Chapter 2
Basic Physical Mechanisms

Abstract There were described the basic mechanisms of thermal exchange in animals: radiation, convection, conduction and evaporation. Radiation laws are discussed together with the radiative properties of the surfaces (absorptance, reflectance, transmittance and emissivity). Thermal exchange by radiation is discussed, and shape factors are described for different geometric structures taken as animal body models. Thermal exchange by conduction is described. There are presented and discussed equations for thermal exchange by free, forced and mixed convection in several cases; the respective equations for nondimensional numbers (Prandtl, Reynolds, Grashof and Nusselt) are presented together with some relevant information. The loss of heat by evaporation is discussed, and equations for nondimensional numbers (Schmidt, Sherwood) are presented. Several examples of application to animals are presented in detail.

Keywords Application to livestock • Conduction • Convection • Evaporation • Heat exchange • Radiation

2.1 Radiation

2.1.1 Definitions and Properties

Thermal radiation is the transfer of heat from a body to another through the displacement of electromagnetic energy by photons, which behave both as particles and waves. As for those electromagnetic waves, they constitute a fraction of the known electromagnetic spectrum (Table 2.1); this band is also known as infrared, from the end of the visible light to the beginning of the microwave band.

The waves are irradiated by atoms and molecules as a result of changes in their energetic content. The amount of radiation emitted depends on the nature of the material, the physical constitution and the absolute temperature of the emitting surface. Every surface whose temperature is above −273.15°C (or 0 K) emits

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Table 2.1 Approximate division of the known electromagnetic spectrum

Radiation band	Wavelength (μm)	Frequency (GHz)
Cosmic rays	$\leq 4 \times 10^{-7}$	$\geq 7.5 \times 10^{11}$
Gamma rays	4×10^{-7} a 1.4×10^{-4}	2.1×10^{9} a 7.5×10^{11}
X-rays	10^{-4} a 2×10^{-2}	1.5×10^{7} a 3×10^{10}
Ultraviolet	5×10^{-3} a 0.39	7.7×10^{5} a 6×10^{7}
Visible light	0.39 a 0.78	3.8×10^{5} a 7.7×10^{5}
Infrared	0.78 a 1,000	3×10^{3} a 3.8×10^{5}
Hertzian waves	3×10^{7} a 10^{11}	3×10^{-6} a 3×10^{3}

The limits of some bands were not yet definitively established (GHz = 10^{9}Hz)

thermal radiation at wavelengths that depend on that temperature. The waves travel at the speed of light or 2.997925×10^{8} m s^{-1} (in the vacuum), and when they attain a surface, a portion of the incoming energy is reflected, a portion is transmitted through the surface, and the rest is absorbed by it. Some important definitions are given as follows:

Radiant flux density. The total flux of radiant energy emitted by a given surface is the *radiant flux* density, which is given in W m^{-2}. The radiant flux density emitted by a surface at a given wavelength per unit area per unit time is its *emissive power* (**R**). In order to know the total amount of emitted radiation, we must know the amounts of radiation effectively emitted at each wavelength, $\mathbf{R}_\lambda$:

$$\mathbf{R} = \int_0^\infty \mathbf{R}_\lambda \, \mathrm{d}\lambda \tag{2.1}$$

Radiosity. However, some of the incoming radiation is reflected and this reflected, radiation leaves the surface together with that is emitted by the surface. Then, we can define *radiosity* (**J**) as the total flux of radiant energy leaving the surface by emission plus reflection per unit area per unit time:

$$\mathbf{J} = \int_0^\infty \mathbf{J}_\lambda \, \mathrm{d}\lambda \tag{2.2}$$

Irradiance. Finally, there is the total radiant flux density that attains the surface coming from the external environment, the *irradiance* (**G**):

$$\mathbf{G} = \int_0^\infty \mathbf{G}_\lambda \, \mathrm{d}\lambda \tag{2.3}$$

The three properties above described are related among themselves, in a way that each one can be deduced from the knowledge of the other two. Thus

$$\mathbf{J} = \mathbf{R} + \rho\mathbf{G} = \mathbf{R} + (1 - \alpha)\mathbf{G} \tag{2.4}$$

where ρ is the reflectance and α the absorptance of the surface, which will be defined as follows. A real surface can behave in four ways with respect to the thermal radiation: (a) reflecting the incident energy, (b) absorbing the energy, (c) transmitting the energy and (d) emitting the energy.

2.1.1.1 Reflectance (ρ)

It is the fraction of the radiant flux striking on a surface at a given wavelength and which is reflected depending on the direction of the incoming radiation and that of the reflected one.

2.1.1.2 Absorptance (α)

Fraction of the radiant flux reaching the surface at a given wavelength, which is absorbed by the surface. It depends on the direction and on the wave length of the incoming radiation.

2.1.1.3 Transmittance (τ)

Fraction of the radiant flux reaching the surface at a given wave length, that is transmitted through the surface.

