# Preface

Stress and strain analysis in rotating disks and cylinders subjected to surface and body loads, as well as to thermal loads deriving from temperature variation along the radius, is a classic subject of machine design. It is a fascinating subject indeed, but also an extremely complex one, as the basic background of the elasticity theory applied to two- or three-dimensional structures is not sufficient for thorough analysis. In fact, it is often necessary to broaden the design spectrum in order to investigate what happens in the case of centrifugal loads that cause stresses beyond the material's yield point, or strong thermal fields that give rise to creep.

The elastic analysis of stresses and strains in rotors is usually the subject of several chapters in standard machine design textbooks. As a rule, these chapters focus on elastic problems for which an exact analytical solution in the form of algebraic formulas for the quantities of interest can be obtained, but seldom deal with inelastic or creep problems. Moreover, attention is limited to disks for which governing equations are solvable in closed form (constant-thickness, hyperbolic-profile, and uniform-strength disks). Only in a few texts is the conical disk taken into consideration, and always under elastic load.

However, very few actual disks have a geometry that is sufficiently simple for such solutions to be directly applicable. Normally, the geometry of rotors is far from that of an axisymmetrically loaded, rotating solid: this is dictated by design requirements. In such cases, numerical solution methods are clearly the only practicable ways for accurately defining the main quantities that are crucial to designers. The most widespread numerical technique for stress and strain analysis of such structures is the finite element method (FEM). Nevertheless, there are some cases where the finite difference method still has several advantages. The boundary integral equation or boundary element method (BEM) can also be used very effectively in a few specific cases.

Designers of disks and rotors are well aware that very few practical engineering issues involve structures for which stress and strain analysis in the linear elastic field and, to an even greater extent, under nonlinear conditions (i.e., plastic or viscoelastic conditions) produces equations to be solved in closed form in order to express relevant parameters through algebraic formulas.

However, formulating the problem in strict analytical terms, and in compliance with the physical phenomenon to be modeled, is essential for arriving at a full understanding of the problem from a continuum mechanics standpoint. Moreover, when a product is still in the design stage, such a formulation is simpler and more versatile than FEM or BEM numerical methods, and makes it possible to quickly define a general configuration which can then be fine-tuned using these numerical methods. Indeed, working with an analytical equation at this stage is much easier than working with a numerical model, even when the latter is expressed in a parametric form. Thus, a more rational design layout is obtained, which makes it possible to save on calculation resources and to streamline the process that leads to the final project.

In a setting where numerical methods, and FEM in particular, are used almost exclusively, a text that returns to an emphasis on theoretical/analytical methods cannot fail to come as something of a surprise. There are several reasons for choosing such an emphasis for the topic covered here, including the following:

- There can be no doubt that these methods are the foundation of numerical techniques. A better knowledge of the former can thus open up new prospects for the latter, while the coordinated use of both can contribute significantly to optimizing the structural calculation procedures used for rotors, bringing major computational advantages.
- Though FEM methods have many advantages during detail design, theoretical/analytical models are much more flexible and versatile during the conceptual development of a new rotor: not only are they entirely general, but they require at most that a differential equation be integrated using simple numerical analysis procedures in the cases where it cannot be solved in closed form.
- When theoretical/analytical models are used, rotor optimization (in terms of profile shape, mass and hence inertia, strength, etc.) is simpler than with FEM modeling which, despite the introduction of new and increasingly sophisticated procedures, still hinges on the initial discretization of the structure in question.
- The theoretical/analytical models presented here are entirely general, as they make it possible to address and solve any singularity problem, including that arising at the rotational axis. During conceptual design, this is another aspect of theoretical/analytical models that makes them highly advantageous from the computational standpoint.
- Lastly, a familiar fact should be borne in mind: the results obtained with FE models are verified experimentally, by creating a simple FEM model whose theoretical solution is known, or by direct comparison with the results that can be obtained with theoretical/analytical model.

It should be borne in mind, however, that the theoretical treatment based on the biaxial stress state leads to results that are more approximate than those of the finite element method, which can account for the effects of triaxial stress. Experimental evidences as well as direct comparison between bidimensional theoretical analysis and FE numerical results demonstrate that the differences are insignificant in the conceptual design of a turbine disk, whose axial dimension is small compared to its outside diameter.

Accordingly, the focus is maintained on analytical-methodological aspects in dealing with each subject. To ensure that analytical models can be even more readily used at the product design stage – thus allowing for extensive operational flexibility – a function is introduced whereby it is possible to define a fourfold infinity of disk profiles, solid or annular, concave or convex, converging or diverging. With this function, even constant-thickness disks or conical disks can be examined as particular cases. Most solutions are expressed in an analytical form, by using linear combinations of hypergeometric functions and particular integrals in closed form. However, actual engineering applications are considered in the analytical formulation of all problems.

