# Preface

This book is based on a three-semester course of lectures delivered by the author at the Faculty of Mathematics of the Vienna University. For preliminary versions of the manuscript, see [113, 114].

The main emphasis of this book is on nonrelativistic and relativistic quantum mechanics with standard applications to the hydrogen atom. Our main intention is to present the quantum mechanics in a comprehensive manner, accessible for a mathematician. The exposition is formalized (whenever possible) on the basis of coupled Maxwell–Schrödinger and Maxwell–Dirac equations. This intention agrees with Hilbert's 6th Problem ("Axiomatize Theoretical Physics"), and Heisenberg's nonlinear programme [83, 84].

Our exposition starts with a chronological analysis of crucial empirical observations and their theoretical systematization, explaining *inter alia* the motivation behind the Heisenberg and Schrödinger equations. The introduction of quantum observables stems from the agreement with corresponding classical observables for short wavelength solutions, Hamilton–Jacobi's theorem being taken into account. Also, the relation between the quantum observables and the Noether symmetry theory is discussed. The Lagrangian formalism is used as a fundamental unifying principle to lay the basis for introduction of the coupled Maxwell–Schrödinger and Maxwell–Dirac equations.

Of course, the equations and observables could be (and formally should be) accepted as axioms. On the other hand, it is crucially important to know the experimental and mathematical facts behind quantum formalism to embed it in the whole of physics.

Moreover, the modern form of quantum theory seems to be far from its completeness, like geometry in pre-Euclidean era. It is therefore particularly important to understand the degree of confidence to individual constituents of the quantum formalism. This is why we pay so much attention to the origin and motivation of the formalism.

The hydrogen spectrum and the atom radiation are calculated with all detail. Parallels between quantum and classical description are traced everywhere to motivate the introduction of quantum differential cross section, magnetic moment, etc.

The scattering problems are solved by application of the perturbation procedure to the coupled Maxwell–Schrödinger equations. We point out some deficiency in the perturbation procedure, which should be fixed with a nonperturbative approach, however this correction is still an open problem.

The introduction of the electron spin is discussed in detail from experimental and theoretical point of view. We calculate the Landé formula for the *gyromagnetic ratio* via the spin-orbital interaction of Russell–Saunders, which explains the Einstein–de Haas experiment and the anomalous Zeeman effect. Further we prove the relativistic covariance of the Dirac equation, obtain the corresponding intrinsic spin momentum, and the corresponding nonrelativistic approximations. Finally, we calculate the hydrogen spectrum via the Dirac equation.

We make explicit invoked assumptions and approximations, and discuss a plausible treatment of some logical leaps in the theory. However, we did not try to establish new rigorous results. Generally, our exposition is not mathematically rigorous. For example, we do not distinguish between Hermitian symmetric and the selfadjoint operators, even though the spectral resolution is used repeatedly.

In appendices (Chaps. 12 and 13), we explain related details from Classical Electrodynamics and Special Relativity, Geometrical Optics, the Hamilton–Jacobi theorem, an updated version of the Noether theory of currents, and the limiting amplitude principle.

In Chap. 14, we collect classical calculations lying in the base of the 'old quantum mechanics'.

**Further Reading** Our main goal is to give a concise explanation of mathematical principles of Quantum Mechanics. More technical details and a systematic comparison with experimental data can be found in [7, 11, 12, 20, 23, 34, 63, 75, 81, 130, 131, 145, 160, 171, 179, 191]. The books [46, 79, 89, 143] and [93, 185], respectively, explain basic concepts of Quantum Mechanics and Classical Electrodynamics. A suitable introduction to the mathematical theory of the Schrödinger equation is contained in [10].

We develop the methods of quantum mechanics for the hydrogen atom which can be extended to other one-electron atoms (lithium, sodium, potassium atoms, etc), and do not touch multi-electron problems of quantum chemistry [34, 41, 178, 187]. We also do not touch the Stability of Matter [31, 134], the Quantum Electrodynamics and Quantum Field Theory [13, 14, 33, 64, 77, 85, 137, 138, 158, 159, 163, 189, 195, 196].

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