Chapter 2
On Logical Connectors (Junctors)

The logical significance of connector or "syncategorema" made visible in the pyramid – Deficiencies of Wittgenstein's and Post's truth-value tables of the logical connectors or so-called logical constants – Proposition-forming connectors characterized by truth values as opposed to expression-forming connectors which lack truth values – How each of them is constructed in the pyramid – The distinct functions of each of the various logical connectors in the pyramid – Equivalence as an expression-forming connector which lacks any truth value. Its function in definitions – Mathematical equations as definitions lacking truth-values – The basic mathematical operators as logical connectors for the construction of arithmetical expressions – The multiplication operator as fusing, rather than connecting concepts – The logical functions of the other mathematical connectors in defining and producing mathematical concepts or expressions: sum, difference, two types of quotient-construction – The logical significance of differential quotients and integrals – The modal connectors "probably" and "possibly" suspend the assertive character of propositions

2. Logical connectors are logical concepts for the description and interpretation of the relationships which obtain among the conceptual positions of the pyramid. As such, logical connectors have definite meanings and distinct scopes of application. The dominant opinion since Aristotle – that logical connectors are themselves "meaningless" and only express meanings in conjunction with expressions and propositions (Greek: synkategoremata; Latin, connotationes) – is therefore false. There are only vertical or horizontal relations. Logical connectors describe such relations in multiple ways. Relations of synonymy among some logical connectors are based on this multiplicity.

2.1. Some logical connectors bind concepts together only into expressions which have no truth values. We name these expression-forming connectors. They have not been sufficiently reckoned with in previous logical investigations..

2.1.1. The expression-forming logical connectors are the "and" (adjunctive connector), the exclusive and non-exclusive "or", the quantifying connectors and the equivalence connector. Negation is employed with

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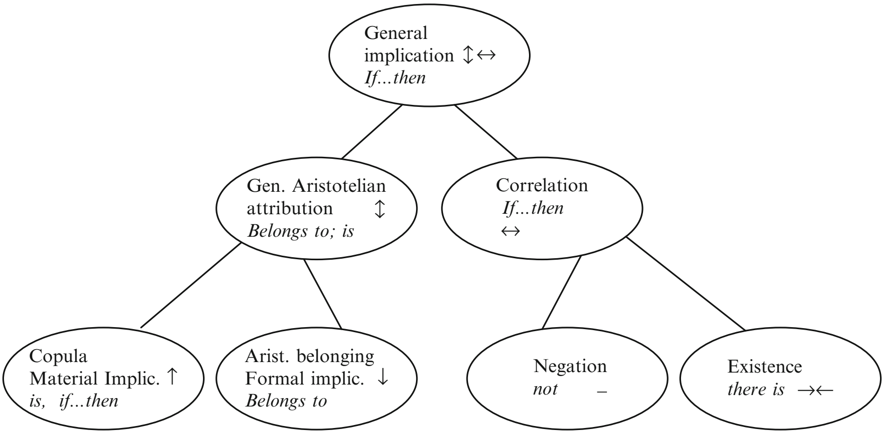

Fig. 2.1 Pyramid of the proposition forming connectors

so-called negative concepts as an expression-forming connector. Definite negation designates a (positive) concept by referring to its negated (dihaeretic) coordinate species (for example, "non-smoker").

2.2. The other logical connectors bind concepts together into propositions with truth values. We name them proposition-forming connectors. They are: The copula ("is"), the general and special (Aristotelian) attribution, the negation of the copula ("is not"), and the existential connector ("there is"). There are also four distinct implicative connectors (all alike formulated as "if…, then...."): 1. material, 2. formal, 3. correlative, 4. general implication. Among these implicative connectors the following are linguistically synonymous and therefore logically equivalent: Copula = material implication. Special Aristotelian attribution = formal implication or inclusion. The general implicative connector is equivalent to all of the other connectors taken together. That is why, in logical practice, the "if…then…" replaces connections which are sometimes true and sometimes false (Fig. 2.1).

As the conceptual pyramid which defines them shows, the proposition-forming connectors are themselves regular logical concepts. They connect regular concepts which are ordered within a pyramid. When they do so in the particular direction which is specific and proper (see the arrows) to each, the result is a true proposition; when the connection runs in any other direction, the result is a false proposition.