To make this text as comprehensive as possible, numerical methods – especially those based on finite differences – are introduced and utilized. The finite element method is used only for verifying and/or validating the analytical approach in the various examples shown, which make the theoretical approach easier to understand and more practical for use in design. With the exception of a few instances, these example cases are developed in full, in order to provide designers with a detailed description of all successive subsequent steps of the analysis process, which is the basis for calculating and verifying disk profiles and configurations of actual practical interest.

This layout mirrors the course in Machine Design offered to final year students of Mechanical Engineering at the University of Rome Tor Vergata. However, this book covers a far broader range of topics than were dealt with in the course. From the outset, in fact, the intention was to provide a textbook for researchers and industry specialists facing actual issues pertaining to rotor conceptual design, as well as for advanced university courses, such as those for Ph.D.s and other postgraduate degrees. This work, which addresses the various subjects in increasing levels of complexity, also covers the basics, including application examples to provide a better understanding of each problem, and is thus also intended for university teaching at master's degree level.

This textbook consists of an introduction which describes and discusses general hypotheses and the associated assumptions underlying theoretical approaches and solution methods for disks, which apply both to elastic conditions and beyond yielding, plus 13 chapters.

The first nine chapters deal with stress analysis in rotating disks in the linear elastic field. More specifically, after outlining a few general considerations, these chapters address the following aspects: mono-dimensional theory of thin disks; equilibrium and compatibility equations; general differential equation for a rotating disk subjected to thermal load; solid and annular disks of constant thickness subjected to various loads – including thermal load – and featuring a fictitious density variation along the radius; hyperbolic-profile annular disk subjected to various loads including thermal load; uniform strength disk; solid and annular conical disks with either variable or constant density along the radius, subjected to various loads including thermal load; disk whose thickness varies according to a power of a linear function and having either variable or constant density along its radius, subjected to various loads including thermal load; disk of arbitrary profile:

Timoshenko-Grammel's method and Manson's method; disk of constant thickness and disk of variable thickness according to a power of a linear function, subjected to angular acceleration; and yield criteria for rotating disks and stress concentration.

Chapter 10 concerns stress analysis in rotating cylinders in the linear elastic field. Specifically, after outlining a few general notions, this chapter addresses the following subjects: fundamental governing equations; cylindrical solid, either with clamped ends or indefinitely extended in the direction of its axis, subjected to centrifugal and thermal loads; cylindrical solid of finite length with free ends, subjected to centrifugal and thermal loads (with edge effect stress on the free ends); and cylindrical solid rod of finite length with free ends, subjected to thermal transient load.

Chapters 11 and 12 deal with stress analysis of disks subjected to centrifugal loads that cause stresses beyond the material's yield point. More specifically, after outlining few necessary general ideas, these chapters analyze the following aspects: approximated basic hypotheses and their limitations regarding disks in non-hardening materials: Millenson-Manson's method as an extension of Manson's method from elastic load to elasto-plastic load; general analytical method; design-related use of plasticization and limit-design factor.

The last chapter contains three appendixes dealing respectively with: calculation of one-dimensional elements subjected to centrifugal load, such as rotating bars, paddles, and blades; in-depth analysis of the solution of the hypergeometric differential equation; and the finite element method for elasto-plastic problems.

To the authors' knowledge, a textbook such as this, which focuses specifically on the analysis and design of rotors (disks and cylinders), does not exist in the international technical literature. It should be pointed out that it not only covers "classic" subjects but also includes advanced scientific contributions by the authors. All these contributions are aimed at designing rotors at the concept stage, i.e., prior to detailed design. Special mention should be made of the following: linear elastic analysis of a conical disk subjected to thermal load and showing fictitious variable density along its radius, by implementing a procedure that simplifies and generalizes the solution put forth by Honegger and Giovannozzi; linear elastic analysis of a disk whose profile varies along the radius according to a power of a linear function, subjected to thermal load and with variable density, which makes the previous calculation method for conical disks more general; analysis of a variable-profile disk subjected to centrifugal load beyond the material's yield point, according to Von Mises' yield criterion, and introducing the completely general law $\sigma = \sigma(\varepsilon)$, expressed by an *n*-grade polynomial; linear elastic analysis of Stodola's hyperbolic disk, subjected to thermal load along its radius; and linear elastic analysis of a disk whose thickness varies according to a power of a linear function, subjected to angular acceleration.

Rome Vincenzo Vullo
March 2012 Francesco Vivio

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Vullo, V.; Vivio, F.

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