Historical Reflections on the Physics Mathematics Relationship in Electromagnetic Theory

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Abstract In this paper I present a historical inquiry on the relationship between physics and mathematics in electromagnetic theory around the nineteenth century. The investigation is within the domain of the history of physics. By essentially following Maxwell's fundamental aspects of physics mathematics in his *A Treatise on Electricity and Magnetism*, some epistemological reflections will be put forth, as well as observations regarding the different scientific approaches between Faraday's *Experimental Researches in Electricity* and Maxwell's science.

1 Physics Mathematics from a Physical Standpoint

1.1 Maxwell's Debate with Faraday

Generally, complete biographical and scientific sketches of Faraday and Maxwell are well documented.[1] Thus, for the sake of brevity, here I avoid discourse on their biographical accounts. I will rather comment on some chapters of *A Treatise on*

[1] Mainly: Everitt (Everitt), Pearce (Pearce), Williams (Williams), Agassi (1971, 2008), Arianrhod (Arianrhod), Mahon (Mahon), Russel (Russel), Harman (1990, 1998, 2004), Hamilton (2002, 2004), Gooding (Gooding), Gladstone (Gladstone), Meurig (Meurig), Bence (Bence), Tyndall (Tyndall), Baggott (Baggott), Cantor (Cantor), Glazebrook (Glazebrook), Heaviside (Heaviside), Hirshfeld (Hirshfeld), Thompson (Thompson), Tolstoy (Tolstoy), Heilbron (Heilbron), Darrigol (2000). Particularly James' studies (James) on Faraday's *correspondence* are indispensable.

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Electricity and Magnetism that are relevant to the aim of this paper, as well as on Faraday's different scientific approach in *Experimental Researches in Electricity.* I will also include some historical–epistemological reflections within physics mathematics.

> I also found that several of the most fertile methods of research discovered by mathematicians could be much better in terms of ideas derived from Faraday than in their original form.[2]

1.2 On Modelling and Processes of Reasoning

In two volumes[3] of *A Treatise on Electricity and Magnetism* (Maxwell 1873) Maxwell's main cultural focus was to propose a new view of electromagnetism and the natural world. The book was also intended to be used as a Cambridge text for students taking the *Tripos examinations*. Maxwell's new thought is most evident in the final section of *A Treatise on Electricity and Magnetism* (Maxwell 1873, II, Pt IV). The physics mathematics aspect starts with his use of mathematics, e.g., the role played by *identity,* such as an equation valid for all values of the variables. We know that a similar aspect in physics is not possible; results in physics depend on measurements and their correspondence to phenomena. According to Simpson (2005, 8–10) the two sides of this identity have been used to propose alternate views of physical reality, that is, an attempt to move the physics to physics mathematics is also represented by vector and scalar concepts. In mathematics, *identities* express the same value in very different ways; in prose, we have *figures of speech* to say the same thing in different ways (*Ivi*). Thus, rhetorical issues in mathematics are essentially *mathematical figures of speech*. When these figures inspire major turns of thought, we can refer to them as *figures of thought* (Simpson 2005, 1–32). Maxwell's diagrams[4] express a cross-section of the electrostatic field of two unequally charged spheres. The electric field is static in the diagram and could be the image of an elastic medium under strain. The passage to a kinetic field (Maxwell 1873, II, Pt IV) is an evident expression of development of mathematical inquiry in physics. Thus the relationship between physics and mathematics in terms of dialectic would be expressed by figures, as well. In this sense, geometry (one of Maxwell's greatest interests) plays an important role in this dialectic. In fact, geometric figures are crucial counterparts to the analytical argument in *A Treatise on Electricity and Magnetism.* For example, part III (Maxwell 1873, II, Pt III) is mostly centered on the magnetic shell as a mathematical physical figure. The same is true for the diagrams.

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[2] Maxwell (1873, I, Preface, xi, line 6).

[3] *A Treatise on Electricity and Magnetism* had three editions in 1873, 1881 and 1891. Only the first and part of the second was edited by Maxwell. Since Maxwell's theory is a pure physics mathematics field (D'Agostino) all editions lack a dichotomy between electric charge and field. These topics will be part of post-Maxwell theories (Larmor 1891, 1892).

[4] E.g., see: Maxwell (1873, I, plate I, plate V, II, 143). See also *Ivi*, 145, 403.

The figure of a shell begins as purely mathematical and in due course becomes a physical quantity. When we envision it, it is painted with a polar medium whose density at any point represents a magnetic strength and we think in terms of a modelling action-at-a-distance theory, with the assumption that this magnetic shell will exert a force on an imagined unit magnetic pole placed at any distance. A potential can therefore be associated with any point by measuring the work required to bring the unit pole to that point, e.g., Gauss's theorem, Green's theorem, Stokes' theorem. Gauss's theorem transforms the physical idea of a shell from a mere summation of parts (as, e.g., indicated by the surface integral in Green's theorem) into an intact whole, represented in its integrity by the solid angle subtended by its boundary. Stokes's theorem works on shell and boundary (Simpson 2005, 15). We also may refer to Maxwell's interest in this subject as a holistic aspect of the potential instead of the complex, detailed action of individual forces.

For both volumes of *A Treatise on Electricity and Magnetism* Maxwell draws mathematical images, diagrams and visual "Electromagnetic instruments" (Maxwell 1873, II, Pt IV, chaps XV) to better explain physical situations that mathematics, solely, sometimes cannot explain. They also refer to illustrations of a more general and mathematical character (e.g., Maxwell 1873, II, Pt IV, chap XVI) compared to, e.g., the previous physical system Maxwell presented (e.g., Maxwell 1873, II, Pt IV, chap XIV). Thus, they help the discourse (especially physical mental models, though experiment) within the physics mathematics domain to clarify the basic physical[5] and complex structure of a natural phenomenon, e.g., the pattern of a field. For example, Ørsted's effect shows (Maxwell 1873, II, 143) a complete electric circuit and the magnetic directionality of the entire surrounding space. The direction of an electric current is generally thought of as passing from a positive electrode to a negative. In order to provide a better physical idea of the phenomenon, Maxwell included a voltaic source. Solid arrows show the direction of current flow in his diagram. The figure represents "[. . .] the same relation between a circular current and the magnetic effect it surrounds" (Simpson, 19, line 3). It should be noted that most of the physical devices and ideas also represented as figures, were born in Faraday's laboratory (Faraday 1839–1855, III, plate II). Generally, the figures contained in *A Treatise on Electricity and Magnetism* are not mere pictures but precise mathematical figures, which are most certainly addressed to the mind to simplify a physical–electrodynamical model (Buchwald). It is interesting to understand the range of Maxwell's figures by comparing some of them in *A Treatise on Electricity and Magnetism*, which, for the sake of brevity, I do not discuss here. Nevertheless, this kind of investigation may refer to

> [. . .] a mathematical construction, but may equally refer to the image of a physical object produced in the laboratory. [Therefore in some figures] [. . .] mathematical construction and physical image coincide [. . .]. They are examples of what Maxwell calls *eye-knowledge*.[6]

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[5] Of course, in order to have a larger view of the behavior of the relationship from other standpoints, one should also study (with respect to previous scientific theories) *L'espace physique entre mathématiques et philosophie* (Szczeciniarz 2006; see also *Id.*, 2008).

[6] Simpson (2005, 21–22, line 17).

