

Contents

1	A Quick Introduction to Gröbner Bases	1
	Takayuki Hibi	
1.1	Polynomial Rings	2
1.1.1	Monomials and Polynomials	2
1.1.2	Dickson's Lemma	3
1.1.3	Ideals	4
1.1.4	Monomial Orders	7
1.1.5	Gröbner Bases	9
1.1.6	Hilbert Basis Theorem	11
1.2	Division Algorithm	13
1.2.1	Division Algorithm	13
1.2.2	Reduced Gröbner Bases	15
1.3	Buchberger Criterion and Buchberger Algorithm	16
1.3.1	S -Polynomials	17
1.3.2	Buchberger Criterion	18
1.3.3	Buchberger Algorithm	22
1.4	Elimination Theory	26
1.4.1	Elimination Theorem	26
1.4.2	Solving Simultaneous Equations	29
1.5	Toric Ideals	33
1.5.1	Configuration Matrices	33
1.5.2	Binomial Ideals	34
1.5.3	Toric Ideals	35
1.5.4	Toric Rings	37
1.6	Residue Class Rings and Hilbert Functions	41
1.6.1	Residue Classes and Residue Class Rings	41
1.6.2	Macaulay's Theorem	46
1.6.3	Hilbert Functions	47
1.7	Historical Background	51
	References	53

2 Warm-Up Drills and Tips for Mathematical Software	55
Tatsuyoshi Hamada	
2.1 Using MathLibre	55
2.1.1 How to Get MathLibre	56
2.1.2 How to Boot and Shut Down MathLibre	56
2.1.3 Various Mathematical Software Packages	56
2.2 File Manager	57
2.2.1 New Folder	58
2.2.2 New Text File	59
2.3 Terminal	59
2.3.1 Files and Directories	60
2.3.2 Text Files	62
2.3.3 Input and Output	62
2.3.4 Character Codes	63
2.4 How to Write Mathematical Documents	65
2.4.1 Writing a \TeX Document	65
2.4.2 Making a PDF File	66
2.4.3 Brief Introduction to \TeX Source Code	66
2.4.4 Math Formulas	67
2.4.5 <code>graphicx</code> Package	68
2.5 Various Math Software Systems	69
2.5.1 KSEG	69
2.5.2 GeoGebra	80
2.5.3 Surf Family	86
2.5.4 Maxima	87
2.5.5 R	91
2.5.6 Sage	92
2.6 Emacs	93
2.6.1 Starting Emacs	95
2.6.2 Cut and Paste	97
2.6.3 Editing Multiple Lines	97
2.6.4 Remove Again	99
2.6.5 Point, Mark, and Region	100
2.6.6 Undo, Redo, and Etc.	101
2.6.7 Command and Shell	102
2.6.8 Math Software Environment	102
2.7 Other Ways of Booting MathLibre	102
2.7.1 Various Virtual Machines	103
2.7.2 Making a USB-Bootable MathLibre	104
2.7.3 How to Install MathLibre to an Internal Hard Disk	105
References	106

3	Computation of Gröbner Bases	107
	Masayuki Noro	
3.1	Improving the Efficiency of the Buchberger Algorithm	108
3.1.1	Elimination of Unnecessary S-Pairs	110
3.1.2	Strategies for Selecting S-Pairs	111
3.1.3	Homogenization	112
3.1.4	Buchberger Algorithm (an Improved Version)	113
3.2	Using Macaulay2, SINGULAR, and CoCoA	115
3.2.1	Getting Started	115
3.2.2	Packages, Libraries, and Files	118
3.2.3	Rings, Term Orderings, and Polynomials	120
3.2.4	Computation of Gröbner Bases	123
3.2.5	Computation of Initial Ideals	125
3.2.6	Computation of Quotient and Remainder	127
3.3	Operations on Ideals by Using Gröbner Bases	130
3.3.1	Elimination Ordering	130
3.3.2	Sum, Product, and Intersection of Ideals	132
3.3.3	Radical Membership Test	133
3.3.4	Ideal Quotient and Saturation	134
3.3.5	Computation of a Radical	137
3.4	Change of Ordering	138
3.4.1	FGLM Algorithm	139
3.4.2	Hilbert-Driven Algorithm	141
3.5	Computation of Gröbner Bases for Modules	142
3.5.1	Term Orderings for Modules	142
3.5.2	Buchberger Algorithm for Modules	143
3.5.3	Computation of Syzygy	144
3.6	Computation in Risa/Asir	145
3.6.1	Starting Risa/Asir	145
3.6.2	Help Files and Manuals	146
3.6.3	Reading and Writing Files	146
3.6.4	Polynomials	146
3.6.5	Term Orderings	147
3.6.6	Computation of Gröbner Bases	148
3.6.7	Computation of Initial Ideals	149
3.6.8	Computation of the Remainder	149
3.6.9	Elimination	150
3.6.10	Computation of Minimal Polynomials	150
3.6.11	Change of Orderings for Zero-Dimensional Ideals	151
3.6.12	Ideal Operations	151
3.7	An Example of Programming in Macaulay2	153
3.7.1	Primary Decomposition of Ideals	153
3.7.2	SYCI Algorithm	154
3.7.3	Implementation in Macaulay2	156

