{L}_{QCD} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L m q_R - \bar{q}_R m q_L - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}, \quad (2.16)$$

where we have omitted the sum over the flavors for simplicity of notation. It is seen in the above equation that, if the quark mass  $m$  approaches 0, the left- and right-handed quarks completely decouple and behave as independent degrees of freedom. Ignoring the mass terms for a moment, it is also observed that this Lagrangian has a global symmetry, corresponding to certain unitary transformations of the quark fields:

$$q'_L = U_L q_L, \quad U_L \in U(N_f)_L, \quad (2.17)$$

$$q'_R = U_R q_R, \quad U_R \in U(N_f)_R. \quad (2.18)$$

Here,  $U_{L,R}$  are unitary  $N_f \times N_f$  matrices, operating in the flavor space of the quark fields. As only the  $u$ - and  $d$ -quarks (and, to a lesser degree, the  $s$ -quarks) can be considered to be light,  $N_f$  here is usually taken to be 2 or 3. Among the symmetries contained in Eqs. (2.17) and (2.18), two have a somewhat special character. Firstly,  $U(1)_V$ , standing for the case, in which  $U_L$  and  $U_R$  are diagonal and equal, represents the quark number conservation in the strong interaction. This symmetry even holds when the finite quark masses are taken into account and is valid at both the classic and quantum level. Secondly, the symmetry of  $U(1)_A$ , in which  $U_L$  and  $U_R$  are diagonal as well, but represent rotations in the opposite direction, is violated by quantum corrections, leading to the axial anomaly (Bell and Jackiw 1969; Adler 1969). Therefore, even if the quark masses are exactly 0, this symmetry is broken.

Removing the two subgroups  $U(1)_V$  and  $U(1)_A$  discussed above, we are left with the symmetries corresponding to  $SU(N_f)_L \times SU(N_f)_R$ , which are usually referred to as chiral symmetry. The respective transformations can be parametrized as

$$q'_L = e^{i\theta_L^a t^a} q_L, \quad e^{i\theta_L^a t^a} \in SU(N_f)_L, \quad (2.19)$$

$$q'_R = e^{i\theta_R^a t^a} q_R, \quad e^{i\theta_R^a t^a} \in SU(N_f)_R, \quad (2.20)$$

with  $a = 1 \sim N_f^2 - 1$  and  $t^a$  being the generators of  $SU(N_f)$ .  $\theta_{L,R}^a$  are arbitrary real parameters.

Even though the QCD Lagrangian possesses chiral symmetry as described in Eqs. (2.19) and (2.20), this symmetry is not fully realized in the QCD vacuum. Specifically, it is instead believed to be dynamically broken (Nambu and Jona-Lasinio 1961a, b), the symmetry breaking pattern being

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V, \quad (2.21)$$

where the transformation corresponding to  $SU(N_f)_V$  is such that both rotations of Eqs. (2.19) and (2.20) are the same ( $\theta_L^a = \theta_R^a$ ). The simplest order parameter for such a partial breaking of chiral symmetry is the quark condensate, expressed as

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R \rangle + \langle \bar{q}_R q_L \rangle. \quad (2.22)$$

Here, contraction and summation over Dirac-, color- and flavor-indices are implicitly assumed. It is clear that the quark condensate (if it has a finite value) generally changes its form under arbitrary transformations of Eqs. (2.19) and (2.20), but is invariant under  $SU(N_f)_V$ , making it an appropriate order parameter for the symmetry breaking pattern of Eq. (2.21).

The value of the quark condensate nowadays can be obtained from first principle lattice QCD calculations (Fukaya et al. 2010). The earliest estimation, however, relied on the Gell-Mann-Oakes-Renner relation (Gell-Mann et al. 1968), based on general considerations of chiral symmetry. It gives (with  $N_f = 2$ ),

$$f_\pi^2 m_\pi^2 = -m_q \langle \bar{u}u + \bar{d}d \rangle, \quad (2.23)$$

where  $f_\pi$  and  $m_\pi$  are the pion decay constant and pion mass, respectively. The values of these parameters can be obtained from experiment.  $m_q$  stands for the averaged quark mass of  $u$ - and  $d$ -quarks, which can not be directly extracted from experimental studies, but has to be estimated by other methods (Gasser and Leutwyler 1982). Finally, assuming that the value for the condensate of the  $u$ - and  $d$ -quarks is the same, one arrives at the following number:

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \simeq -(240 \text{ MeV})^3. \quad (2.24)$$

In addition, QCD sum rule studies provide estimates for the  $s$ -quark condensate  $\langle \bar{s}s \rangle$ , which gives an about 20 % reduced value, compared to Eq. (2.24) (Reinders et al. 1985).