Finally, the relationship between physics and mathematics (and geometry) is created by figures, as well. Another important aspect related to the physics mathematics in Maxwell's work is the role played by *analogies*, which abound in the *Treatise*. Maxwell presented early ideas on the use of physical analogies in science in his *On Physical Lines of Forces* (Maxwell 1855–1856; 1861–1862) where he specified that:

> The present state of electrical seems peculiarly unfavourable to speculation. [. . .]; some part of the mathematical theory of magnetism are established, while in other parts the experimental data are wanting; [. . .]. Such a theory must accurately satisfy those laws [on electrical theory], the mathematical form of which is known, and must afford the means of calculating the effects in the limiting cases where the known formulae are inapplicable. [. . .]. The first process therefore is the effectual of study of the science, must be one of simplification and reduction of the results [. . .]. The results of this simplification may take the form of a purely mathematical formula or of a physical hypothesis.[7]
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> In order to obtain physical ideas without adopting a physical theory we must make ourselves familiar with the existence of physical analogies. By a physical analogy I mean that partial similarity between the laws of one science and those of another which makes each of them illustrate the other. Thus all mathematical sciences are founded on relations between physical laws and laws of numbers, so that the aim of exact science is to reduce the problems of nature to the determination of quantities by operations with numbers.[8] [. . .].
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> It is by the use of analogies [. . .] that I have attempted to bring before the mind, in a convenient and manageable form, those mathematical ideas which are necessary to the study of the phenomena of electricity. The methods are generally those suggested by the *process of reasoning* [my emphasis] which are found in the researches of Faraday*, [. . .] are very generally supposed to be of an indefinite and unmathematical character, when compared with those employed by the professed mathematicians.[9]

The concept of *analogy,* which he called "processes of reasoning" (Maxwell 1856, 157, line 29), is at the foundation of many of much of Maxwell's reasonings and figures.[10] In this case, the link between mathematics and physics is amplified. In fact, in physics mathematics, e.g., we may refer to physical analogies in which processes of measurements or foundations (Beth; Lindsday) or formulations in one physical domain, such as that of mechanics, optics or fluids, may be compared to corresponding processes of measurements in another domain, such as electricity or magnetism, and establish relationships within each domain. This is one of the main aims of the history of foundations and historical epistemology of science for shared knowledge. For instance, one can also consider the relationships between charged bodies and elastic bodies under strain; or, electrostatic phenomena (Coulomb) in terms of an elastic medium. From a physics mathematics point of view, Maxwell used these processes or reasoning to discuss the use of the infinite in his mathematics. He tried to incorporate an elastic continuum as the seat of potential energy of the electric charge in the completed account of the electromagnetic field.

[7] Maxwell ([read 1855 and 1856] 1855) 155, line 1.

[8] *Ivi*, 157, line 7.

[9] *Ivi*, 157, line 29. (Author's symbol).

[10] See also Maxwell (1860).

> Maxwell's methodology had more original components. He developed the classification of quantities as short-cut through the method of formal analogies.[11]

Particularly, if we delve into the depths of the historical foundations of science, we can see an interesting analogy between electric theory and mechanics as the following table concisely demonstrates:

Table 1 Mechanics and electricity . Some general analogies among fundamental magnitudes of the theories[12]

(Lazare Carnot's) mechanics[13]	Electric theory
Quantity of motion Ip_i	Electric charge Q
Flux of quantity of motion Ip_i	Flux of electriccharge of current I_Q
Velocity v_i	Electric potential V
Viscosity η	Electric conductance σ
Mechanical resistance R_p	Electric resistance R_{Qr}
Mass of a body, m	Capacity of a charged body, C
Mechanical inductance*1/k* (k of a spring)	Electric inductance I

E.g., *how to explain analogies with attraction and repulsion* (see also Fufay)?

> In a philosophical point of view, moreover, it is exceedingly important that two methods should be compared [. . .] while at the same time the fundamental conceptions of what actually takes place, as well as most of the secondary conceptions of the quantities concerned, are radically different.[14]

In general one can claim:

Mechanical actions decrease the potential (electrodynamics)	Pressing by fluids on walls (hydrodynamics)

Particularly among theories:

Refraction of light	Reflection of trajectories of particles in a force field
Light propagation	Propagation of vibrations in an elastic body
Heat propagation in a uniform body	Action at-a-distance

An analogy between fluid flow and a magnetic field seems appropriate since flow is somehow involved in the electric current that produces the magnetic field. We can also think of the *current* in the Voltaic pile and the *heat* in a heat machine (Pisano 2004; Gillispie and Pisano 2012). Maxwell followed this line of thinking to include

[11]Darrigol (2000, 175, line 31).

[12]Gillispie and Pisano (2012) and related references on Drago (1988). See also Pisano (2011a, b).

[13]Carnot (1786, 1803a, b).

[14]Maxwell (1873, I, xii, line 20).

a kinetic account of magnetism in his approach to the electromagnetic theory. For the aim of this paper, we can see *On Faraday's Lines of Force* (Maxwell [read 1855 and 1856] 1855, 155–229) where Maxwell showed his intention of bypassing formal theory with analogy in order to show mathematical relations in ways that non-mathematicians could understand, an idea put forth by experimentalists like Faraday.

> [...] to have been in reality a mathematician of very high order–one from whom the mathematicians of the future may derive valuable and fertile methods.[15]

> [Maxwell to Faraday] Now as far as I know you are the first person in whom the idea of bodies acting at a distance by throwing the surrounding medium into a state of constraint has arisen, as a principle to be actually believed.[16]

This can also help us to appreciate Maxwell's character when he produced his set of physics mathematics equations, which were reasonably described as a mathematical version of Faraday's experiments and main ideas in physics. Maxwell was interested in natural phenomena and attempted a mathematical interpretation, raising the following question: "Are there Real Analogies in Nature?" (Campbell and Garnett [1882] 1969, 235–244). An answer to this question would be given through the idea of a connected mechanical system (expressed in Lagrange's equations).

1.3 On Physical Ideas and Mathematical Quantities

It is scientifically significant for the development of *authentic science* and for the history of science that in *A Treatise on Electricity and Magnetism* Maxwell showed his admiration for Faraday by dedicating a chapter of 26 pages (Maxwell 1873, II, Pt IV, chap I, 128–145) to Michael Faraday's physical ideas. Moreover, he proposed theoretical arguments to describe the basic phenomena of electricity and magnetism (Maxwell 1873, II, Pt IV, chap I). He freely demonstrated his scientific devotion to Faraday, wishing to induce readers to share his views. From a physics mathematics point of view and its dialectic, this chapter is emblematic.

> As I proceeded with the study of Faraday, I perceived that his method of conceiving the phenomena was also a mathematical one, though not exhibited in the conventional form of mathematical symbols. I also found that these methods were capable of being expressed in the ordinary mathematical forms, and thus compared with those of the professed mathematicians.[17]

Here, Maxwell's main intention was to show that the widespread scientific concerns with regard to this concept could be reduced.

[15] Maxwell ([1890] 2003, II, 360, line 11).

[16] Part of a letter written by Maxwell to Faraday on 9 November 1857 (Faraday 2008, Letter 3354, 301, line 24).

[17] Maxwell (1873, I, *Preface*, x, line 7).

At the beginning of this first chapter, Maxwell proposed a new interpretation of Ørsted's effect (Ørsted) from physics mathematics operators. He examined two circuits carrying two currents and sought to characterize the electric force concept of their interaction in terms of energy. Thus, he avoided any foundational discussion on the nature of the force, focusing on the concept of energy, that is, an integration. A crucial concept in physics mathematics is that, together with its inverse, differentiation is one of the two main operations in calculus. Particularly, his new point of view (including energy) will allow us to understand the action of one current upon another. Maxwell was careful to formulate two interacting currents in terms of potential[18] and mutual energy.