2.2.1. The truth-value tables of the connectors or so-called logical constants of L. Wittgenstein (Tractatus logico-philosophicus 5.101) and Post have become generally accepted standards of mathematical logic and especially of propositional logic. But they do not distinguish between expression-forming and proposition-forming connectors. They also attribute “meta-truth-values” for allegedly connected true and false elementary propositions to each of the 16 connectors constructed in the tables.

2.2.2. The truth-value table method of defining connectors involves several deficiencies. To begin with: “p” is supposed to signify “true proposition” and “not-p” is supposed to signify “false proposition”. But “true proposition” and “false proposition” are not themselves true and false propositions; instead, they are specified concepts. Now, concepts have no truth values. The meta-truth values ascribed to their combinations are therefore not based on the truth values of (elementary) propositions, as is supposed by the method.

2.2.3. The meaning and application of almost half of the connectors defined in this way are dubious, and the proposals for naming and characterizing them are very unclear. There is no model or example for their meaningful application.

2.2.4. There is no definition of the copula, the proposition-forming connector (“is”). Instead the function of the copula is confused with the (expression-forming) function of equivalence. There is also no definition of the expression-forming connectors for quantification.

2.2.5. Tautology is “defined” as a joining together (adjunction) of two supposed truths (“if p then p, and if q then q”), but this amounts to the expression “one truth and another truth”, which doesn’t constitute any proposition at all. Contradiction is “defined” as an adjunction of two contradictions (“p and not-p, and q and not-q”), which means “a contradiction and another contradiction”. Actually, these aren’t definitions nor propositions at all.

2.2.6. Accordingly, the truth-value tables of propositional logic are incomplete, redundant, and misleading because they do not take account of the distinction between proposition-forming and expression-forming connectors.

2.3. The general implication is the highest level general concept among the proposition-forming logical connectors. Its meaning is the omnidirectional connection of concepts within a pyramid; that is, it is a connector which links concepts together in any direction. (If A then AB, if AB then A, if AB then AC, if AC then AB). It’s truth values are those of the three subordinate forms of implication to the extent that these are true. It does not possess any truth value or falsity in its own right, but only to the extent it supplants one of the special implications.

2.3.1. The general implication is by no means tautological in the sense in which the tautology is defined in the truth-value tables as “always true” (“if p, then p” and “if q, then q”). Rather, the tautology cannot properly be considered a logical connector at all, simply because it doesn’t connect. To “connect” something with itself is not a connection!

2.3.2. Unrestricted use of general implication would result in the apparent truth of any proposition in the form of an "if.., then…" statement. But such a use would contradict one of the false truth values of one of the forms of special implication. For that reason, such an unrestricted use could at most apply in cases where one lacks understanding of the thematic (pyramidal) connections of the concepts or propositions involved, as for example in the textbook piece of wisdom "If a butterfly beats its wings in Paris, than there will be a typhoon in Japan."

2.4. The two dihaeretic species concepts subordinate to the generic concept of general implication are the general Aristotelian "attribution," which connects vertically in both directions, and the correlating implication, which connects horizontally in both directions.

2.4.1. The general "attribution" is therefore ambiguous. It means that a generic concept or its characteristics as generic characteristics are intensional attributes of the species and subspecies which are subordinate to that generic concept. At the same time, it means that those species and subspecies are extensionally attributed to or subsumed under a common generic concept.

2.4.2. The correlative implication can be read as "if one, than the other." (If AB, then AC, and the reverse.) In substantive applications it serves as the logical formulation of correlative and in particular of causal relationships. In some Stoic inferences, it represents (as a conjecture which is not an assertion) the "non-demonstrable" premise for the causal inference.

2.4.3. The correlative implication presupposes a generic concept which is common to the correlated concepts. The common generic characteristics of the correlated concepts – in other words, that which is identical in each of them, also often called the tertium comparationis – are contained in this general concept, while the specific differences of the correlated concepts express that which distinguishes them.

2.4.4. When the correlative implication is applied to causal connections ("if cause, then effect" or "if effect, then cause") the temporal difference (cause before, effect after) of the linked substantive parts of the causal correlation must also be expressed in these specific differences. As Sextus Empiricus (flourished 180–200 A.D.) emphasized, cause and effect cannot be "simultaneous". In that case, the common superior concept designates a substantial identity between cause and effect which persists through time. It falls to the particular ontological theory in which this logical connector is used to specify in what this identity could consist.

