2
Photonic Metamaterials and Transformation Optics: A Very Brief Introduction and Review

Martin Wegener

2.1 Introduction

The major aim of optics & photonics is to obtain complete control on light propagation and light-matter interaction. In this context, materials play a crucial role. In materials, the propagation of light is influenced by the local refractive index n. The refractive index tells us by what factor thfe phase velocity of light inside the material is slower than the vacuum speed of light. Thus, one should actually rather call the refractive index *the slowness factor* of light. Microscopically, in usual materials at optical frequencies, the phase velocity is modified by electric dipoles (formed by the negatively charged electrons and the positive nuclei) that are excited by the electric component of the electromagnetic light wave. These electric dipoles re-radiate waves just like an antenna in radio engineering. The re-emitted wave excites further electric dipoles in the material that again re-radiate, etc. Thus, it is intuitively clear that, inside the material, light will propagate with a velocity different from that in free space. Usually, it is slower than in free space. Equivalently, the refractive index is larger than unity, *i.e.*, $n > 1$. Under these conditions, the refractive index is the square root of the electric permittivity ε. $n > 1$ clearly implies that $n > 0$, which means that the phase-velocity vector and the vector of the electromagnetic energy flow, the Poynting vector, point into the same direction. Waves with this property are called *forward waves*.

At optical frequencies, magnetic dipoles play no role at all. Mathematically, this can be expressed by stating that the magnetic permeability, μ, is equal to one.

M. Wegener (✉)
DFG-Center for Functional Nanostructures (CFN), Institute of Applied Physics (AP), Institute of Nanotechnology (INT), Karlsruhe Institute of Technology (KIT), Wolfgang-Gaede-Straße 1, Karlsruhe, 76131 Germany

Institut für Angewandte Physik, Universität Karlsruhe, D-76128 Karlsruhe, Germany
e-mail: martin.wegener@physik.uni-karlsruhe.de

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This obviously limits the opportunities in optics: One can only directly influence the electric component of the electromagnetic light wave but not its magnetic component. In other words, one half of optics has been missing. Just one example is that *backward waves* (equivalent to $n < 0$) were unheard of in optics for many years. Artificial materials called metamaterials have changed that in the last decade (Sect. 2.2).

In more general, one can also consider intentionally spatially inhomogeneous magneto-dielectric metamaterial structures – a generalization of graded-index optics. This field called transformation optics (Sect. 2.3) is strongly inspired by the concepts and the mathematics of Albert Einstein's theory of general relativity.

The present brief paper gives a first flavor for these ideas and is based on the much more thorough and fairly recent reviews [1–4], in which numerous references to the literature have been given (especially see [4] regarding photonic metamaterials).

2.2 Photonic Metamaterials

Effective magnetic properties can be obtained in artificial man-made structures called metamaterials. In these structures, the usual atoms of materials are replaced by sub-wavelength-scale functional building blocks that are sometimes called *meta-atoms*. These meta-atoms can be packed densely into an effective material. In usual materials at optical frequencies, the lattice constant, a, is about thousand times smaller than the wavelength of light λ ($\lambda/a = 1{,}000$). Thus, the light wave effectively averages over the fine lattice periodicity. The same holds true in metamaterials, however, the lattice constant is typically only about ten times smaller than the wavelength of light here ($\lambda/a = 10$). As a result, the averaging is less perfect. It is not possible to unambiguously define a sharp borderline, where a description in terms of effective materials completely breaks down. However, the well-known fundamental Bragg condition corresponds to a wavelength of light that is only twice as large as the lattice constant ($\lambda/a = 2$). Under these conditions, pronounced Bragg reflection/scattering is likely to occur. If the wavelength of light equals the lattice constant itself ($\lambda/a = 1$), an incident plane wave of light generally even leads to diffracted orders emerging from the structure (giving rise to the Wood or Rayleigh anomaly). At this point, one can certainly no longer treat the periodic structure as an effective homogeneous material.

