Preface

The present volume is an English translation of the Japanese mathematics book "Gröbner Dojo" (Kyoritsu Shuppan Co., Ltd., September 2011). The *dojo* is a Japanese traditional term, which represents, in general, the place for the training of the *judo*, an Olympic sport. Our book "Gröbner Dojo" invites the reader to the Gröbner world, a fascinating research area of mathematics, where three aspects of Gröbner bases, viz., theory, application and computation, are linked effectively and systematically. A beginner including a first year graduate student can learn the ABC's of Gröbner bases from "Gröbner Dojo." In addition, "Gröbner Dojo" can be a how-to book for users of Gröbner bases such as scientists engaging in statistical problems as well as engineers being active in industrial society. This is the reason why we select the term *dojo* for the title of our book.

An idea of Gröbner bases was apparently studied by Francis Sowerby Macaulay in 1927; he succeeded in finding a combinatorial characterization of the Hilbert functions of homogeneous ideals of the polynomial ring. Later, current definition of Gröbner bases was independently introduced by Heisuke Hironaka in 1964 and Bruno Buchberger in 1965. However, after the discovery of the notion of Gröbner bases by Hironaka and Buchberger, no activity had been done for about twenty years. A first breakthrough was done by David Bayer and Michael Stillman in the middle of 1980s, who created the computer algebra system Macaulay with the help of Gröbner bases. In 1995 the second breakthrough was achieved by Bernd Sturmfels, who discovered the fascinating relation between regular triangulations of convex polytopes and Stanley–Reisner ideals of initial ideals of toric ideals. Furthermore, the third breakthrough arose in 1998 when Persi Diaconis and Bernd Sturmfels demonstrated an exciting application of Gröbner bases to algebraic statistics.

With these backgrounds, in October 2008, the JST[1] CREST[2] Hibi project started toward the progress of theory and application of Gröbner bases together with the

[1] Japan Science and Technology Agency.

[2] Core Research for Evolutional Science & Technology.

development of their algorithms. The publication of "Gröbner Dojo" was already announced in the original research plan of the project.

"Gröbner Dojo" is a comprehensive textbook to learn algebraic statistics based on Gröbner bases. First, in Chap. 1, starting from Dickson's Lemma, a classical result in combinatorics, we explain the division algorithm, Buchberger criterion and Buchberger algorithm. Then the theory of elimination follows and toric ideals are introduced. In addition, the basic theory of Hilbert functions is discussed. Moreover, the historical background of Gröbner bases is surveyed.

Chapter 2 is a warming-up drill for learning the basic ideas of using mathematical software. We choose the mathematical software environment named "MathLibre." It is a collection of mathematical software and free documents. MathLibre is a kind of Live Linux system. Linux is a system compatible with UNIX, a traditional OS for specialists. Many mathematical research systems are developed on UNIX. The basic usages and fundamental ideas of UNIX are introduced.

Chapter 3 discusses how to compute various objects related to Gröbner bases explained in Chap. 1. After introducing fundamental tools for efficient Gröbner basis computation, we illustrate fundamental computations related to Gröbner bases by using Macaulay2, SINGULAR, CoCoA and Risa/Asir.

In writing Chaps. 1–3, we do not assume that the reader is familiar with theory and computation of Gröbner bases. If the reader has an experience of handling Gröbner bases, then these chapters may be skipped partly. On the other hand, since the latter Chaps. 4–6 are written independently, after reading the former Chaps. 1–3, the reader can read Chaps. 4–6 in any order.

Chapter 4 is devoted to algebraic statistics. This field was initiated by the work of Diaconis and Sturmfels in 1998 and the work of Pistone and Wynn in 1996, both applying Gröbner basis theory to statistics. Since then the field has been developing rapidly with providing challenging problems to both statisticians and algebraists.

Chapter 5 plays the introduction to two fascinating rainbow bridges between the world of Gröbner bases and that of convex polytopes. One is the big theory of Gröbner fans and state polytopes. The other is the reciprocal relation between initial ideals of toric ideals and triangulations of convex polytopes.

Recently, Gröbner bases of rings of differential operators turn out to be useful to numerical evaluations of a broad class of normalizing constants in statistics. The method is called the holonomic gradient method, which is a rapidly growing area in algebraic statistics. Chapter 6 is a self-contained exposition to invite readers to these topics. Nobuki Takayama, the author of Chap. 6, thanks Professor Francisco Castro-Jiménez for providing useful comments to a draft of Chap. 6.

Finally, Chap. 7 provides a collection of rich problems and their answers by utilizing various software systems, such as Risa/Asir, 4ti2, polymake, R, and so on. Chapter 7 complements Chaps. 4–6, and is helpful for readers to understand how to use software systems to study or apply Gröbner bases.

On behalf of the JST CREST Hibi project, I would express our thanks to JST for providing financial support, which made it possible to organize international conferences and to employ promising young researchers. Finally, I am grateful to Ms. Kaoru Yamano for her administrative job for Hibi project.[3]

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[3] All computer programs appearing in this volume and a list of corrections are available at http://www.math.kobe-u.ac.jp/OpenXM/Math/dojo-en

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