2.4.5. The dihaeretic species concepts under the general attribution are: special attribution, which connects only from above to below (this is the connector which Aristotle employed most); and the copula, which connects only from below to above. The copula is synonymous with material implication (of generic characteristics in subordinate concepts).

Special attribution is synonymous with formal (extensional) implication or inclusion (of subordinate concepts in the extension of a superior one).

2.4.6. The dihaeretic species concepts combined in the concept of correlative implication are: negation (of the copula), on the one hand, and the existential connector, which connects horizontally in both directions, on the other. Negation occurs only between coordinate concepts (AB is not AC). Its image in the pyramid is the empty space between coordinate concepts. The existential connector, as negation's counterpart, cancels the distinction between coordinate concepts and converts their common (generic) characteristics into a higher level concept ("there is A"). That is why the existential connector serves to introduce general concepts into the logical formalism.

2.5. The negation (of the copula) is the distinguishing connector of coordinate concepts.

2.5.1. The negation of the copula is only reversible (commutative) between dihaeretic (mutually exclusive) coordinate species of a conceptual pyramid. It is not reversible between multiple coordinate species concepts, or between concepts drawn from different conceptual pyramids. In such cases it is transitive. Many logical mistakes of so-called double negations are due to the failure to observe this distinction.

2.5.2. The negation of the negation (so-called double negation) refers back to the initial concept only between dihaeretic species concepts. In all other applications double negation refers indeterminately to any and all concepts outside the scope of the general concept which includes the initially negated concept. The so-called "four corners" (Sanskrit: "catuskoti") of the Indian-Buddhist logician Nagarjuna (Second century A.D.) ("not A" and "not not-A" and "not: A and not-A" and "not: A or not-A") in their traditional understanding make use of this indeterminacy for the "proof" of the so-called "void" or "emptiness" (shunyata) of concepts.

2.6. The existential connector ("there is") designates directly individual conceptual positions within a pyramid or introduces their designations. When applied to content-filled examples, it asserts the existence of an object which corresponds to the concept. It also introduces negative expressions (negative terms) ("There are non-smokers"). Assertions concerning existence can be false if the existence of an existing (formalized) conceptual position is denied or a non-existent position is asserted. The conceptual position in the pyramid is itself the object of the logical assertion that it exists in relation to all other concepts.

2.6.1. The existential connector is often combined with negation, most often in the form of the negation of the extension of a concept. ("There are no ....") This mostly occurs, as a matter of both linguistic formulation and of content, within limited contexts. ("There are no fresh vegetables in winter," although of course there are in other seasons!). Absent such restrictions, the problem called "Meinong's paradox" arises, to the extent that the existence and non-existence of a concept (and in some cases of the object to which it refers) is asserted at the same time. In logic, however, this occurs only in the case of contradictory concepts. What is actually negated in the case of the negative existential connector is only the extension of the contradictory concept. What the negated existential connector expresses is that the contradictory concept has no extension of its own, that is, separate and apart from the combined extensions of its constituent concepts, and the same holds for all irregular (contradictory and dispositional) concepts. These concepts have traditionally been thought of as having no extension of their own and consequently referring to no object or state of affairs. Of course, a contradictory concept must be recognized as such in order to be introduced by means of a negated existential connector. For example: "There are no zombies"; that is, no living dead people.

2.7. Each connector which forms an expression is constructed by fusing together two of the dihaeretic connectors which form propositions, and which were discussed above, into contradictory connecting concepts (of the connector). For that reason, they possess no extensions of their own, but have at the same time both extensions of each of the original connectors which were combined to form them. Through this fusion they reciprocally cancel their respective functions, which generated truth values. That is why expression-forming connectors have no truth values. They can be inscribed, in rectangles, in the pyramid of the proposition-forming connectors as contradictory concepts. The expression-forming logical connectors also supply the logical basis of the mathematical connectors used in calculating (Fig. 2.2).

