

Contents

1	Genesis of Quantum Mechanics	1
1.1	Atoms and Spectra	1
1.1.1	Concept of Atom	1
1.1.2	Early Spectra Observations	2
1.1.3	Resonance Nature of Spectra	2
1.1.4	Combination Principle	3
1.2	Electrodynamics	3
1.2.1	Ampère Theory of Magnetization	4
1.2.2	Maxwell Electrodynamics	4
1.2.3	Cathode Rays: Thomson's Discovery of Electron	5
1.2.4	Elementary Electric Charge	6
1.2.5	The Zeeman Effect	7
1.2.6	Lorentz's Theory of Electrons	7
1.2.7	Abraham: Mass–Energy Identification	8
1.3	Thermodynamics	9
1.3.1	Equilibrium Radiation	10
1.3.2	Experiments and Theory	11
1.3.3	The Rayleigh–Jeans Law: Ultraviolet Divergence	13
1.3.4	Planck's Law: Quantization of Energy	15
1.3.5	Photoelectric Effect: Einstein's Rules	17
1.3.6	Einstein and Debye: Specific Heat of Solids	17
1.4	'Old Quantum Mechanics'	18
1.4.1	Planetary Model and Inconsistency of Classical Physics	18
1.4.2	Bohr's Postulates	19
1.4.3	Debye Quantum Rules	19
1.4.4	Boltzmann Distribution in Old Quantum Mechanics	20
1.4.5	Einstein Theory of Radiation: Counting Photons	21
1.4.6	Bohr's Correspondence Principle	23

2	Heisenberg's Matrix Mechanics	25
2.1	Heisenberg's Matrix Formalism	25
2.1.1	Classical Oscillator	25
2.1.2	Quantum Oscillator	27
2.2	Early Applications of Heisenberg Theory	30
2.2.1	Eigenvalue Problem	30
2.2.2	Intensity of Spectral Lines	32
2.2.3	The Normal Zeeman Effect	32
2.2.4	Quantization of Maxwell Field and Planck's Law	33
3	Schrödinger's Wave Mechanics	35
3.1	Wave-Particle Duality: de Broglie and Schrödinger	35
3.1.1	De Broglie Waves of Matter	35
3.1.2	De Broglie Wavelength and Dispersion Relations	37
3.1.3	Canonical Quantization I: Free Particles	39
3.1.4	Canonical Quantization II: Bound Particles	39
3.1.5	Quantum Stationary States: Eigenvalue Problem	41
3.1.6	Stationary Perturbation Theory	42
3.2	Quasiclassical Asymptotics	43
3.2.1	Geometrical Optics	43
3.2.2	Application to Schrödinger Equation	44
3.2.3	Generalizations	45
3.2.4	Hamilton Optical-Mechanical Analogy	45
3.2.5	Conclusions	46
3.3	Quantum Observables and Conservation Laws	46
3.3.1	Quantum Observables	47
3.3.2	Conservation Laws	47
3.3.3	Correspondence Principle	49
3.4	Charge Continuity Equation	50
3.5	Equivalence of Heisenberg's and Schrödinger's Theories	52
3.5.1	Heisenberg Observables	52
3.5.2	Heisenberg's Picture	53
3.5.3	Heisenberg's Equation	54
3.5.4	Correspondence Between Heisenberg's and Schrödinger's Pictures	55
4	Lagrangian Formalism	57
4.1	Hamiltonian and Lagrangian Formalism	57
4.1.1	Hamiltonian Formalism	57
4.1.2	Lagrangian Formalism	58
4.1.3	Variational Principle and Euler-Lagrange Equations	58
4.2	Maxwell-Schrödinger Equations	60
4.2.1	Lagrangian Density	60
4.2.2	Gauge Invariance and Charge Continuity	62
4.2.3	Gauge Transformation	62
4.2.4	Perturbation Theory	63
4.3	Klein-Gordon Equation	64

