Preface

This book is designed as an advanced undergraduate or a first-year graduate course for students from various disciplines like applied mathematics, physics, engineering. It has evolved while teaching courses on partial differential equations during the last decade at the Politecnico di Milano.

The main purpose of these courses was twofold: on the one hand, to train the students to appreciate the interplay between theory and modelling in problems arising in the applied sciences and on the other hand to give them a solid background for numerical methods, such as finite differences and finite elements, also through numerical simulations for selected problems. Accordingly, this textbook is divided into two parts.

The **first one**, Chapters 2 to 6, has a rather elementary character with the goal of developing and studying basic problems from the macro-areas of *diffusion, propagation and transport, waves and vibrations.* A knowledge of advanced calculus and ordinary differential equations is required to this part. Also, the repeated use of the method of separation of variables assumes some basic results from the theory of Fourier series. All this background material is summarized in the introductory Chapter 1 and in the Appendices.

Chapter 2 is devoted to first order equations and in particular to first order scalar conservation laws. Simple models from traffic dynamics are used to introduce concepts as characteristics lines, rarefaction and shock waves.

Chapters 3 and 5 deal with diffusion/reaction diffusion models, respectively. The heat and the Fisher-Kolmogoroff equations constitutes the reference models to illustrate the qualitative properties of the solutions and the asymptotic behavior towards equilibria.

In Chapter 4, the main properties of solutions to the Laplace/Poisson equation, Maximum principle, mean value properties, Green's function and Newtonian potential are the main topics.

In Chapter 6 the fundamental aspects of waves propagation are examined, leading to the classical formulas of d'Alembert, Kirchhoff and Poisson.

The **second part**, Chapters 7,8 and 9, develops the Hilbert spaces methods for the *variational formulation* and the analysis of *linear boundary* and *initial-boundary value problems.*

The understanding of these topics requires some basic knowledge of Lebesgue measure and integration, summarized in Chapter 7. This chapter contains tools from functional analysis in Hilbert spaces. The main theme is the solvability of abstract variational problems, leading to the Lax-Milgram Theorem. Then, we present a brief introduction to the theory of distributions of L. Schwarz and the most common Sobolev spaces, necessary for a correct variational formulation of the most common boundary value problems.

Chapter 8 is devoted to the variational formulation of elliptic boundary value problems and their solvability. The development starts with one-dimensional problems, continues with Poisson's equation and ends with general second order equations in divergence form. The last section contains an application to a simple control problem, with both distributed observation and control.

The issue in Chapter 9 is the variational formulation of initial-boundary value problems for second order parabolic operators in divergence form.

At the end of each chapter, a brief account of numerical methods is included, with a discussion of some particular case study, to complete a *model-theory-simulation* path.

Also a number of exercises is presented. Some of them can be solved by a routine application of the theory or of the methods developed in the text. Other problems are intended as a completion of some arguments or proofs in the text. Also, there are problems in which the student is required to be more autonomous. Most problems are supplied with answers or hints at the end of the volume.

In the first part the exposition if flexible enough to allow substantial changes in the order of presentation of the material, without compromising the comprehension. All chapters are in practice mutually independent, with the exception of Chapter 5, which presumes the knowledge of Chapters 3 and 4.

In the second part, which, in principle, may be presented independently of the first one, more attention has to be paid to the order of the arguments.

A huge number of books on partial differential equation has been written. At the end of this volume we have indicated some of the most popular ones, to which the reader can refer for a more advanced comprehension of the subject.

Acknowledgments. While writing this book we benefitted from comments, suggestions and criticisms of many colleagues. In particular, we express our gratitude to Cristina Cerutti, Michele Di Cristo, Maurizio Grasselli, Alessandro Veneziani and Gianmaria A. Verzini.

Milan, September 2012

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Springer

http://www.springer.com/978-88-470-2861-6

A Primer on PDEs
Models, Methods, Simulations
Salsa, S.; Vegni, F.; Zaretti, A.; Zunino, P.
2013, XIV, 489 p., Softcover
ISBN: 978-88-470-2861-6