> [. . .] the magnetic force in a field can be deduced from a [mathematical] potential function, as in several former instances, but the potential is in this case a function having an infinite series of values whose common difference is $4\pi i$. The differential coefficients of the potential with respect to the coordinates have, however, definite and single values at every point. The existence of a potential function in the field near an electric current is not a self-evident result of the principle of the [total] conservation of energy [. . .]. We must therefore for the present consider the law of force and the existence of a potential as resting on the evidence of the experiment already described.[19]

In this sense, when focusing on scalar functions to describe a physical system, an eventual discussion of the electric force concept should take place *a posteriori*. Of course, this physics mathematics approach can leave a physicist with some doubt. In fact, Maxwell's mathematical reasoning produced local results where a point, in its physical system, can become one of the infinite results of a differential equation,[20] as opposed to the role played by *an electric pole at point Z, the charge at point Z,* etc. Therefore, an infinitesimal point (in infinitesimal analysis) cannot describe the entire physical system and its interactions. In this sense, the field concept and its mathematical framework would help the discussion. When considering this, Simpson noted that Faraday has already used an *exploring coil* (Faraday 1855, II, Plate II) in order to sample magnetic action at multiple locations near a magnet. Simpson suggested that Maxwell also followed this practice in his *A Treatise on Electricity and Magnetism* with the two circuits. While one circuit becomes "[. . .] an infinitesimal exploring loop, a new and richer "point P" to replace the old: a physical loop, not a mathematical monopole" (Simpson 2005, 43, line 20, author's marks). Thus the loop becomes a dynamic entity in its infinitesimal form and not just an exploratory body. As previously mentioned, in this part of his *A Treatise on Electricity and Magnetism,* Maxwell established the foundations and relationship between electricity and magnetism, that is, a new scientific theory within the physics

[18] Of course, as always occurs in science, the electromagnetic theory was developed thanks to many other related and correlated studies (Fox; Giannetto) with Faraday and Maxwell. For instance, one may consider Neumann's contribution (Schlote 2005, 123–140).

[19] Maxwell (1873, II, Pt IV, 130, line 19).

[20] A similar situation happened to the Newtonian second law of motion.

mathematics domain. Therefore, he required a new dynamical theory, for this new phenomenon. New physics mathematics theory also means new magnitudes, which should be both mathematical and physical. The same situation occurred in mechanics (e.g.: velocity, acceleration, space time). Therefore, in order to build a new dynamical theory, Maxwell needed to build something unlike Newtonian mathematics. It is not by chance that Maxwell established this new theory by using Lagrange's equations. He freely referenced them in Chapter V, *On the equations of motion of a connected system* (Maxwell 1873, II, 183–194).

1.4 On Electric Induction

Maxwell also dedicated most of Chapter III to Ampère's (1826; Darrigol 2000) differing theory of electrodynamics (Maxwell 1873, II, 146–161). Moreover, a logical comparison contrasting Ampère (e.g., *Ivi*, 146–156) and Faraday (e.g., *Ivi*, chap III)'s methods emerges, together with his evident preference for Faraday's ideas. This intellectual contrast is important for the aim of this paper both for understanding the role of mathematics in his physics mathematics and for understanding the kind of mathematics and scientific approach he preferred, with respect to distinguished mathematicians and physics scholars such as André-Marie Ampère[21] (1775–1836). For example, Maxwell's opinion on the complex mathematical way of arriving at Ampère's fundamental equation is noted. The equation can be found in two ways[22]:

$$d^2 f = i i' \frac{ds ds'}{r^2} \left(\sin\alpha \sin\beta \cos\gamma - \frac{1}{2} \cos\alpha \cos\beta \right)$$

$$d^2 f = i i' \frac{ds ds'}{r^2} \left(\cos\omega - \frac{3}{2} \cos\alpha \cos\beta \right)$$

By considering the advanced mathematics that one should employ to arrive at Ampère's equations and by considering his convincing argument on the crucial role played by Newtonian mechanical force to explain these new electric phenomena as well, it is clear that Maxwell could not accept the Newtonian paradigm[23] to explain

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[21] Ampère developed a mathematical theory to describe previously observed electromagnetic phenomena and he proposed many new foundations to study.

[22] The argument and its formula were read by Ampère at *Académie des Sciences* (Ampère 1826, 151, ft 1, 204; see also: Ampère 1822a, 293–318; see 317 and his comment ft 1; Ampère 1822b, XX, 187; 188; Ampère 1822c, XX, 405–419).

[23] It is known that Ampère clearly tried to mathematically describe the physical world, and new electric-magnetic phenomena as well, by using Newtonian laws. He conducted many studies to achieve this result in a new physical domain. Let us think, e.g., of his physics mathematics where a distance r appears in the denominator as a square. In this case, the law of the inverse square describes the force (like Newton for gravity) between current elements. Thackray proposes an interesting essay on Newton and his ideas in the history of science (Thackray 1970; Pisano 2007).

and mathematically describe the new kind of interactions between two elementary bodies. Moreover, the Newtonian law of universal gravitation could not be used since no interaction (attractive–repulsive) existed: one action was conceived, solely. In chapters II and III (Maxwell 1873, II, Pt IV) these intellectual and foundational aspects of Maxwell's physics mathematics theory clearly emerge; most notably the discussion on the impossibility of accepting Ampère's result (Ampère 1822a, b, 1827) since his reasoning was scientifically too weak to show true electromagnetic actions between one infinitesimal element and another. Here, the physics mathematics relationship changes since the mathematical interpretation of the physical phenomenon changes. Nevertheless, history teaches us that Maxwell's framework reconsidered Ampère's theory of forces between current elements. Ampère's law with Maxwell's correction establishes that a magnetic field can be generated both by electrical current (*Ampère's law*) and by changing the electric field (also known as *Ampère's law with Maxwell's correction*). These corrections are particularly important since they show that not only does a changing magnetic field induce an electric field, but a changing electric field also induces a magnetic field: the definitive physics mathematics interaction between electric and magnetic domains without, e. g., measuring and discussing the physical nature of magnitudes. One of the reasons may be related to (a) a consolidated mechanical Newtonian theory, (b) the authorship of Ampère in the international scientific panorama of the nineteenth century, (c) the assumption of new theoretical elements in his *A Treatise on Electricity and Magnetism* would allow Maxwell to review some reasoning and try to produce a new style of thinking in this newly debated domain of science, (d) gravitation law, weaker than electric law, is applied to all bodies, while electromagnetic laws for charged bodies only, etc. Therefore, new studies could be done.

A new discussion immediately follows the aforementioned discussion and it is connected to one of the main elements of Maxwell's physics mathematics (Maxwell 1873, II, Pt IV, III). This idea is indebted to Faraday's experimental physics: electric induction where one circuit, carrying a current, induces a current in another nearby circuit. The difference (with respect to Ampère's physics mathematics) in methods of reasoning and mathematical procedure concerning *electric induction* phenomena emerged. For instance, (ca. Faraday-) Maxwell's physics mathematics considered the action between two bodies A, and B, letting an intervening medium act on body B. The equations are in terms of mutual energy.[24] Ampère considered a direct action between two bodies A, and B, leaving the mathematical equations in terms of forces of any intervening physical medium.

Particularly, *Induction of Electric Currents* (read November 24, 1831) was one of Faraday's first discoveries in electricity[25] (Sweetman) looking back to similar electrostatic phenomena, where one can observe the production of an unbalanced electric charge on an uncharged conductive body as a result of a charged body (opposite sign) being brought close to it without touching it. Finally, only a

[24]Maxwell also discussed electric forces in terms of *electromotive force* (Maxwell 1873, II, Pt IV, IV–V).