5	Wave-Particle Duality	67
5.1	Electron Beam and Uncertainty Principle	67
5.1.1	Plane Wave as Electron Beam	67
5.1.2	The Heisenberg Uncertainty Principle	68
5.2	Diffraction of Electron Beams	69
5.2.1	Experimental Observations	69
5.2.2	The Davisson–Germer Experiment	70
5.2.3	The Double-Slit Experiment	71
5.2.4	The Aharonov–Bohm Effect	71
5.2.5	Diffraction of Electrons via Schrödinger Theory	72
5.3	Probabilistic Interpretation	74
5.3.1	Reduction of Wave Packets and Probabilistic Interpretation	74
5.3.2	Soliton-Type Asymptotics	74
5.3.3	Solitons and Reduction of Wave Packets	75
5.3.4	The Aharonov–Bohm Paradox	76
5.3.5	The Known Results on Soliton Asymptotics	76
5.3.6	Particle-Like Behavior of Solitons	77
6	The Eigenvalue Problem	79
6.1	The Hydrogen Spectrum	79
6.1.1	The Eigenvalue Problem	79
6.1.2	Spherical Symmetry and Separation of Variables	80
6.1.3	Spherical Coordinates	82
6.1.4	The Radial Equation	83
6.1.5	Eigenfunctions and Quantum Numbers	85
6.2	The Spherical Spectral Problem	86
6.2.1	Hilbert–Schmidt Argument	86
6.2.2	Lie Algebra of Angular Momenta	87
6.2.3	Irreducible Representations	87
6.2.4	Spherical Harmonics	89
6.2.5	Angular Momenta in Spherical Coordinates	91
7	Atom Radiation	93
7.1	Atom in Thermodynamical Equilibrium	93
7.1.1	Relaxation to Equilibrium Distribution	93
7.1.2	Perturbation Theory	94
7.1.3	The Dirac ‘Interaction Picture’	95
7.1.4	Thermodynamical Equilibrium in Schrödinger Theory	96
7.2	Atom Radiation	97
7.2.1	Radiated Maxwell Field	97
7.2.2	The Rydberg–Ritz Combination Principle	98
7.2.3	The Dipole Approximation	99
7.2.4	Energy Flow at Infinity	101
7.2.5	Intensity of Spectral Lines	101
7.2.6	The Correspondence Principle	103

7.2.7	Selection Rules	105
7.3	Black Body Radiation	106
7.3.1	Correlations of the Radiation Field	106
7.3.2	The Dipole Approximation	107
7.3.3	Equilibrium Correlations and Spectral Density	108
7.3.4	Nonstationary Perturbation Theory	109
7.3.5	Correlations in First Approximation	109
7.3.6	The Dirac–Einstein Theory of Transitions	111
8	Scattering of Light and Particles	117
8.1	The Classical Scattering of Light	117
8.1.1	The Incident Plane Wave	117
8.1.2	The Scattering Problem	118
8.1.3	Neglecting Self-Action	119
8.1.4	The Dipole Approximation: The Thomson Formula	120
8.2	Quantum Scattering of Light	122
8.2.1	The Scattering Problem	122
8.2.2	The Atomic Form Factor	123
8.2.3	The Energy Flux	125
8.3	Polarization and Dispersion	126
8.3.1	The First Order Approximation	126
8.3.2	The Modified Ground State	128
8.3.3	The Kramers–Kronig Formula	129
8.4	Photoelectric Effect	131
8.4.1	Radiation in Continuous Spectrum	133
8.4.2	The Limiting Amplitude	134
8.4.3	Angular Distribution: The Wentzel Formula	135
8.4.4	Derivation of Einstein’s Rules	135
8.4.5	Further Improvements	137
8.4.6	Atomic Ionization and Photoelectric Effect	137
8.5	The Classical Scattering of Charged Particles	138
8.5.1	The Rutherford Scattering	138
8.5.2	The Angle of Scattering	139
8.5.3	Differential Cross Section: Rutherford Formula	140
8.6	The Quantum Scattering of Electrons	141
8.6.1	The Radiated Wave	141
8.6.2	The Differential Cross Section	143
9	Atom in Magnetic Field	145
9.1	The Normal Zeeman Effect	145
9.1.1	The Hydrogen Spectrum in a Magnetic Field	146
9.1.2	The Normal Splitting of Spectral Lines	146
9.2	Diamagnetism and Paramagnetism	148
9.2.1	The Magnetic Moment	148
9.2.2	The Langevin Formula	149
9.2.3	Paramagnetism	150