In general, $\rho + \alpha + \tau = 1$. However, most gases present high τ but low α and ρ values. For example, the air under normal pressure conditions is virtually transparent to the thermal radiation, in such a way that for the atmosphere $\tau = 1$ and $\alpha \cong \rho \cong 0$. Other gases, as CO_2, present high absorptance values for thermal radiation.

On the other hand, solid surfaces in general (except glass) are opaque to the thermal radiation, and for them, $\tau = 0$ and $\rho + \alpha = 1$.

2.1.1.4 Emissivity (ε)

It is the ability of a surface to emit thermal radiation, as compared to a standard surface (*black body*). A black body would be any surface which can emit as radiation the entire energy amount contained in it, independently of the wavelength. By definition, $\varepsilon = 1$ for such a surface.

There has no perfect black bodies in the nature, except perhaps the astronomical phenomena known as *black holes*. An approximate model would be a great container whose inner walls were at a uniform, constant temperature. If it has a small hole in the wall, any radiation entering through the hole into the container will reflect successively on the inner surfaces; at each time, a fraction of this energy is absorbed and other fraction is reflected; at the end, it will be almost null the probability that any radiation amount escapes through the hole to the external environment. Thus, the virtual "surface" of the open hole is a black body, because it absorbs every incoming radiation and reflects none.

However, most real surfaces – named as *grey bodies* – behave in a different way, because α is always <1 for them; in addition, there has a dependence on the incoming radiation wavelength. As $\rho > 0$ in real surfaces, they always reflect some portion of the incoming radiation. It is interesting to note that $\alpha = \varepsilon$ for the grey bodies, a fact that can be explained as follows.

Turning back to the hole in the container, suppose that within the cavity the irradiance is equal to the emissive power of the inner walls, $\mathbf{G} = \mathbf{R}_b$. If in the cavity is placed a small object whose surface is a grey body with emissivity ε and absorptance α, being maintained at the same temperature T as the cavity, then the thermal balance will require that the emitted energy be equal to that absorbed. Thus

$$\alpha\mathbf{G} = \mathbf{R} = \varepsilon\mathbf{R}_b \tag{2.5}$$

where $\mathbf{R}_b$ is the emissive power of a black body. But as $\mathbf{G} = \mathbf{R}_b$, then we have $\alpha\mathbf{R}_b = \varepsilon\mathbf{R}_b$ and finally $\alpha = \varepsilon$. As a consequence, *every real surface with a high absorptance has an equally high emissivity with respect to a given wavelength and a given temperature*. On the other hand, for the most natural surfaces, $\rho=1 - \alpha$ and $\rho = 1 - \varepsilon$.

2.1.2 Radiation Laws

2.1.2.1 Stefan-Boltzmann's Law

The emissive power of the radiation emitted by a black body is proportional to the fourth power of its absolute temperature:

$$\mathbf{R}_b = \sigma T^4 \ \ \mathrm{W\,m^{-2}} \tag{2.6}$$

where $\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$ is the *Stefan-Boltzmann constant*. By considering this law, it is also possible to define the emissive power of a grey body as

$$\mathbf{R}_\lambda = \varepsilon_\lambda \sigma T^4 \ \ \mathrm{W\,m^{-2}} \tag{2.7}$$

in which ε_λ is the emissivity of the surface to given wavelength λ. Table 1.3 shows the emissivity values for some surface types.

2.1.2.2 Planck's Law

The emissive power is a function of the wavelength and the surface temperature. Max Planck applied to this case his quantum theory to explain radiation by the model:

$$\mathbf{R}_{\lambda,T} = \frac{a}{\lambda^5\left[e^{b/(\lambda T)} - 1\right]} \ \mathrm{m}^{-2}\,\mu\mathrm{m}^{-4} \tag{2.8}$$

Where:

$a = 2\pi h c_0 = 3.741775 \times 10^8$ W m^{-2} μm^4
$b = hc_0/k = 14{,}387.8$ μm K
$h =$ Planck's constant $= 6.626075 \times 10^{-34}$ J s
$k =$ Boltzmann's constant $= 1.380658 \times 10^{-23}$ J K^{-1}
$c_0 =$ light speed in the vacuum $= 2.99792458 \times 10^8$ m s^{-1}

The Stefan-Boltzmann constant is related to the above given constant values in a way that

$$\sigma = \left(\frac{\pi}{b}\right)^4\left(\frac{a}{15}\right) = 5.67 \times 10^{-8}\ \ \mathrm{W\,m^{-2}\,K^{-4}}$$

By using Planck's law it is possible to determine a curve for the variation of emissive power of a surface, according to the wavelength. In animals, the skin behaves as a black body for the thermal radiation (infra-red), and Eq. 2.8 can be easily applied. Suppose a swine whose hairless skin is at a temperature of 35°C or 380.15 k. The emissive power is calculated for the several wavelengths from 0.5 to 100 μm by means of formula 2.8, thus obtaining the curve shown in Fig. 2.1.

In this example there was no power emission at wavelengths $\lambda < 2$ and >75 μm. In order to estimate the total radiation energy emitted, one would calculate by integration the area under the curve between the specified limits (2 and 75 μm).