### 2.3.3 Dilatational Symmetry

The dilatational symmetry, similarly to chiral symmetry, only holds in the limit of vanishing quark masses. In this limit, the QCD Lagrangian involves no explicit energy scale and the theory is therefore scale invariant. This means that the energy dependence of any physical quantity is fixed by its dimension,

$$F(E, p_1, p_2, \dots) = E^{d_F} f\left(\frac{p_1}{E}, \frac{p_2}{E}, \dots\right), \quad (2.25)$$

where  $d_F$  stands for the dimension of the quantity  $F$  and  $f$  is a dimensionless function.

Infinitesimal scale transformations can be parametrized by the following coordinate redefinition

$$x'_\mu = x_\mu + \varepsilon x_\mu, \quad (2.26)$$

with the infinitesimal parameter  $\varepsilon$ . The N other current derived from this transformation reads as

$$j_\mu = x_\nu T_\mu^\nu, \quad (2.27)$$

$T_\mu^\nu$  being the energy momentum tensor. This gives

$$\partial^\mu j_\mu = T_\mu^\mu. \quad (2.28)$$

Therefore, the dilatational symmetry is valid if the right hand side of the above equation vanishes. Classically, the trace of the energy momentum tensor  $T_\mu^\mu$  only receives non-zero contributions from terms involving finite quark masses. However, quantum fluctuations lead to additional effects due to the so-called trace anomaly (Crewther 1972; Chanowitz and Ellis 1972; Collins et al. 1977). Taking this contribution into account, one obtains

$$T_\mu^\mu = \frac{\beta}{2g} G_{\mu\nu}^a G^{a\mu\nu} + \sum_f m_f \bar{q}_f q_f, \quad (2.29)$$

where the first term originates from the trace anomaly.  $\beta$  is the  $\beta$ -function of QCD, which has already appeared in Eq. (2.4). It therefore follows from Eqs. (2.28) and (2.29), that the dilatational symmetry is not only broken by the finite quark masses, but also by quantum effects. This can be understood from the fact that in quantum field theory, a renormalization point  $\mu$  has to be introduced, thus leading to a new scale that violates the symmetry of Eq. (2.25).

### 2.3.4 Center Symmetry

The center (or  $Z(N_c)$ ) symmetry of QCD has a somewhat different character from the ones discussed so far, as it is a symmetry of QCD at finite temperature, and is only exactly valid when all quarks are infinitely heavy. As will be discussed below, this symmetry is related to the confinement-deconfinement transition of QCD at finite temperature (McLerran and Svetitsky 1981; Svetitsky and Yaffe 1982).

For discussing the center symmetry, it first has to be remembered that in quantum field theory at finite temperature one takes the time axis to be imaginary ( $t \rightarrow -i\tau$ ,  $\tau$  being a real parameter) and all bosonic fields have to satisfy periodic boundary conditions with respect to this axis, the period being  $1/T$  (Le Bellac 1996). Now, it is noticed that one can gauge transform the periodic gluon field according to Eq. (2.10) with a transformation matrix  $U(x)$ , which does not necessarily need to be periodic:

$$U(\tau + 1/T, \mathbf{x}) = \mathbf{z}U(\tau, \mathbf{x}). \quad (2.30)$$

Here,  $\mathbf{z}$  must be an element of  $SU(N_c)$ . Substituting this gauge transformation matrix into Eq. (2.10), one obtains

$$A'_\mu(\tau + 1/T, \mathbf{x}) = \mathbf{z}A'_\mu(\tau, \mathbf{x})\mathbf{z}^\dagger + \frac{i}{g}\mathbf{z}\partial_\mu\mathbf{z}^\dagger. \quad (2.31)$$

In order for this transformed gauge field to be periodic, the right-hand side of the above equation should be equal to  $A'_\mu(\tau, \mathbf{x})$ , and this can only happen if  $\mathbf{z}$  can be interchanged with any other  $SU(N_c)$  matrix and does not depend on the space-time coordinates at all. Therefore,  $\mathbf{z}$  has to be proportional to the identity matrix  $\mathbf{1}$  with a constant coefficient. As  $\mathbf{z}$  is an element of  $SU(N_c)$ , its possible realizations turn out to be

$$\mathbf{z} = e^{2\pi i n/N_c} \mathbf{1}, \quad (n = 0, 1, \dots, N_c - 1). \quad (2.32)$$

These matrices commute with any member of the  $SU(N_c)$  group, are called the center of  $SU(N_c)$  and are denoted as  $Z(N_c)$ .