The paradigm building block of metamaterials is the metallic split-ring resonator (SRR), which is depicted in Fig. 2.1. This metallic ring with a slit can be viewed in different ways. First, one can think of it as a rolled-up half-wavelength antenna. In this picture, the SRR eigen-wavelength is given by 2π times the diameter of the ring. If densely packed, the lattice constant is only slightly larger than the ring's diameter and the operation wavelength is more than six times larger than the lattice constant ($\lambda/a = 2\pi$). Secondly, one can think of the SRR as a tiny LC-circuit. Here, the ring forms (almost) one winding of an inductor with inductance L. The ends of the wire form the plates of a capacitor with capacitance C. The resulting LC

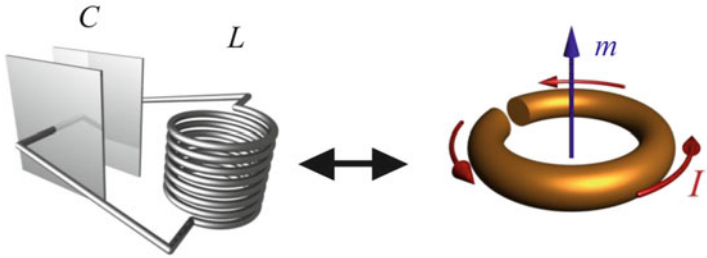

Fig. 2.1 Illustration of the split-ring resonator (*SRR*), a paradigm building block of metamaterials (Figure provided by Stefan Linden)

eigen-wavelength is about ten times larger than the ring's diameter for the structure shown in Fig. 2.1. However, one can influence the eigen-wavelength by the width of the slit. If the slit becomes very narrow, the capacitance eventually goes to infinity, and so does the eigen-wavelength.

The magnetic component of the electromagnetic light wave can induce a circulating and oscillating electric current I in the ring (see Fig. 2.1). This current gives rise to a magnetic-dipole moment, m, which is oriented normal to the plane of the ring. When excited with light at a frequency above the LC eigen-frequency, the current develops a 180-degree phase shift with respect to the excitation – just like any harmonic oscillator. For many densely packed SRR, this means that the local magnetic field can be opposite to the external magnetic field of the incident light wave. Mathematically, the magnetic permeability becomes negative, *i.e.*, $\mu < 0$.

In optics, the LC-circuit has to be so small that its eigen-frequency lies in the optical frequency range. Thus, it is interesting to ask how the eigen-frequency scales with the size of the LC-circuit. In the macroscopic world, both the inductance and the capacitance are simply proportional to SRR size. Thus, the eigen-wavelength is proportional to the SRR size as well. This finding is an immediate consequence of the scalability of the Maxwell equations. However, scalability implies that the constituent material properties (*i.e.*, those of the metal) must not change significantly. In the electric-circuit language, this means that the resistance must be negligible. Interestingly, the resistance of the SRR wire is inversely proportional to the SRR size. This means that the resistive contribution increases upon decreasing the SRR size to approach optical frequencies, whereas L and C decrease. Ohm's law for the resistance $R(\omega) = \mathrm{R} + \mathrm{i}\omega L_{\mathrm{kin}}$ has two contributions. The first one is the usual resistance R, which is frequency-independent. The second contribution is frequency dependent and stems from the fact that the metal electrons acquire a phase lag for frequencies above the collision frequency. This 90-degrees phase lag is cast into the imaginary unit, i, and means that a usual metal wire also acquires an inductive response – regardless of its shape. The corresponding inductance L_{kin} is connected to the kinetic energy of the electron system and scales inversely with SRR size. Thus, it eventually overwhelms the usual (Faraday) inductance L at small SRR sizes, hence at high operation frequencies. As a result, the LC eigen-frequency saturates and becomes size independent below a certain SRR size range. For SRRs shaped like the one shown in Fig. 2.1, the saturation occurs at around 1-μm SRR resonance wavelength [1]. For variants of the SRR, *e.g.*, with two slits to reduce capacitance and, hence, increase the eigen-frequency, visible operation frequencies have been achieved [1, 2].

Furthermore, by combining $\mu < 0$ with $\varepsilon < 0$, negative refractive indices (more precisely, backward waves) at visible frequencies have been accomplished [1]. In addition, fabrication advances have meanwhile made truly three-dimensional structures experimental reality as well [2–4]. For example, this allows for realizing compact circular polarizers based on arrays of three-dimensional gold helices exhibiting a bandwidth around one octave – an early real-world application of the far-reaching ideas of photonic metamaterials [2, 5]. Three-dimensional helices can be viewed as a transformed version of planar SRR [2, 5].

Reduction or even complete elimination of the very large photonic metamaterial losses remains to be an important and demanding future challenge [3].