[25]For a *historiography reassessment* see Agassi (2008, 466–468).

concise reaction in the second wire when the current in the first circuit was being turned on or off was produced. Faraday named this state the "*electro-tonic* state" (Faraday 1839–1855, I, Is, §3, p 16, line 28) but, even though he found a way around this problem by reasoning in terms of lines of force, he never mathematically demonstrates it. Therefore, from a physics mathematics standpoint, Maxwell first pointed out this concept, which was so important in Faraday's physics, by attempting a geometrical and mathematical solution in *On induction of electric current* (Maxwell 1873, II, Pt IV, III, 172–173). He began by stating his unsatisfied equation and reasoning on the matter (Maxwell 1873, II, Pt IV, III, 172–173). Then, he continued his studies in a short chapter, *Induction of a current on itself* (Maxwell 1873, II, Pt IV, IV, 180–183) specifically on *Induction of a current on itself* (Faraday 1839–1855, I, IXs). From a physical standpoint, Faraday noted (*Ivi*) a new form of inductive effect should (in fact) exist for a battery. Sparks take place when the current is unexpectedly interrupted. Nevertheless, the effect is lacking when this same wire is uncoiled and extended out in a straight line. According to Faraday, this effect is, at first glance, similar to mechanical *inertia*:

> 1077. Returning to the phenomenon, the first thought that arises in the mind is, that the electricity circulates with something like *momentum* or *inertia* in the wire, and that thus a long wire produces effects at the instant the current is stopped, which a short wire cannot produce.[26]

The analogy is interesting from a Newtonian mathematical point of view. If we go on to think from a physical viewpoint, Faraday's ideas contain an important assumption: the foundation of the theory is shifting from static to kinetics and moving systems. From a physics mathematics point of view, Maxwell produced analogies with images of the force of (matter) water in a pipe etc. These analogies are not adequate since, in effect, the water flow is unaffected by changes in the pipe and effects of self-induction, (and related phenomena) depend entirely on the configuration of the conductor. Maxwell stated that when motion is not occurring within the wire, then it should belong to the surrounding space. With regard to the correlated matter concept in his physics mathematics electromagnetic theory, Maxwell stated his fundamental dynamical idea of matter; it cannot be seen or touched, and since it is connected to energy and momentum[27] (Maxwell 1873, II, Pt IV, §550, 181–182) concepts, calculations should be advanced:

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[26] Faraday (1839–1855, I, IXs, 330, line 9).

[27] He referred to Torricelli's relation on the idea of matter: "Torricelli [. . .] has expressed the relation between the idea of matter on the one hand and those of force and momentum on the other, neither of which can exist without the other." (Maxwell [1890] 2003, II, 812). "[. . .] as Torricelli remarked 'is a quintessence of so subtle a nature that it cannot be contained in any vessel except the inmost substance of material things ["La forza poi, e gl'impeti, sono astratti tanto sottili, son quintessenze tanto spiritose, che in altre ampolle non si posson racchiudere, fuor che nell'intima corpulenza de' solidi naturali" (Torricelli 1715), *Lezioni Accademiche*, p 25; author's quotation marks]. Hence all these theories lead to the conception of a medium in which the propagation takes place, and if we admit this medium as an hypothesis, I think it ought to occupy a prominent place in our investigations, and that we ought to endeavour to construct a mental representation of

> It is difficult, however, for the mind which has once recognized the analogy between the phenomena of self-induction and those of the motion of material bodies, to abandon altogether the help of this analogy, or to admit that it is entirely superficial and misleading. The fundamental idea of matter, as capable by its motion of becoming the recipient of momentum and energy, is so interwoven with our forms of thought that, whenever we catch a glimpse of it in any part of nature, we feel that a path is before us leading, sooner or later, to the complete understanding of the subject.[28]

For Maxwell, a dynamical theory is strictly founded on the concept of energy. The matter appears to be very different from the Newtonian mass concept since, here, it moves toward a new dynamical style of thinking. Here, concerning energy, Maxwell is convinced that it is present in the space surrounding the current-carrying conductor.

> Again, when the current is left to itself, it may be made to do mechanical work by moving magnets, and the inductive effect of these motions will, by Lenz's law, stop the current sooner than the resistance of the circuit alone would have stopped it. In this way part of the energy of the current may be transformed into mechanical work instead of heat. It appears, therefore, that a system containing an electric current is a seat of energy of some kind; and since we can form no conception of an electric current except as a kinetic phenomenon[* footnote to "Faraday, *Exp. Res.* (283)"], its energy must be kinetic energy, that is to say, the energy which a moving body has in virtue of its motion.[29]

Nevertheless, since we cannot have evidence for the existence of moving bodies, rather than Newtonian matter, in the open space near a conductor, Maxwell's seems to have a conception of physical matter (Maxwell 1920) very closely related to a kind of material–mathematical modelling for interpreting a physical system. Thus, his electromagnetic theory is something more than his final physics mathematics equations. It is related to an existent state of matter: "a physical system bearing energy associated with every [mathematical] point in [geometrical] space" (Simpson 2005, 58, line 8). Faraday called it *electrostatic induction*.

Finally, Maxwell provided a mathematical form for Faraday's physical reasoning, but we should specify that he did not obtain the satisfaction of resolving it. On the other hand, in this case, Faraday's aim was more than a physical explanation. He was looking for an *electronic state* that truly exists in nature.

2 Physics Mathematics from a Mathematical Standpoint

In the previous sections, we dealt with physics mathematics evidence, from a physical point of view in relation to Faraday's physics. Here, we will focus on physics mathematics evidence but from a strictly mathematical point of view (McAuly).

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all the details of its action, and this has been my constant aim in this treatise." (Maxwell 1873, II, §866, p 438, line 9). On Torricelli see: (Capecchi and Pisano 2007).

[28]Maxwell (1873, II Pt IV, IV, 181, line 36).

[29]Maxwell (1873, II Pt IV, IV, 182, line 30).

2.1 On Physics Mathematics in Electromagnetic Theory

In this section, we will mainly be dealing with *On the equation of motion of a connected system* (Maxwell 1873, II, Pt IV, V). Maxwell's effort passed through Newtonian science to reach a new physics mathematics based on the concept of energy instead of that of force.[30] However, a question may be: *what kind of force concept, energy are we dealing with?* In analytical mechanics, *equations of motion* are equations that describe the behaviour of a system in terms of its motion as a function of space and time (e.g., the motion of a particle under the influence of a force). Sometimes the term refers to the differential equations that the system satisfies (e.g., Newton's second law or Euler–Lagrange equations[31]), and sometimes to the solutions to those equations. With regard to Maxwell, a question may be: *what kind of mathematics did he prefer to use for his physics mathematics of the electromagnetic theory?* Maxwell needed an adequate mathematics vastly different from that used in the Newtonian paradigm. The alternative mechanical science (until Laplace's[32] mechanics and just before Lazare Carnot's mechanics) was Lagrange's analytical mechanics (Capecchi and Drago 2005; Panza 2003b; Blay).

Maxwell used Lagrange's mechanics as a new approach to physical theory, thinking that it might be the closest to Faraday's physical problems expressed in a non-mathematical way. Thus, Maxwell reasoned in broader terms about the connected mechanical system. Since Lagrange's equations of motion are expressed in terms of rates of change, a kind of differential equation is obtained. Nevertheless, Maxwell first accepted Lagrange's method and then his equations to describe electromagnetic problems (Maxwell 1873, II, Pt IV, V, 193–194). His preliminary ideas on the role played by physical magnitudes in this Lagrangian approach are evident in some parts of *Treatise on Electricity and Magnetism* (Maxwell 1873, II, Pt IV, V, 184–186, 191).