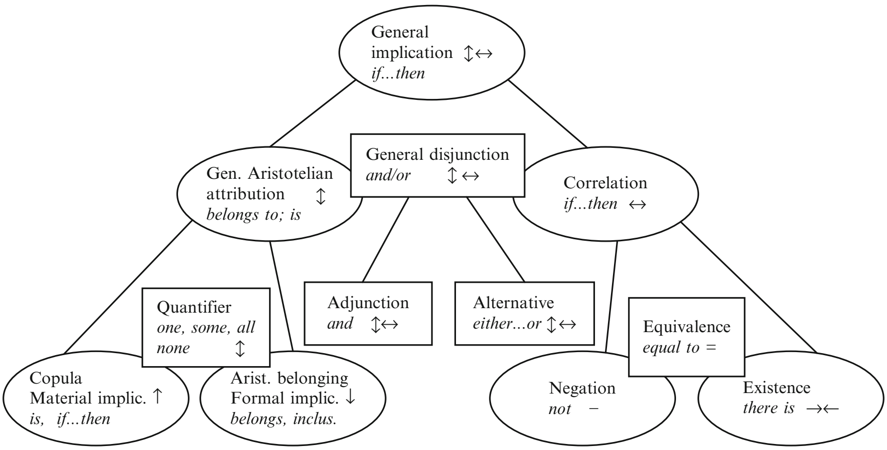

Fig. 2.2 Pyramid of both the proposition-forming and the expression-forming connectors

2.8. The non-exclusive (general) disjunction (“and/or” or Latin “vel”) is formed by fusing general attribution and correlative implication and connects in all directions. Adjunction (“and”) and the alternative (“either…or…”, Latin “aut… aut”) differentiate and specify these modes of connection. One sees that adjunction and the alternative are coordinate species concepts. They define each other reciprocally through negation: “And” means “not or” and “or” means “not and.”

2.9. The (positive) quantifying expressions (a …, some …, all …) are formed by fusing the copula and special attribution and they connect vertically in both directions. “all” connects a general concept with each of the subordinate concepts which come within its scope of application. “some” connects a general concept in an indeterminate form with one or more of its subordinate concepts, but does not express with which of two or more concepts the connection is made. “a” (one of) connects a general concept in an indeterminate fashion with one of its subordinate concepts or with one of the individuals which lie within its scope of application, but does not express, again, with which concept or individual of “all” of them, the connection obtains. Particular and individually quantified concepts are for that reason incompletely specified and require for their precise definition the exhibition of their intensions (through equivalence or in the form of an equation).

2.9.1. The logical “none” (negative quantifier) is usually formed by combining the negation and “a” or “one” (“not one”) and is used in logic in that sense. But more precisely, what is meant is “not even one of” or “all not.” “None” occurs frequently in the propositions of Aristotle’s syllogisms (For example: “No animal is a plant”). But Aristotle’s use can be understood throughout as the negation of a predicate (“All animals are non-plants”). In contrast, in mathematical logic the negative quantifier produces “empty concepts”, that is, concepts which are said to have definite intensions but no extension. Such structures are in each case not logical concepts, but specifically mathematical ones, which are also imported into physics. That shows itself in the fact that they are, in contrast to logical concepts, subject to what is called Meinong’s paradox: That which is thereby represented in consciousness doesn’t exist, but if it did exist, it couldn’t be represented!

2.10. Equivalence is often described as “reciprocal implication” (“then and only then when” or “if and only if”). As a logical matter, this usage can only refer to the generic characteristics of the correlative implication which is contained in, but constitutes only one aspect of the equivalence connector. In this context, to speak of implication is misleading to the extent that it suggests that equivalence involves two distinct coordinate concepts. That is not the case at all. For this implication exhibits, in addition to the generic characteristics of the (horizontal) implication, the specific characteristics of negation and the existential connector, each of which excludes the other.

Through this mutual exclusion their truth values neutralize each other, and it is this which the equivalence expresses. Negation signifies that both sides of the equivalence are not the same and therefore must be distinguishable. The existential connector, however, introduces both of the distinguished sides into the logical context of the formalism as "one concept." This equivalence is precisely what occurs when an incompletely specified concept is defined by means of one which is precisely specified (and the reverse).

2.10.1 Equivalence is therefore a connector which forms expressions, and that is why it cannot have truth values.

2.10.2. Since Leibniz it has been customary to consider all propositions with the same truth values as equivalent. Leibniz believed that all true propositions – simply because they were true – could be substituted for one another within a logical formalism without affecting either the validity of the inferences which contained them or, consequently, the truth values of the conclusions drawn from those inferences. (Leibniz's Latin formulation was "salva veritate" – "under preservation of truth"). But one doesn't see in what this equivalence between propositions which are true (and, accordingly, also between false propositions) – but which are also distinguishable and therefore distinct – could consist. The fact that they possess the common characteristic of being true (or false) does not render them any more equivalent than does the possession of any other common characteristic. Leibniz's view and its widespread application in logical formalisms is also false and misleading because "true proposition" or "false proposition" are, within the formalism itself, not propositions but specified concepts, which themselves can have no truth value. Genuine equivalence between propositions with the same truth values results only when they express the same propositional meaning (For example: "All animals are living things" = "Being alive is an attribute of all animals").