While the action for the pure  $SU(N_c)$  theory is invariant under the center symmetry, one can consider other gauge invariant operators constructed from gluonic fields, for which this is not the case. Among them, the Polyakov loop (Polyakov 1978), defined as the path-ordered product the gauge field, directed in the imaginary time direction from 0 to  $1/T$ , is most simple:

$$L(\mathbf{x}) = \frac{1}{N_c} \text{Tr} \left\{ \text{P exp} \left[ ig \int_0^{1/T} d\tau A_4(\tau, \mathbf{x}) \right] \right\}. \quad (2.33)$$

It can be shown that the Polyakov loop transforms as

$$L'(\mathbf{x}) = \mathbf{z}L(\mathbf{x}), \quad (2.34)$$



under the center symmetry transformation, where  $z$  stands for the constant factor in front of the identity matrix in Eq. (2.32). Hence, as long as it does not vanish, the Polyakov loop is not invariant under the  $Z(N_c)$  transformation and therefore serves as an order parameter of the corresponding center symmetry. In other words, if  $L(\mathbf{x})$  takes a finite value, the center symmetry is spontaneously broken.

Additionally, it should be mentioned here that the Polyakov loop of Eq. (2.33) also has significant implications related to deconfinement of quarks. Namely, the expectation value of  $L(\mathbf{x})$  can be related to the free energy of a single quark (Polyakov 1978; Susskind 1979),

$$\langle L(\mathbf{x}) \rangle = e^{-\Delta F_q/T}, \quad (2.35)$$

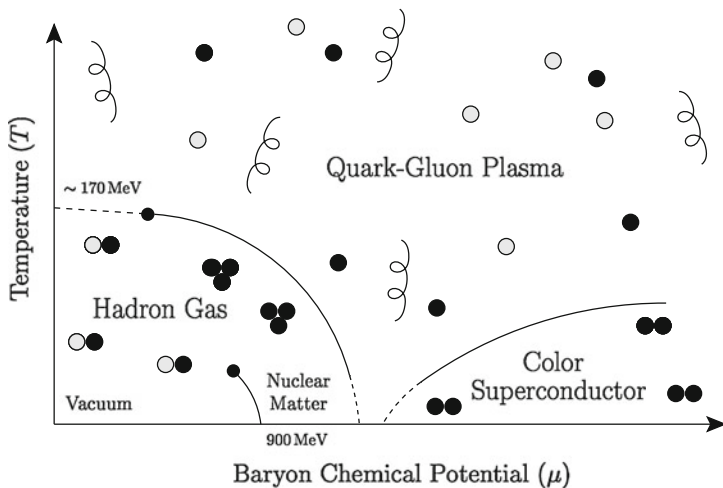
where  $\Delta F_q$  is the difference between the free energy of a system with and without a single deconfined quark. As long as the quarks are confined, such a free energy of a quark should be infinitely large, and the Polyakov loop should thus vanish. On the other hand, for a system in the deconfined phase, the respective quark free energy should have a finite value, meaning that the Polyakov loop will have a value larger than 0.

## 2.4 Phases of QCD

The phases of QCD at various values of temperature and density continue to be intensively studied both theoretically and experimentally. For a recent review of the current statuses in theory, see Fukushima and Hatsuda (2011). However, despite of these efforts, there are still many open questions and fully established facts are rather rare. In this short introduction, we will not discuss all open issues in detail, but can only give a broad overview about what is known about the properties of QCD in a hot or dense medium.

In Fig. 2.2 a sketch of the QCD phase diagram is given. One can see in this figure that there are essentially three phases. Firstly, there is the hadron gas phase at low temperature and density, where both the vacuum in which we live in and nuclear matter are located. Secondly, the quark-gluon plasma phase is realized at high temperature (Cabibbo and Parisi 1975), in which quarks and gluons are deconfined and behave as weakly interacting particles. Thirdly, the color superconductor phase is expected to appear at high density and low temperature, where quarks are believed to form Cooper pairs, leading to color superconductivity (Barrois 1977; Bailin and Love 1984). It however has to be noted here, that many features of this phase diagram are not well understood. Especially for the region of moderately high chemical potential and low temperature, where the three phases meet, there is no conclusive picture available, yet. This is so because in this domain, neither perturbative methods nor lattice QCD calculations (Wilson 1974) can be reliably applied, and one therefore has to resort to model calculations.