3.8	Additional Problems	160
3.9	Answers to Problems	160
	References	162
4	Markov Bases and Designed Experiments	165
	Satoshi Aoki and Akimichi Takemura	
4.1	Conditional Tests of Contingency Tables	166
4.1.1	Sufficient Statistics	166
4.1.2	2×2 Contingency Tables	169
4.1.3	Similar Tests	176
4.1.4	$I \times J$ Tables	180
4.2	Markov Basis	192
4.2.1	Markov Basis	192
4.2.2	Examples of Markov Bases	195
4.2.3	Markov Bases and Ideals	200
4.3	Design of Experiments and Markov Basis	204
4.3.1	Two-Level Designs	205
4.3.2	Analysis of Full Factorial Designs	206
4.3.3	Analysis of Fractional Factorial Designs	213
4.4	Research Topics	217
4.4.1	Topics with Markov Bases for Models without Three-Factor Interactions for Three-Way Contingency Tables	217
4.4.2	Topics Related to the Efficient Algorithm for a Markov Basis	218
4.4.3	Topics on Modeling Experimental Data	219
	References	220
5	Convex Polytopes and Gröbner Bases	223
	Hidefumi Ohsugi	
5.1	Convex Polytopes	223
5.1.1	Convex Polytopes and Cones	224
5.1.2	Faces of Convex Polytopes	225
5.1.3	Polyhedral Complices and Fans	230
5.2	Initial Ideals	230
5.2.1	Initial Ideals	231
5.2.2	Weight Vectors and Monomial Orders	231
5.2.3	Universal Gröbner Bases	234
5.3	Gröbner Fans and State Polytopes	235
5.3.1	Gröbner Fans of Principal Ideals	235
5.3.2	Gröbner Fans and State Polytopes of Homogeneous Ideals	237
5.4	State Polytopes of Toric Ideals	244
5.4.1	Circuits and Graver Bases	244
5.4.2	Upper Bounds on the Degree	245

5.4.3	Lawrence Liftings	250
5.4.4	Computations of State Polytopes	251
5.5	Triangulations of Convex Polytopes and Gröbner Bases	252
5.5.1	Unimodular Triangulations	252
5.5.2	Regular Triangulations	254
5.5.3	Initial Complices	256
5.5.4	Secondary Polytopes and State Polytopes	261
5.6	Ring-Theoretic Properties and Triangulations	264
5.6.1	Lexicographic Triangulations and Unimodular Configurations	264
5.6.2	Reverse Lexicographic Triangulations and Compressed Configurations	268
5.6.3	Normality of Toric Rings	270
5.7	Examples of Configuration Matrices	272
5.7.1	Configuration Matrices of Finite Graphs	272
5.7.2	Configuration Matrices of Contingency Tables	275
	References	277
6	Gröbner Basis for Rings of Differential Operators and Applications	279
	Nobuki Takayama	
6.1	Gröbner Basis for the Ring of Differential Operators with Rational Function Coefficients R	279
6.2	Zero-Dimensional Ideals in R and Pfaffian Equations	286
6.3	Solutions of Pfaffian Equations	289
6.4	Holonomic Functions	299
6.5	Gradient Descent for Holonomic Functions	300
6.6	Gröbner Bases in the Ring of Differential Operators with Polynomial Coefficients D	304
6.7	Filtrations and Weight Vectors	311
6.8	Holonomic Systems	314
6.9	Relationship Between D and R	316
6.10	Integration Algorithm	318
6.11	Finding a Local Minimum of a Function Defined by a Definite Integral	326
6.12	A -Hypergeometric Systems	330
6.13	Notes	342
	References	343
7	Examples and Exercises	345
	Hiromasa Nakayama and Kenta Nishiyama	
7.1	Software	346
7.2	Markov Bases and Designed Experiments	347
7.2.1	Conditional Tests of Contingency Tables (Sect. 4.1)	348
7.2.2	Markov Basis (Sect. 4.2)	350
7.2.3	Design of Experiments and Markov Basis (Sect. 4.3)	363

7.3	Convex Polytopes and Gröbner Bases	367
7.3.1	Convex Polytopes (Sect. 5.1)	368
7.3.2	Initial Ideals (Sect. 5.2)	376
7.3.3	Gröbner Fans and State Polytopes (Sect. 5.3)	379
7.3.4	State Polytopes of Toric Ideals (Sect. 5.4)	384
7.3.5	Triangulations of Convex Polytopes and Gröbner Bases (Sect. 5.5)	391
7.3.6	Ring-Theoretic Properties and Triangulations (Sect. 5.6) ..	403
7.3.7	Examples of Configuration Matrices (Sect. 5.7)	407
7.4	Gröbner Basis of Rings of Differential Operators and Applications	413
7.4.1	Gröbner Basis for the Ring of Differential Operators with Rational Function Coefficients R (Sect. 6.1)	414
7.4.2	Zero-Dimensional Ideals in R and Pfaffian Equations (Sect. 6.2)	426
7.4.3	Solutions of Pfaffian Equations (Sect. 6.3)	432
7.4.4	Holonomic Functions (Sect. 6.4)	439
7.4.5	Gradient Descent for Holonomic Functions (Sect. 6.5)	440
7.4.6	Gröbner Bases in the Ring of Differential Operators with Polynomial Coefficients D (Sect. 6.6)	441
7.4.7	Holonomic Systems (Sect. 6.8)	445
7.4.8	Relationship of D and R (Sect. 6.9)	450
7.4.9	Integration Algorithm (Sect. 6.10)	452
7.4.10	Finding a Local Minimum of a Function Defined by a Definite Integral (Sect. 6.11)	462
	References	465
	Index	467

Gröbner Bases

Statistics and Software Systems

Hibi, T. (Ed.)

2013, XV, 474 p. 123 illus., Hardcover

ISBN: 978-4-431-54573-6