2.10.3. Equivalence expresses synonymy between concepts and terms. Equivalences are logical expressions without truth values. They serve as definitions, which can be "arbitrarily posited".

2.10.4. Lexica and dictionaries use equivalent expressions to define concepts. If equivalences were true propositions, all entries in (good) lexica and dictionaries would also be true. Anyone who wanted to rely on the exemplary mathematical "truth" of "two times two = four" could rely, with the same justification, on the exemplary philological "truth" that "poverty = (German) Armut."

2.10.5. Negated equivalences, for example those in etymological dictionaries (in which words with a current meaning are assigned to words with different historical meanings) similarly cannot have truth values.

2.11. Apart from multiplication (the formation of a product), the basic mathematical operations used for calculating are, from a logical point of view, logical connectors which form expressions. They are partly synonymous with logical connectors, and partly reducible to them. Multiplication is not a logical connector at all, but a fusion of concepts.

2.12. The principal mathematical connector is the equation. It is uncontroversial that equations are equivalences between that which stands to the left and to the right of the equals sign. Consequently, as a logical matter, an equation can have no truth value. Nevertheless, in mathematics equations are considered true propositions. Compare, for instance, Kant's example of a "synthetic judgment a priori": $5+7=12$, and Frege's example of an "analytically true function": $5+7=8+4$. Since elementary arithmetic equivalences have been, for centuries, memorized in the form of addition and multiplication tables and beyond that the expression "is equal to" (or simply "is") has usually been confused or identified with the copula, such elementary equivalences have come, as a matter of linguistic usage, to seem obviously true.

2.12.1. As a logical matter, inequalities are negated equations. Consequently, they too have no truth values.

2.12.2. The specifically mathematical relation "less than/equal to/greater than" (< = >) similarly has no truth value. Logically, it is synonymous with the nonexclusive disjunctive connector ("…or…or…or…" or "and/or....and/or....and/or...."). It unites an equation with inequalities in a single expression.

2.12.3. If equations and inequalities were true or false assertions, there would have to be also equations and inequalities in mathematics which would be both true and false propositions at the same time (that is, contradictions). But that does not exclude that mathematical equations contain sometimes contradictory expressions, as is the case in the formalization of geometrical functions as equations which are in fact correlative implications (see also 2.16.1).

2.13. Calculations in the form of equations define one or multiple numerical values (the "solutions" of the equation) and, in reverse, numbers define expressions used in calculations. The elementary calculations and multiplication tables learned in school are essentially memorization of some of these definitions and of the ways in which they are formed.

2.14. Beyond that, certain expressions for calculations define specific kinds of number concepts. For example, subtraction beyond zero results in the negative numbers, repeated division yielding continually recurring remainders results in the irrational numbers, finding roots of negative numbers results in the imaginary numbers.

2.14.1. Addition is a form of combination which is restricted to concepts which are coordinate species, that is, concepts at the same level of the pyramid. Subtraction is (in the positive range) a combination with a negated element ("and not"). Products are not formed by means of connectors but are instead specified quantities (for example, $3 \cdot 2$ = "a doubled three or "a tripled two").

2.15. The other kinds of calculations define concepts or expressions as extended number concepts and metric concepts (particularly in geometry and physics).

2.15.1. Subtractions resulting in zero form the "empty concept" ("x and not x") and define it as zero (for example, 3–3 = 0). In analytical geometry, in which geometrical relationships (for example, curves in three-dimensional space) are defined arithmetically, each coordinate on the x, y-, or z-axis represents a particular continuum of positive and negative numbers with a zero point. The three dimensions of space intersect at right angles to each other in a common zero point. At the same time, however, each axis represents a continuum of zero points with respect to the coordinates located on the other two axes. This method of representing numbers is commonly called Cartesian system. But Descartes did not mention nor made use of it in his work "La géometrie" of 1637. Through this system zero is defined as the "common zero point" of the three number-continua as well as infinitely many zero points on each of the axes with respect to the other two axes. By this means, zero also becomes "definable" in functional equations by means of positive and negative numbers, which sounds quite dialectical. In a plane, for example, the points at which a curve intersects the x-axis (so that y = 0) are included among the "solutions" of the functional equation y = f(x) of the curve. Applying this to the physical measurement of quantities permits one to equate quantities with zero value with positive or negative measured quantities; that is, to define the latter as equivalent to the former, which also sounds quite dialectical. However, this shows that the so-called geometrical functional equations, although formalized as and named equations, aren't really equations at all. Instead, they are mathematical propositions which correlate different numerical values of different Cartesian dimensions. This is the reason why the so-called solutions of such geometrical functions have truth-values. They are therefore to be formalized and read as implicative propositions, E.g.: "If y = 0 then x = n".