The region of the QCD phase diagram that is perhaps best known is located around zero chemical potential, as here lattice QCD calculations are available. Particularly

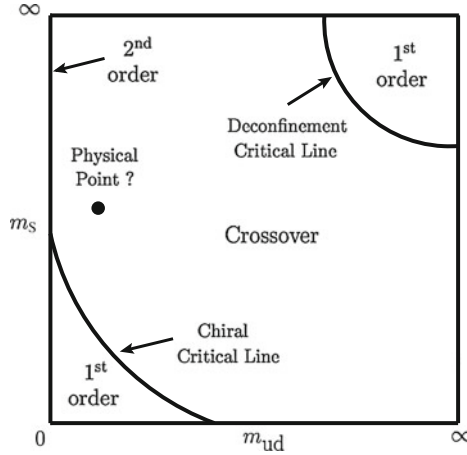


**Fig. 2.2** Sketch of the various phases of QCD, as a function of the temperature and the baryon chemical potential, which is related to the baryon density

interesting is the transition region between the hadron and quark-gluon plasma phase, which has been investigated in detail in past studies. The behavior of this transition depends on the flavor content of the theory, as illustrated in the Columbia plot, shown in Fig. 2.3. For the purely gluonic case with no active flavors, which is most easily studied on the lattice because the quenched approximation can be used here, it is found that the transition is of first-order (Fukugita et al. 1990), with a critical temperature of about  $T_c \simeq 260\text{--}270$  MeV. This situation corresponds to the top-right corner of Fig. 2.3.

On the other hand, due to technical difficulties related to the description of quarks on the lattice, massless quarks can at present not be reliably treated in lattice QCD and one has to consider other methods in this case. A quite general method for handling this problem is the Ginzburg-Landau approach, in which one writes down a general effective Lagrangian in terms of an appropriate order parameter of chiral symmetry (Pisarski and Wilczek 1984). Furthermore, taking into account the effect of the  $U_A(1)$  or axial anomaly (Kobayashi and Maskawa 1970; 't Hooft 1976), one obtains

$$\begin{aligned}
 \mathcal{L}_{\text{eff}} = & \frac{1}{2} \text{Tr} \partial \Phi^\dagger \partial \Phi + \frac{a}{2} \text{Tr} \Phi^\dagger \Phi \\
 & + \frac{b_1}{4!} (\text{Tr} \Phi^\dagger \Phi)^2 + \frac{b_2}{4!} \text{Tr} (\Phi^\dagger \Phi)^2 \\
 & - \frac{c}{2} (\det \Phi + \det \Phi^\dagger).
 \end{aligned} \tag{2.36}$$



**Fig. 2.3** The Columbia plot for QCD at zero density, depicting the nature of the confinement-deconfinement or chiral transition at finite temperature for various flavors. The *top-right* corner describes the purely gluonic transitions with infinitely heavy quarks, while the *bottom-left* corner stands for the chiral transitions with three massless quarks

Here,  $\Phi$  is a  $N_f \times N_f$  matrix and stands for the order parameter of chiral symmetry. Under a chiral  $SU(N_f)_L \times SU(N_f)_R$  transformation, it changes as

$$\Phi' = V_L \Phi V_R, \quad (2.37)$$

while under the axial  $U_A(1)$  transformation, it changes as

$$\Phi' = e^{i\alpha} \Phi. \quad (2.38)$$

It can be seen from the above equations that the first four terms of the Lagrangian of Eq. (2.36) are invariant under both  $SU(N_f)_L \times SU(N_f)_R$  and  $U_A(1)$  transformations, while the last terms breaks the  $U_A(1)$  symmetry. Therefore, this last term explicitly incorporates the effect of the chiral anomaly.

Analyzing now the thermal properties of Eq. (2.36), one finds that for  $N_f = 2$  this Lagrangian is equivalent to the  $\phi^4$  model, which possesses the  $O(4)$  symmetry. This model is known to have a second order phase transition. On the other hand, for  $N_f = 3$ , due to the cubic interaction introduced by the axial anomaly term, the model exhibits a first order transition. These considerations lead to the picture shown in Fig. 2.3, where on the right side the transition changes from first order at the bottom ( $N_f = 3$ ) to second order on the top ( $N_f = 2$ ). For a more detailed discussion of this issue, see Chap. 6 of Yagi et al. (2005).