In all of chapter V (Maxwell 1873, II, Pt IV, V, 184–194) differing from Newtonian mechanics, Maxwell expressed a motion and energy relations within the system as a whole, rather than in terms of laws of motion governing the actions of forces. Here, after proposing several methods, he announced a third method related to Lagrange's[33] equations:

> 563.] There is a third method of expressing the kinetic energy, which is generally, indeed, regarded as a fundamental one. By solving the equations (3) [$\dot{q} = \frac{dT_p}{dp}$. (*Ivi,* 189)] we may express the momenta in terms of the velocities, and then, introducing these values in (13)

[30]Thus, mass is another Newtonian concept Maxwell does not use.

[31]Interesting new approaches to the use of mathematics for physical variables have recently been established within complexity theory; they are also very interesting from an historical/philosophical standpoint (Longo).

[32]Gillispie (1997).

[33]It is interesting to remark that in chapter V (Maxwell 1873, II) Maxwell used Lagrange's equations (Lagrange) to formulate his own general equation for electromagnetic theory, but only in chapter VI would it describe a particular physical system.

> $[T_{p\dot{q}} = \frac{1}{2}(p_1\dot{q}_1 + p_2\dot{q}_2 + \&c.)$. (*Ivi,* 191)], we shall have an expression of T involving only the velocities and the variables. When T is expressed in this form we shall indicate it by the symbol $T_{\dot{q}}$. This is the form in which the kinetic energy is expressed in the equations of Lagrange.[34]

With regard to equations of motion in his electromagnetic theory, I note that in the Lagrange system,[35] the field is a *continuum*; it remains whole and in substance undivided. Since Maxwell aimed at formulating a dynamical justification to field equations, in this part, he focused on the fact that the magnetic field appeared as a completely kinetic system. Thus, he assumed that the energy of the system should be totally kinetic (Maxwell 1873, 189–192). Particularly,

> In the case of a system with several variables an expression such as *dT/dqi,* whose purpose is in effect to test the dependence of T on the variable *qi* exclusively, does not denote a simple derivative. Since it is the variation of *qi alone* that is wanted, the configuration of the rest of the system must be held unchanged while *qi* is varied. This evidently does not represent an actual, or even possible, motion of the system but rather, in effect, a sort of thought-experiment. [In effect] The derivative taken in this purely conceptual manner is today termed the *partial* derivative of T with respect to *qi*.[36]

Regarding the use of the partial differential in his physics mathematics, Maxwell mainly referred to partial differentiation of the energy function, which, in his calculations, represented his mathematical–dynamical idea to solve physical operations related to his *dynamical relations of thought* between physics and mathematics and his aim to propose a dynamical justification for field equations. Maxwell formulated his mathematical field concept through several phases, e.g.: (a) a geometrical study of Faraday's hydrodynamic analogical[37] model, based on lines of systems of forces – imagined by Faraday – and "[. . .] the collection of imaginary properties [. . .]" (Maxwell 1856, 160, line 4) of the *theory of motion of an incompressible fluid* (*Ivi*), (b) a concrete mechanical model (Maxwell 1861–1862, part I), based on the production of magnetic forces from electric current, called molecular vortexes (Kragh), (c) a dynamical justification for field equations (Maxwell 1865, 1873).

2.2 On the Field Concept and Mathematical Operators

The field concept is a part of Maxwell's theory, which constitutes a defining moment in *A Treatise on Electricity and Magnetism*. In order to include Faraday's insight,

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[34]Maxwell (1873, II, Pt IV, V, 191, line 12).

[35]Of course, a strict relationship between Lagrange's equations and the second law becomes evident for a single moving body.

[36]Simpson (2005, 65, line 10). Author's italics.

[37]By means of an analogical model he could establish mathematical relationships between some physical quantities defined (yet not very well, e.g., intensity of electricity) by Faraday and hydrodynamic quantities (e.g., force).

Maxwell projected and realized one of the basic structures of his new physics mathematics: Faraday's early physical concept of the field:

> Faraday had no mathematical or mechanical preconceptions, and his theory mostly reflected patient experimental explorations.[38]

> Faraday and Thomson invented field theory: they introduced theoretical entities in space between electric and magnetic sources, and they elaborated powerful techniques for investigating the properties of these entities.[39]

> Proceeding from Faraday's and Thomson's writings, Maxwell reached the essentials of his electromagnetic field theory stepwise, in three great memoirs. In *On Faraday's line of force* his aim was to obtain a mathematical expression of Faraday's field conception.[40]

In Maxwell's opinion, the field is part of a new theory, physics mathematics within the domain of a physical theory.

In 1831, Michael Faraday demonstrated (without using mathematics) the reciprocal effect, in which a moving magnet in the vicinity of a coil of wire produced an electric current. Ørsted's experiment on a magnetic needle led Faraday, around the 1830s, to conceive the law of electromagnetic induction (Faraday 1839–1855; see also Williams) and a notion of the magnetic field.[41] It should be noted that Faraday, even though he previously used the terms *magnetic curves*, and *lines of magnetic forces* (e.g., Faraday 1839–1855, I, §114, 32; see footnote), officially used the word "field" in his *Diary* for the first time on November 7, 1845, (Nersessian 1989, note 7, 6). Other crucial definitions were proposed in 1845 (Faraday 1839–1855, III, §2149, 2) and late 1852. The later definition is the most important and decisive field concept (Faraday 1839–1855, III, §3071, 328). Faraday introduced the *magnetic field* concept in 1845 but published it in 1846 (Faraday 1839–1855, III, §2247, 29; see also: *Ivi*, §2252). On other occasions, he used the term *magnetic field* (e.g.: Faraday 1839–1855, III, §2463–§2475, §2806–§2810, §2831, §3171). Faraday presented a very clear definition of the *magnetic field* in 1850 at *The Royal Society of London,* which was published *posthumously* in 1851 (Faraday 1839–1855, III, §2806, 203). It is a crucial concept that Maxwell consequently studied from a physics mathematics point of view[42] (mainly: Maxwell 1865, 460, 1873, I, §44, 44; II, §476, 128). This idea would be followed without a priority of physical measurements, that is, the idea was to remain within the physics mathematics domain.

[38] Darrigol (2000, 136, line 4).

[39] Darrigol (2000, 134, line 10).

[40] Darrigol (2000, 172, line 29). Author's quotations marks.

[41] For the sake of brevity, here I avoid fully discussing the *history of field concept*. On details concerning the history of *The field conceptsof Faraday and Maxwell,* see a recent work by Assis, Ribeiro and Vannucci (Assis, Ribeiro and Vannucci; see also Nersessian).

[42] Later, its theory essentially became a field theory making use of Lagrange's equations of motion. I refer the reader to secondary literature (Darrigol 2000, 2005; Siegel 1981).