2.15.2. Problems in subtraction which yield negative results form the "negative numbers" (in logical terms: numbers which are negated numbers, that is: not-numbers, but which nevertheless at the same time are numbers!). Their application to concepts of physics leads to negative concepts (minus degrees in temperature scales, for example, or positive and negative forces, positive and negative matter). The formation of such negative concepts has also become current in our "mathematicized" ordinary language. There is a plausible prototype for such usage in the language of banking and accounting. Debits and credits (liabilities and assets) are negative and positive concepts of financial value. The loan amount of the borrower, which for him is "in the red" is at the same time an asset or "in the black" for the bank, and the reverse is also true. In economics, one now speaks of "negative growth", which means a "positive decrease."

2.15.3. Mathematical powers represent the fusion of numerical concepts with themselves. This specific way of forming a product (by means of like factors) is used to define numerical values (which means other concepts). The procedure has been rendered plausible since antiquity through the model of geometrical squares and cubes with "sides of the same length." But this analogy fails to take account of the fact that these spatial "sides," considered as vectors, have different directions and are therefore different factors – a fact which stands in contradiction to the initial assumption of their identity. Moreover, any number raised to a power of one is defined as equal to itself, whereas geometrical square planes and cubes, the sides of which are equal to a single unit are plainly distinct from this single unit of length. In analytical geometry powers higher than the third power can be formulated whenever we wish to do so (in the so-called Minkowski spaces). But that which is to be represented as a fourth, fifth or higher power is only explained as a procedure analogous to combining a third Euclidian spatial dimension to the two dimensions of a plane. The physicist constructs concepts formed by means of powers "perceptible" only by means of models. Since he simply uses them as a formal factor in his calculations, he can, in principle, dispense with any sensory perception and hold out the "sensory imperceptibility" of such concepts as used in physics as their distinguishing feature. And that sets them apart from the concepts of other disciplines. Nevertheless, in other sciences and in philosophy there are concepts of the same type: That is to say, concepts formed by means of powers or "reflexive concepts," although their structure has scarcely been recognized. These concepts are characterized by the same dialectical doubling or self-multiplication, under simultaneous preservation of their unity. "Consciousness of self" (or self-consciousness) is a concept which satisfies these criteria. It has its prototype in the Aristotelian divine attribution of "thinking of thinking" (Greek: "noesis noeseos") and denotes both a unity and, at the same time, two separate things united with each other.

2.15.4. Quotients are ambiguous and have for that reason a double function. As proportions of quantities (some : some) they represent commutative expressions which cannot be further simplified by calculation (for example, the relationship between the number of goals scored by each team in a soccer match). As expressions for calculations in problems of division they define a part which results from a completed process of division. For that reason they are not commutative. (For example, distance/time = speed.). In the case of divisions which are "incomplete," that is, in which continued division of "the remainder" cannot be repeated with a finite succession of numerals, the quotient defines a specific kind of number, the irrational number. (For example, with respect to a circle, π (Pi) = circumference/diameter). The so-called "basic fractions" (1/2, 1/3, etc.) are themselves expressions used in ordinary language (a half, a third, etc.) and are therefore not problems requiring calculation. When they are represented in the form of equations (for example 1/2 = 0,5) what is shown is simply an equivalence between the representation of the expression as a quotient and as a decimal. In the case of 1/3 = 0,333…, where the number value of the right side of the equation is called irrational, one may benevolently speak of an equation but logically of non-equation, because nobody has ever shown what the exact decimal value really is. The mathematical term "limes" (Latin: boundary, German: Grenzwert) for the endless (infinite) series of points (which were never defined in mathematics as genuine mathematical signs) disguises this logical contradiction.