Finally, let us consider more realistic cases, which lie close to the physical point, indicated by the black dot in Fig. 2.3. Even though still challenging due to the light  $u$  and  $d$  quark masses, lattice simulations are now at the stage of becoming possible in

such a regime. Most of these simulations employ the staggered fermions (Susskind 1977; Sharatchandra et al. 1981), while some also use the Wilson fermion formalism (Wilson 1975). Recent results of such studies suggest that the transition at the physical point is smooth crossover (Aoki et al. 2006). Furthermore, the value of the critical temperature has been evaluated by various groups, the latest results giving an averaged value of roughly 170 MeV, with a scatter of about 20 MeV (Aoki et al. 2009; Bazavov et al. 2009; Bornyakov et al. 2010; Cheng et al. 2009; Borsanyi et al. 2010).

## References

- Adler SL (1969) *Phys Rev* 177:2426
- Aoki Y, Endrodi G, Fodor Z, Katz SD, Szabo KK (2006) *Nature* 443:675
- Aoki Y, Borsanyi S, Durr S, Fodor Z, Katz SD, Krieg S, Szabo KK (2009) *JHEP* 06:088
- Bailin D, Love A (1984) *Phys Rep* 107:325
- Barrois BC (1977) *Nucl Phys B* 129:390
- Bazavov A et al (2009) *Phys Rev D* 80:014504
- Bell JS, Jackiw R (1969) *Nuovo Cimento* 60A:47
- Le Bellac M (1996) *Thermal field theory*. Cambridge Monographs on Mathematical Physics, Cambridge
- Bethke S (2009) *Eur Phys J C* 64:689
- Bornyakov VG et al (2010) *Phys Rev D* 82:014504
- Borsanyi S, Fodor Z, Hoelbling C, Katz SD, Krieg S, Ratti C, Szabo K (2010) *JHEP* 09:073
- Cabibbo N, Parisi G (1975) *Phys Lett B* 59:67
- Callan CG (1970) *Phys Rev D* 2:1541
- Chanowitz M, Ellis J (1972) *Phys Lett B* 40:397
- Cheng M et al (2009) *Phys Rev D* 81:054510
- Collins J, Duncan L, Joglekar S (1977) *Phys Rev D* 16:438
- Crewther R (1972) *Phys Rev Lett* 28:1421
- Faddeev LD, Popov VN (1967) *Phys Lett B* 25:25
- Fukaya H et al (2010) JLQCD collaboration. *Phys Rev Lett* 104:122002
- Fukugita M, Okawa M, Ukawa A (1990) *Nucl Phys B* 337:181
- Fukushima K, Hatsuda T (2011) *Rep Prog Phys* 74:014001
- Gasser J, Leutwyler H (1982) *Phys Rep* 87:77
- Gell-Mann M, Oakes RJ, Renner B (1968) *Phys Rev* 175:2195
- Greiner W, Schramm S, Stein E (1994) *Quantum chromodynamics*. Springer-Verlag, Berlin
- Gross DJ, Wilczek F (1973) *Phys Rev Lett* 30:1343
- Ioffe BL, Fadin VS, Lipatov LN (2010) *Quantum chromodynamics, perturbative and nonperturbative aspects*. Cambridge Monographs on Particle Physics Nuclear Physics and Cosmology, Cambridge
- Kobayashi M, Maskawa T (1970) *Prog Theor Phys* 44:1422
- McLerran LD, Svetitsky B (1981) *Phys Rev D* 24:450
- Nambu Y, Jona-Lasinio G (1961a) *Phys Rev* 122:345
- Nambu Y, Jona-Lasinio G (1961b) *Phys Rev* 124:246
- Pisarski RD, Wilczek F (1984) *Phys Rev D* 29:338
- Politzer HD (1973) *Phys Rev Lett* 30:1346
- Polyakov AM (1978) *Phys Lett B* 72:477
- Reinders LJ, Rubinstein H, Yazaki S (1985) *Phys Rep* 127:1
- Sharatchandra HS, Thun HJ, Weisz P (1981) *Nucl Phys B* 192:205
- Susskind L (1977) *Phys Rev D* 16:3031

- Susskind L (1979) Phys Rev D 20:2610
- Svetitsky B, Yaffe LG (1982) Nucl Phys B210:423
- Symanzik K (1970) Comm Math Phys 18:227
- 't Hooft G (1976) Phys Rev Lett 37:8
- Wilson KG (1975) Quarks and strings on a lattice. In: Arnowitt R, Nath P (ed) Gauge theories and modern field theory. MIT Press, Cambridge
- Wilson KG (1974) Phys Rev D 10:2445
- Yagi K, Hatsuda T, Miyake Y (2005) Quark-Gluon plasma. Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, Cambridge
- Yndurain FJ (1983) The theory of quark and gluon interactions. Springer-Verlag, Berlin

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