By following differentiation of the energy function and by using Lagrange's equations of motion, Maxwell was able to formulate his momentum concept (Maxwell 1873, II, Pt IV, V, 190–191), one of the basic concepts for building his *Dynamical theory of electromagnetism,* which he presented in chapter VI (Maxwell 1873, II, Pt IV, VI, 195–205). In order to proceed toward his dynamical project, he needed to establish the mathematical conditions to write a form of the equations capable of describing a particular[43] physical system within electromagnetic theory. In his words:

> 567.] In this outline of the fundamental principles of the principles of the dynamics of a connected system, we have kept out of view the mechanics by which the parts of the system are connected. We have not even written down a set of equations to indicate how the motion of any part of the system depends on the variation of the variable [. . .].[44]
>
> Our only assumptions are, that the connexions of the system are such that time is not explicitly contained in the equations of conditions, and that the principle of the conservation of energy is applicable to the system. Such a description of the methods of pure dynamics is not unnecessary, because Lagrange and most of his followers, to whom we are in debt for these methods [. . .].[45]
>
> As the development of the ideas and methods of pure mathematics has rendered it possible, by forming a mathematical theory of dynamics, to bring to light many truths which could not have been discovered without mathematical training, so, if we are to form dynamical theories of other sciences, we must have our minds imbued with these dynamical truths as well as with mathematical methods.[46]

In chapter VI (Maxwell 1873, II, Pt IV, VI) Maxwell focused on this point, seeking adequate coefficients to convert previous general equations (Maxwell 1873, II, Pt IV, VI, 198–199). From a strictly mathematical point of view, he pointed out three categories among coordinates q_i (Maxwell 1873, II, Pt IV, VI, 197–198) obtaining the total energy T divided into three parts, generating terms of three forms: $x_i x_j, y_i y_j$, and $x_i y_j$. Non-homogeneous coordinates are combined in terms of the form $x_i y_j$. From a physical point of view, they are mechanical (x) and electrical (y) variables. These products give rise to three corresponding sources of kinetic energy when they appear in the equation for T.

> 571.] [. . .]. We may therefore divide T into three portions, in the first of which T_m, the velocities of the coordinates x only occour, while in the second, T_e, the velocities of the coordinates y only occour, and in the third, T_{me}, each term contains the product of the velocities of two coordinates of which one is x and the other y.

[43]Previously, in chapter V, Maxwell pointed out (from Lagrange) a general form of the equations of motion which are not able to describe a particular physical system.

[44]Maxwell (1873, II, Pt IV, V, 193, line 34).

[45]Maxwell 1873, II, Pt IV, V, 194, line 5).

[46]Maxwell 1873, II, Pt IV, V, 194, line 22).

We have therefore $T = T_m + T_e + T_{me}$

$$T_m = \frac{1}{2}(x_1x_1)\dot{x}_1^2 + \&c. + (x_1x_2)\dot{x}_1\dot{x}_2 + \&c.,$$

Where $$T_e = \frac{1}{2}(y_1y_1)\dot{y}_1^2 + \&c. + (y_1y_2)\dot{y}_1\dot{y}_2 + \&c.,$$

$$T_{me} = \frac{1}{2}(x_1y_1)\dot{x}_1\dot{y}_1 + \&c.^{47}$$

These physics mathematics arguments dealt with three methods "[. . .] of detecting the existence of the terms of the form T_{me}, none of which have hitherto led to any positive results" (Maxwell 1873, II, Pt IV, VI, 205, line 27). Thus, the equations from chapters VI were made to bear on a simple electromagnetic system consisting of only two circuits in *Theory of electric circuits* (Maxwell 1873, II, Pt IV, VII). His reasoning would be extended to more complex systems, which involve various conductors and a variety of mechanical motions and corresponding equations, interpreted in electrical terms in accordance with Faraday's experimental discoveries in electromagnetic induction. Finally, electromagnetic phenomena are very well connected with equations of motion and their forms generated the foundation for Maxwell's theory of the electromagnetic field.

2.3 On Magnetic Vector Potential and Electric Displacement

As previously mentioned, when establishing the *electronic state* as a state of momentum, Maxwell was guided by Faraday's studies. Particularly, in Faraday's scientific panorama, Lagrange's method does not entail electromagnetic theory. Therefore, in *Exploring of the field by means of the secondary circuit* (Maxwell 1873, II, Pt IV, VIII, 211–226), Maxwell returned to Faraday's experimental methods of the moving wire to explore the magnetic field in its new form. A consequence of this choice, of course, was not making the relationship with field's momentum a priority. His return to Faraday's physical style of thinking certainly concerned his main scientific ideas and methods, which he should have converted into physics mathematics reasoning. In fact to accomplish this, Maxwell introduced a vector, which in some way referred to a quantity of momentum at every point in space. The vector is the very intricate *magnetic vector potential* (Maxwell 1873, II, II, §405, 27; Bork) which assumed an important role in Maxwell's reasoning on his field concept. His reasoning is significant for physics mathematics and for his related translation of Faraday's thought. In chapter[48] VIII (Maxwell 1873, II, VIII) Maxwell argued on the very mathematical structure of the electromagnetic

[47]Maxwell (1873, II, Pt IV, V, 197–198, line 36).

[48]And in (II and) IX (*Ivi*).

field, often using Faraday's studies and experimental data to accomplish this goal. In order to do succeed in his endeavour, Maxwell needs

> 594.] [...] to deduce from dynamical principles the expressions for the electromagnetic force acting on the conductor carrying an electric current through the magnetic field, and for the electromotive force acting on the electricity within a body moving in the magnetic field.[49]

For the sake of brevity, Faraday's physics and Maxwell's physics mathematics dialectic (Maxwell 1873, II, Pt IV, VIII, 215–218) regarding the field concept may be listed as the following: (a) initially no current (b) by means of the field a current inducted is (measured-) calculated, (c) a constant current passes, (d) a mechanical force is then calculated. We can observe that (a) calculating the electromotive force acts on the generalized electrical variable and (b) this determines the electromagnetic (mechanical) force (Maxwell 1873, II, Pt IV, VIII, 218–221) on the wire. This is Maxwell's *affectionate* tentative to develop a "[...] mathematical method which we shall adopt [that] may be compared with the experimental method used by Faraday"[50] (Maxwell 1873, II, Pt IV, VIII, 217, line 5). At the end of this reasoning (*Ivi*), a different and evident scientific approach emerges with respect to early reasoning proposed by Maxwell in the first chapters (*Ivi*), in which, e.g., he used Gauss's theory of scalar potentials to discuss general elements of the magnetic field. In chapter VIII (*Ivi*), by using Faraday's wire, Maxwell once again selected Lagrangian mechanics for his momentum. (Simpson 2005, 108–109). Maxwell's physics mathematics style thinking emerges for both methods and concepts related to the old scalar theory of magnetism (Maxwell 1873, II, Pt IV, VIII, 224–225).

Maxwell provides a dynamical explanation of the field theory (Maxwell 1873, II, Pt IV, IX, 227) in *General equations of the electromagnetic field* (*Ivi*) where the terms *General equations* in the title of the chapter would be (possibly) a *definitive form of physics mathematics equations*. He listed them from "A" to "L" (Maxwell 1873, II, Pt IV, IX, 227–233) which, for the sake of brevity and since there are various works related to this matter, I do not reproduce them here. In writing these equations, Maxwell established the same differential cell established in the previous chapter, however, they are now connected by an elastic medium and potential energy.

A physics mathematics interaction between the magnetic and electric domain is once again presented, but now this interaction is thanks to a new theoretical element which Maxwell called *Electric displacement* (Maxwell 1873, II, Pt IV, IX, §608, 232). It is a quantity that is defined in terms of the rate of change of electric displacement as a field. *Electric displacement* has the units of electric current density, and it has an associated magnetic field just as actual currents do. However, it is not an electric current of moving charges, but a time-varying

[49] Maxwell (1873, II, Pt IV, VIII, 217, line 1).