2.15.5. Differential quotients are proportions of infinitesimals, that is, "quantities" or "magnitudes" which cannot be expressed in numbers. Which sounds rather dialectical. Their meaning is customarily made clear – as it was by Leibniz when he invented them – through numerical proportions of the two smaller sides of a right-angled triangle, the hypotenuse of which as secant is fitted as closely as possible into a short segment of a curved line becoming in "infinitely short segments" a tangent. The differential quotient defines the so-called gradient of this curve at a point at which the shorter sides of the triangle and the hypotenuse are said to converge, approximating zero, in a way that can't be numerically determined. The contradictory nature of the differential quotient shows itself geometrically in the following way: Although this "point" is defined as "without extension," nevertheless, it is assumed that there are infinitesimal extensions at the same point (on the x- and y- axes of the Cartesian plane). This contradiction persists in the purely arithmetic application of the differential quotient, since there the expressions ∂x and ∂y serve as variables for "quantities" which are both numerical and infinitesimal. The usual explanation for this is that the infinitesimal "quantities" can approximate to zero without reaching it. This explanation camouflages the logical contradiction that such quantities are "simultaneously zero and different from zero."

2.15.6. Integrals add up certain sums – which are not capable of being added and which consist of numeric quantities and infinitesimal non-numbers – to form numbers. Since there isn't a method of calculation for the definition of integrals, their numerical values are defined by using sums of "exhausted" (that is, empirically derived) or of approximated quantities (of the content of planes or volumes) which are empirically measured and recorded in tables.

2.16. Functional equations (usually called simply "functions") are equivalences of expressions for calculations containing variables (so-called unknown magnitudes). The usual formal notation, $y = f(x)$, expresses in logical terms that the general concept or numerical value y means the same as the higher-level concept x, to the extent that x is qualified by f as its specific difference. For example, “square = quadrangle with right angles and sides of the same length”, or “bachelor = unmarried man”, or “$4 = 2 \cdot 2$”. That is the reason they are called equations. They were developed by Descartes as arithmetic definitions of geometric structures, specifically as definitions of strait or curved lines as rows of points in a plane. In Frege’s terminology, one can formulate these relationships as follows: The sense of a quantified or specified value of x and the sense of a (likewise quantified or specified) value of y have the common reference of a point in a Cartesian plane. The replacement of the variables x and y by numerical values then defines a line as a sequence of points in the plane.

2.16.1. If one abstracts from these geometrical applications, functions lose, together with their geometrical meaning, their descriptive character. If x and y receive purely arithmetic meanings, which as a rule represent different numerical values, then the functional equations become correlative implications with truth values, that is assertive propositions. The art of “analysis” then consists in isolating the zero-value(s) of one of the variables and correlating the values of the other variable. Descartes, in his “Géometrie” recommended this procedure as “the best to consider” the equations.

2.17. Mathematical quantifiers which refer to units are synonymous with the logical “a” (in the sense of “one”). That applies as well to the so-called infinitesimal (infinitely small) which represents a non-numerical unit. Mathematical quantifiers which refer to definite numerical values (for example, “greater than one”) are specifications of the logical “some.” Mathematical quantifiers which refer to the infinite (large) are synonymous with the logical “all.”

2.17.1. One sees from mathematical texts that in mathematics logical and mathematical connectors are used side by side. One moves freely in speaking of numbers and other mathematical entities from “one” to “some” or “all” or “none.”

2.18. “Probably” or “possibly” are not logical connectors. But they do in fact play a significant role in the study of propositions and inferences. Their meaning and functions, however, have been mistaken, because it has generally been assumed, in both classical and mathematical logic, that even conjectures and predictions can only be expressed in the form of unequivocal assertions (since logic does not speak in the mode of the grammatical subjunctive!). When “probably” or “possibly” are added to logical connectors which form propositions, they suspend the assertive character of the proposition.

2.18.1. The use of the logical connectors which form implications also compels logicians to clothe conjectures in the form of true assertions. The result is that sentences properly expressed in the grammatical subjunctive (the counter-factual or the conditional) are formalized as propositions in the indicative, which convey an assertion. Logic would have avoided many mistakes and paths which have led it astray had it not followed Aristotle in logically formalizing linguistic connecting particles which, in ordinary language, express the subjunctive, such as “if … would have been (the case), then … would also have been (the case)” as implications expressing assertions, such as “if … is (the case), then … is (also the case)”. This was a mistake which the Stoics did not make.

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