10	Electron Spin and Pauli Equation	151
10.1	Concept of Electron Spin	151
10.1.1	The Anomalous Zeeman Effect	151
10.1.2	The Einstein–de Haas Experiment	152
10.1.3	The Stern–Gerlach Experiment	153
10.1.4	Borh’s Theory of Periodic Table	153
10.1.5	The Spin Conjecture	154
10.1.6	The Sommerfeld Theory of Metals	154
10.2	The Pauli Equation	155
10.2.1	The Additional Magnetic Moment	155
10.2.2	The Spin Momentum	155
10.2.3	Uniform Magnetic Field	156
10.2.4	General Maxwell Field	157
10.2.5	The Stern–Gerlach Double Splitting	158
10.3	The Anomalous Zeeman Effect	159
10.3.1	The Spin-Orbital Coupling	159
10.3.2	Quantum Numbers	160
10.3.3	The Landé Formula	161
10.3.4	Applications of the Landé Formula	163
11	Relativistic Quantum Mechanics	165
11.1	The Free Dirac Equation	165
11.2	Pauli’s Theorem	168
11.3	Lorentz Covariance	170
11.4	The Angular Momentum	171
11.5	Negative Energies	173
11.6	The Maxwell–Dirac Equations	174
11.6.1	The ‘Minimal Coupling’ to External Maxwell Field	174
11.6.2	Gauge Transformation	175
11.6.3	The Hamiltonian and Lagrangian Formalism	175
11.6.4	Charge and Current	176
11.6.5	The Coupled Equations	177
11.6.6	Gauge Transformation for Coupled Equations	178
11.6.7	Charged Antiparticles	178
11.7	Nonrelativistic Approximations	179
11.7.1	The Order $1/c$	181
11.7.2	The Order $1/c^2$	182
11.8	Spinor Spherical Harmonics	184
11.8.1	Spherical Symmetry	184
11.8.2	Separation of Variables	185
11.8.3	Clebsch–Gordan’s Theorem	186
11.9	The Hydrogen Spectrum	189
11.9.1	The Radial Equation	189
11.9.2	Spectrum	191

12	Electrodynamics and Special Relativity	195
12.1	The Maxwell Equations and Potentials	195
12.1.1	Synthesis of Maxwell Equations	196
12.1.2	Maxwell Potentials	198
12.2	Maxwell's Theory of Light, and Einstein's Postulate	199
12.2.1	Maxwell's Theory of Light	199
12.2.2	'Luminiferous Ether': The Michelson and Morley Experiment	200
12.2.3	Einstein's Postulate	200
12.3	The Lorentz Group	202
12.3.1	The Lorentz Transformations	202
12.3.2	Properties of the Lorentz Transformations	203
12.3.3	Examples	204
12.4	The Lagrangian Formalism for Maxwell Field	205
12.4.1	The Maxwell Equations in Gaussian Units	205
12.4.2	4D Vector Potential	206
12.4.3	Tensor Field	206
12.4.4	The Lagrangian Density and Current	207
12.5	Covariant Electrodynamics	208
12.5.1	Transformation of the Convection 4-Current	208
12.5.2	Transformation of Maxwell Field	209
12.6	The Lorentz Force	210
12.6.1	The Lorentz Equation	210
12.6.2	The Lagrangian Formalism	211
12.6.3	The Hamiltonian Formalism	212
12.7	Energy of the Maxwell Field: Poynting's Theorem	213
12.8	Momentum of the Maxwell Field	215
12.9	Abraham: Electromagnetic Mass	216
12.10	The Hertzian Dipole Radiation	218
12.11	The Initial Problem for the Maxwell Equations	220
12.12	The Long Time Asymptotics: Retarded Potentials	224
12.13	The Lorentz Theory of Polarization and Magnetization	226
12.13.1	Stationary Molecular Fields in Dipole Approximation	227
12.13.2	Multipole Expansions of Non-Stationary Fields	228
12.13.3	Molecule in a Stationary State: Magnetic Moment	229
12.13.4	The Maxwell Equations in Matter	230
12.13.5	The Refraction Coefficient	232
13	Mathematical Appendices	233
13.1	The Lagrangian and Hamiltonian Mechanics	233
13.1.1	The Lagrangian Mechanics	233
13.1.2	The Legendre Transformation and Hamiltonian Mechanics	234
13.1.3	The Hamilton–Jacobi Equation	235
13.2	Geometrical Optics	238

13.3	The Noether Symmetry Theory	241
13.3.1	Field Symmetry	241
13.3.2	The Noether Current and the Continuity Equation	243
13.4	Application of Noether's Theorem	246
13.4.1	The Energy, Momentum, and Angular Momentum Conservation	246
13.4.2	The Energy-Momentum Tensor	248
13.4.3	Phase Invariance and the Charge Continuity Equation	249
13.4.4	Gauge Invariance	250
13.5	The Limiting Amplitude Principle	251
13.5.1	Harmonic Source and Spectrum	252
13.5.2	The Limiting Absorption Principle	253
14	Exercises	257
14.1	The Kepler Problem	257
14.2	The Bohr–Sommerfeld Quantization	260
14.3	Electromagnetic Plane Waves	263
14.4	Polarization and Dispersion	264
14.5	The Normal Zeeman Effect	266
14.6	Diamagnetism and Paramagnetism	268
14.7	The Vector Model	270
14.7.1	Precession of the Angular Momentum	270
14.7.2	The Vector Model	271
14.8	Quantization of Harmonic Oscillator	272
	Bibliography	275
	Index	283



<http://www.springer.com/978-94-007-5541-3>

Quantum Mechanics: Genesis and Achievements

Komech, A.

2013, XVIII, 286 p., Hardcover

ISBN: 978-94-007-5541-3