[50] I suggest examining "Fig. 38" (Maxwell 1873, II, Pt IV, VIII, 217) presented by Maxwell during his reasoning.

electric field. It is the "Equation of True Currents (H)" (Maxwell 1873, II, Pt IV, IX, §610, [232–]233), where the *true electric current* (*Ivi*, 232), *density of the current of conduction* with *time-variation of electric displacement* and therefore total movement of electricity are taken into account. Maxwell also wrote that equation in terms of its components where the relations between physical quantities and mathematical ones are more evident. The dynamical relation of thought between mathematical operators and physical quantities concerning the *electric displacement* was first conceived by Maxwell in his *On Physical Lines of Force*[51] in connection with the displacement of electric particles in a dielectric medium where Maxwell added the *displacement current* to the electric current term in *Ampère's Circuital Law* (Maxwell [1890] 2003, I, pp. 471–474). Another account was discussed in *A Dynamical Theory of the Electromagnetic Field* (Maxwell 1865). In this case, he used this amended version of *Ampère's Circuital Law* to obtain[52] the electromagnetic wave equation. The displacement current term is now seen as a crucial addition that completed Maxwell's equations and is necessary for explaining many phenomena, in particular the existence of electromagnetic waves. Maxwell, as announced in his previous works, has now, by means of the *electromagnetic field*, completed his research program and life project by establishing a new style of thinking and conceiving a new science through the interaction of electric and magnetic phenomena.

2.4 A Road to Physics Mathematics?

In general, a unit of measurement is effectively a standardised quantity of a physical (and chemical) property, used as a factor to express occurring quantities of that property. Therefore, any value of a physical quantity is expressed as a comparison to a unit of that quantity. In the physics mathematics domain one generally precedes by means of calculations, therefore the units of measurement are not a priority in terms of a solution to an analytical problem (Lindsay, Margenau and Margenau). In this sense, the physical (and chemical) nature of the quantities is not a priority.[53]

[51]Maxwell firstly wrote *On Faraday's Lines of Force* (Maxwell [1890] 2003, I, 155–229) which was completed in1856. In 1861(−2) he wrote another ambitious paper, *On Physical Lines of Force* (Maxwell [1890] 2003, I, 451–513) where he elaborated his theory on mechanical vortex (Thomson 1883; see also *Id.,*1881, 1885, 1891).

[52]Generally, this derivation now seems commonly reasonable in physics mathematics, however, it combines the general idea of uniting electricity, magnetism and optics into one single unified theory. I do not comment on this derivation.

[53]For instance, one can see an analogous situation concerning heat and temperature concepts in the analytical theory of heat (Fourier 1807, 1822, Lamé 1836, 1861; see also Pisano and Capecchi) with respect to Sadi Carnot's thermodynamic theory (Pisano 2010, 2011a, b; Gillispie and Pisano 2012). I briefly note that physics considers the indispensable agreement between theoretical data

One may discuss the role played by a certain science in history (e.g., physics), focusing solely on the historical period and the kind of mathematics adopted. For my aim, the most important aspect is the role played by the relationship between physics and mathematics adopted in a scientific theory in order to describe mathematical laws – e.g., the second Newtonian mathematical law of motion[54] (Panza 2002). Time is a crucial physical magnitude in mechanics, but in the aforementioned law, it (with space) is also a mathematical magnitude since it is involved in derivative operations. Most importantly, if we lose their mathematical sense, we would lose the entire mechanical paradigm. Nevertheless, the approaches to conceive and define foundational *mechanical–physical quantities* and their *mathematical quantities* and interpretations change both within a physics mathematics domain and a physical one. One could think of mathematical solutions to Lagrange's energy equations, rather than the crucial role played by collisions and geometric motion in Lazare Carnot's algebraic mechanics or Faraday's experimental science with respect to Ampère's mechanical approach in the electric current domain and finally the physic mathematics choices in Maxwell's electromagnetic theory.

Physical science makes use of experimental[55] apparatuses to observe and measure physical magnitudes. During and after an experiment, this apparatus may be illustrated and/or designed. Generally, this procedure is not employed in pure mathematical studies. Thus, one can claim that experiments and their illustrations can be strictly characterized by physical principles and magnitudes to be measured. A modelling of results of the experimental apparatus allows for the broadening of the hypotheses and the establishment of certain theses. If one avoids study-modelling experimental results, one may generate an analytical scientific theory since there is no interest in the nature of physical magnitudes and their measurements. For example, a *quasistatic process* is a thermodynamic process that happens infinitely slowly. However, it is very important to note that *no* current–real process is quasistatic. Therefore, such processes can only be approximated by performing them infinitesimally slowly. *But what does it mean from an empirical physical standpoint?* A process cannot be static (equilibrium situation?) and process (non-equilibrium situation or dynamical one?) at the same time ... and vice-versa. However this reasoning assumes a certain quantification and *scientificity* if one considers the dynamical-thought standpoint in the use of an infinitesimal concept in mathematics. In this case an infinitesimal point may express a (dynamical-thought) idea of quantities (e.g., in a differential equation) so small that there is no way

and observations/experimental data (including the properties of magnitudes) to establish a physical theory. Generally, such arguments are not considered rigorous by physics mathematics.

[54]In this reasoning I consider the Newtonian science and its development in the history.

[55]From a physical standpoint and for the aim of this paper, I remark that the *Physicist and Natural Philosopher* (Everitt) was the founder (1874) of *Cavendish Laboratory,* thanks to William Cavendish – 7th Duke of Devonshire who was also Chancellor of Cambridge University and donated money for the construction of the physics laboratory. Maxwell became the first *Cavendish Professor of Physics* (1871–1879) for a tenure in *experimental physics* (Gmellin).

to see them or to measure them or, in common speech, an infinitesimal point is a quantity which is smaller than any feasible measurement, but not zero in size; and at the same time, so small that it cannot be distinguished from zero by any available means.[56] *Again, what does it mean in an empirical physics?* And, *if we do not use strict empirical procedures and instruments*, *what kind of physics are we talking about?* For example, *is distance s and time t the same magnitudes (numerator and denominator) in* $v = ds/dt$*?* And, *if we delete the friction from theory of heat (or thermodynamics) should we restore and come back to the mechanics of the seventeenth century?* And above all, *what about perpetual motion and the law of inertia?*

This mentioned mathematical formalism radically changed Faraday's basic concepts and explained them in abstract concepts in a different order: first the local ones around a point and then the global ones.[57] In these types of theories there are various mathematical aspects so the theory appeared entirely mathematical, e.g. the analytic theories. In fact, the content was a sophisticated mathematics (differential calculus by partial derivatives, integral calculus, series, etc.) in order to interpret each field of phenomena. Therefore, a physical theory in origin was absorbed by mathematics, producing two crucial consequences: the birth of a new analytical theory and, at the same time, the scientific-cultural demise of the previous classical physical theory of heat. This analytical approach is one of the reasons why in physics mathematics (in terms of a discipline), e.g., a result of an integration (e.g., related with an energy concept) rather than a differential equation (e.g., related with a dynamical force concept) can be correctly accepted without discussing physical–chemical properties, physical significance, and units of measurements of the interested quantities. Instead, it refers to the use of advanced mathematical methods to solve mathematical aspects of problems in physics (i.e., one can see McAulay (McAulay)). This should be accomplished by studying and solving problems inspired by physics within a mathematically rigorous framework.

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[56]Generally speaking in constructive mathematics a point is a range (not an infinitesimal point) typical of empirical measurements in physics. Of course this mathematical approach did not properly obtain the entire *power of calculus* such as is possible to have by means of infinitesimal analyses.

[57]There is not enough space in this paper, but it would be interesting to discuss the alternative theories that came about before Maxwell's physics mathematics and those that continued after. For example, Weber and Riemann (Pisano and Casolaro) based their theory on those entities that charges and magnetic dipoles interact with and act on, that is to say, the material supports of electromagnetism. They succeeded in repeating a great part of the theory but not all of it; for example, the displacement current, since it is without material supports, remains mostly outside. Other highly developed theories of electromagnetism were that of Helmholtz and that of Poincaré (1890, 1897, 1900). Only the birth of relativity put an end to their rivalry because only Maxwell was in agreement with this new innovation.