2.3 Transformation Optics

Further interesting opportunities arise for intentionally *inhomogeneous* magneto-dielectric metamaterial structures. Transformation optics is a corresponding design tool: Suppose you take the rubber sheet shown in Fig. 2.2 with a Cartesian grid drawn onto it [2]. An observer looking normal onto the rubber sheet will see an

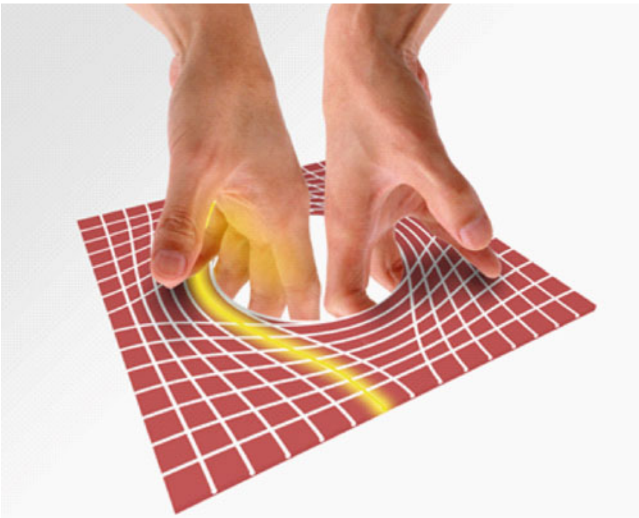

Fig. 2.2 Transformation optics connects the geometry of curved space with propagation of light in inhomogeneous magneto-dielectric materials (Taken from Ref. [3])

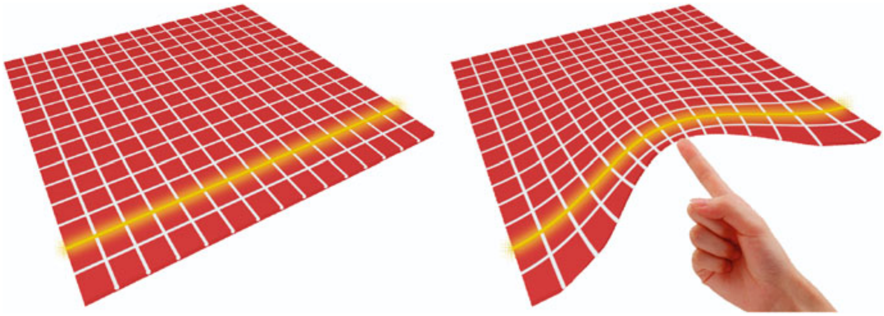

Fig. 2.3 Illustration of the carpet-invisibility-cloak transformation (Figure provided by Michael S. Rill)

undistorted rectangular grid. Upon stretching the rubber sheet within the plane or by even pulling and distorting the rubber sheet in the third dimension (mathematically, by performing a coordinate transformation), the observer will see a distorted set of lines. Any of these lines represents the potential path of a light ray [2]. By appropriate pulling on the rubber sheet, essentially any light path can be tailored. For example, if we take a screw driver and punch a hole into the rubber sheet and then open up this hole to macroscopic size in Fig. 2.2, no grid line will pass this hole. Hence light does not enter the hole, we have created an invisibility cloak [2]. Any person within the hole cannot be seen and cannot look outside his/her hole either. Amazingly, by a purely mathematical manipulation on Maxwell's equations, the effect of any such transformation on electromagnetic light waves can be shown to be exactly the same as in usual Cartesian space with a certain spatial distribution of the (real) tensor quantities $\varepsilon = \mu$. Transformation optics tells us explicitly and constructively how to derive the distribution $\varepsilon = \mu$ from the metric tensor for the complete Maxwell equations [2]. The condition $\varepsilon = \mu$ ensures that the material impedance Z is equal to the vacuum impedance everywhere, avoiding any undesired reflections of light.

Experimental realization of such complex anisotropic three-dimensional low-loss magneto-dielectric structures is presently not quite possibly yet. Often, singularities occur at the edges of the structures. However, the simple and special case of the carpet cloak illustrated in Fig. 2.3 avoids singularities, requires only locally isotropic refractive indices, and approximately gets away without a magnetic response (*i.e.*, $\mu = 1$).

Basically, the carpet cloak in Fig. 2.3 does not start from a single point (compare Fig. 2.2), but rather from the edge of a fictitious two-dimensional space. In practice, the edge corresponds to a mirror. Thus, an object can be hidden underneath a metallic carpet. To make the resulting bump in the carpet disappear (which can be viewed as a particularly demanding example of aberration correction), a graded-index structure needs to be put on top of it. Transformation optics allows for designing this graded-index profile, metamaterials for realizing any refractive index. Corresponding experiments in three-dimensional space have recently been published at telecom [2, 6] and at visible [7] frequencies.

However, invisibility cloaking is just a demanding benchmark example for the strength of the design tool called transformation optics. After all, invisibility cloaking was believed to be impossible by many just a few years ago. Presently, several researchers are searching for real-world applications designed by transformation optics.

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