3 Conclusion

Among physicists, mathematicians, historians and philosophers[58] who are credited width the study of *mathematical physical quantities*[59] by means of experiments, modelling, properties, existences, structures etc. one can strictly focus on how physics and mathematics work in a unique discipline *physics mathematics* (or, if one prefers, *mathematics physics*). Of course, I do not mean a mathematical application in physics and vice-versa but rather a new (at that time) way to consider this science. Here it is a new discipline *physics mathematics* and not mathematical physics. A new methodological approach to solve physical problems (origin) where the quantities can be physical and mathematical at the same time (first novelty) and measurements are not a priority or a prerogative (second novelty) that makes, however, a coherent and valid physical science. Thus, the emergency in physics mathematics (discipline) belonged to physics, not in an advanced use of mathematics to solve physical problems, since physics has changed its *face* since, within this new physical mathematical discipline, it changed its foundations. With regards to Maxwell:

> Maxwell's challenge was to expound a new doctrine and at the same time to establish a new standard in the treatment of current problems. In order to meet these two conflicting requirements, he carefully separated the basic mathematical and empirical foundations of the subject from more speculative theory. [...] Maxwell defined the basic physical quantities in a neutral manner that could be accepted both by fluid and field theorists. For example, he introduced the quantity of electric charge of a body by means of Faraday's hollow conductors [...] with these neutral definitions, Maxwell could conduct much of mathematical analysis without deciding the nature of electricity and magnetism.[60]

Thus, I refer to a structured discipline, having its own hypothesis, methods of demonstration, an internal coherent logic etc., where the change in the kind of infinity in mathematics produces a change in both significant physical processes (Pisano 2010) and interpretations of *physical quantities*. One of the possible scientific approaches to understanding the history of the foundations of science may be to combine historical and epistemological aspects (primary sources, historical hypothesis, shared knowledge and epistemological interpretations) by means of a logical and mathematical inquiry called the *historical epistemology of science* (Pisano 2011b).

Below, I present a possible key of investigation,[61] *a mò* of conclusion related to Faraday and Maxwell's accounts:

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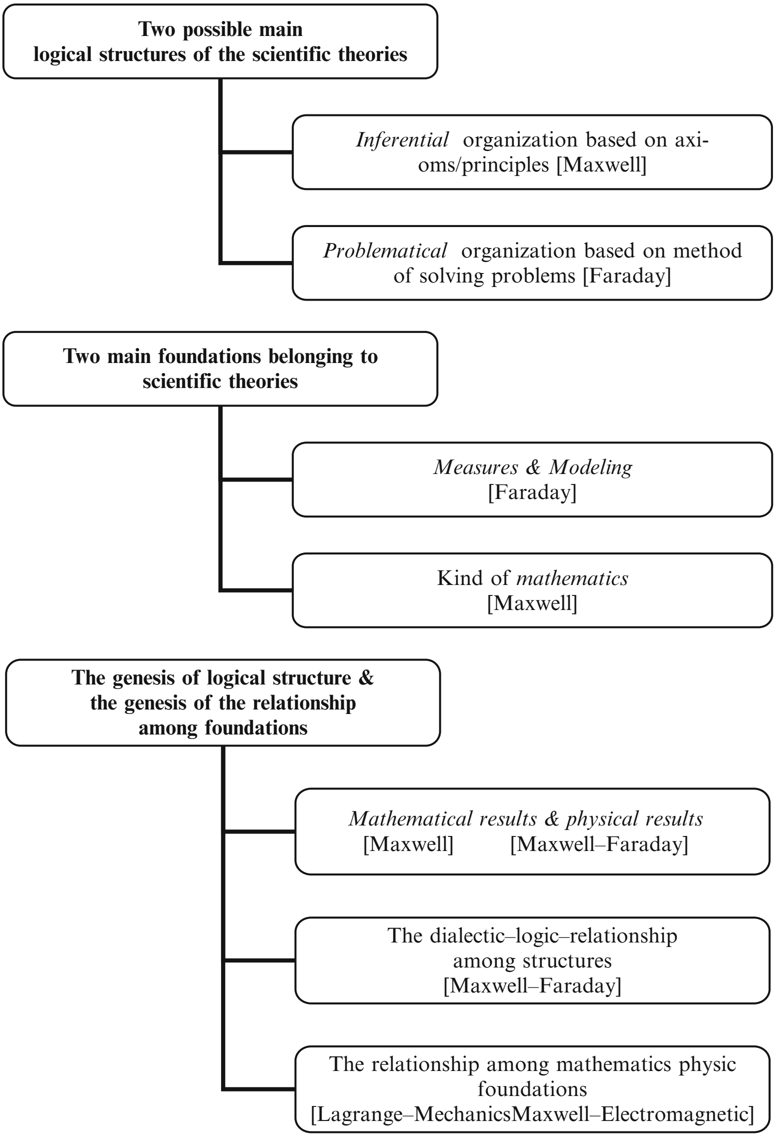
[58] Whewell conducted an interesting study on inductive processes in science and the philosophy of science during Maxwell's time (Whewell 1837, 1840).

[59] A recent study on the role played by *mathematical objects* in philosophy of mathematics is presented in (Panza 2003a).

[60] Darrigol (2000, 167, line 1).

[61] A previous interesting account on the "options in the science and history of science" was presented in (Drago 1988, 2007; Pisano and Gaudiello 2009a, b).



A new theory may essentially mean new magnitudes, new instruments to measure, new results recognized from a physical and mathematical standpoint. The new physics mathematics theory was not simply an advanced mathematical procedure applied to a previous experimental theory; it was more. It was an improvement of the same contents from a new physics standpoint, using new mathematical magnitudes and new quantities. Thus, it was necessary to establish a new theory having new magnitudes to justify both the *new nature of ideas* and to avoid hiding physical reasoning with a "[...] terrific array of symbols [...]".[62] For my aim, the novelty is the new conception of physical quantities; they are not solely physical, nor solely mathematical, but physics mathematics quantities since they, altogether, represent *relationships of thought* among *mathematical quantities* and *physical structures* (including logic and language) in order to foster "[...] reducing these [experimental electric and magnetic] phenomena into scientific form [...]" (Maxwell 1865, 459, line 6). Mathematical techniques of derivatives and integration are the instruments to link new mathematical results to the new physical *process of reasoning*.

Finally, Maxwell mainly based his work on Faraday's experimental genius regarding how science works and characteristics of scientific methods, together with Lagrange's mechanics. Maxwell's formidable approach, physically and mathematically integrated, constructed a new theory for a new discipline, physics mathematics, crucial for both science and the history (epistemology–philosophy) of science.

> I have confined myself almost entirely to the mathematical treatment of the subject, but I would recommend the student, after he has learned, experimentally if possible, what are the phenomena to be observed, to read carefully Faraday's *Experimental Research in Electricity*.[63]

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[62] It concerned the famous debate (1886) between Peter Guthrie Tait (1831–1901) and Ludwig Eduard Boltzmann (1844–1906). On this, see the editorial *On some question in the kinetic theory of gases. Reply to Prof. Boltzmann* (1888) in Philosophical Magazine, 5s, 25/154. The polemic on the advanced use of mathematics with respect to previous mechanical theories involved many other scholars; leading physicists considered the findings with scepticism.

[63] Maxwell (1873, I, *Preface*, xiii, line 14) (Author's italic).

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