2.1.2.3 Wien's Law

Deduced from Eq. 2.8, this law states that the emissive power of a surface varies inversely to the wavelength and proportionally to the surface temperature. Its maximum value is found for a wave length given by the relation

$$\lambda_{max} = 2897.8\,T^{-1}\ \ \mu\mathrm{m} \tag{2.9}$$

where T is the surface temperature (K).

2.1.2.4 Cosine Law

When a radiant beam reaches a surface with an area so small that the incoming rays can be considered as parallel ones, then the surface irradiance will depend on its angle with respect to the direction of the rays.

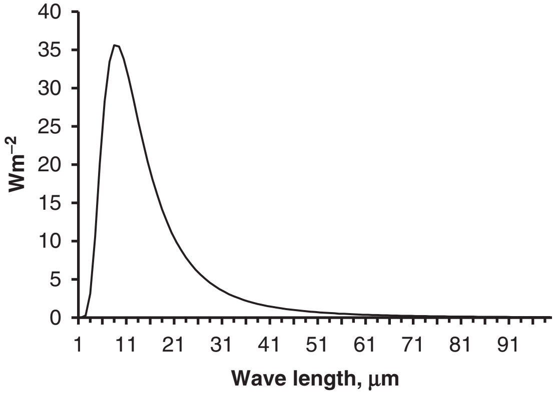

Fig. 2.1 Variation of the emissive power at the skin surface of an animal for a cutaneous temperature of 35°C

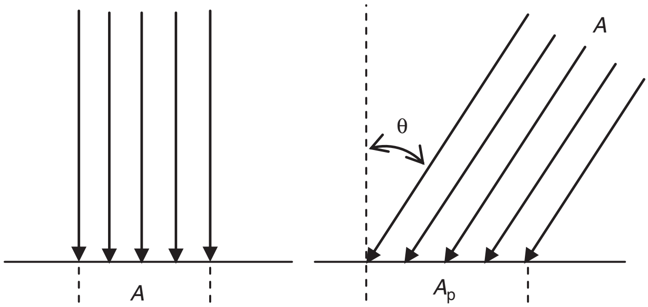

Fig. 2.2 The area of a horizontal surface illuminated by a radiation beam increases in proportion to its zenith angle to the surface, θ

This fact can be illustrated as follows. Suppose that a light beam is perpendicular (i.e. at a zenith angle of 90°) to a plane horizontal surface and results in a bright focus with area A_p. Then, as the zenith angle of the beam is reduced, the surface area of the beam focus increases proportionally to the beam angle, as it is shown in Fig. 2.2.

If A is the area of the focus relative to the angle $\theta = 0°$, then it follows that

$$A_p/A = \cos\ \theta \tag{2.10}$$

and the radiant flux density attaining effectively the surface will be given as

$$\mathbf{R} = \mathbf{R}_0 \cos\theta\ \mathrm{W\,m^{-2}} \tag{2.11}$$

where $\mathbf{R}_0$ is the radiant flux density normal to the surface.

2.1.2.5 Beer's Law

If the radiation beam passes through a given material before reaching a surface, then its intensity will be decreased at an exponential rate, depending on the thickness of the crossed layer and on its absorptance relative to the wavelength. Considering that the radiant flux reaches a given x point during its path across the layer, it will have there a flux density $\mathbf{R}_x$, and then, the absorption of radiation by a layer with thickness dx will be estimated as

$$\mathrm{d}\mathbf{R} = -\kappa\,\mathbf{R}\,\mathrm{d}x \tag{2.12}$$

where κ is the *attenuation* or *extinction coefficient*, defined as the probability a ray be intercepted along a distance dx through the layer. By integration, we can obtain the radiation flux density at any point x:

$$\mathbf{R}_x = \mathbf{R}_0\,\mathrm{e}^{-\kappa x} \tag{2.13}$$

by considering that $\mathbf{R}_0$ is the flux density at $x = 0$.

This law is applied strictly to the case in which the wave length band is sufficiently narrow so that the κ value is constant for the entire band. However, it is often used for the solar radiation crossing the atmosphere (in this case, absence of dispersion must be assumed); then, the distance dx is evaluated in terms of *atmospheric mass number*, which is given by:

$$m = \frac{P_\mathrm{a}}{101.325\cos\theta}$$

as defined previously in Chap. 1 (Eq. 1.7a).

2.1.3 Radiation Geometry

2.1.3.1 Effective Irradiation

It is often desired to evaluate the amount of thermal radiation received on a body's surface. In order to do this, the relationship between surface geometry and directional properties of the radiation must be taken into account.

Suppose a sphere exposed to a radiant flux that comes into it at an angle β with respect to the horizontal plane. The surface area of the sphere is given by $4\pi r^2$, where r is its radius; however, the radiant source "sees" the apparent area of the sphere, which is a circle whose area is $A_\mathrm{p} = \pi r^2$. This apparent area is projected over a horizontal surface below, producing a shadow whose form and area (A_s) depend on the angle of the radiation to the horizontal surface. Then

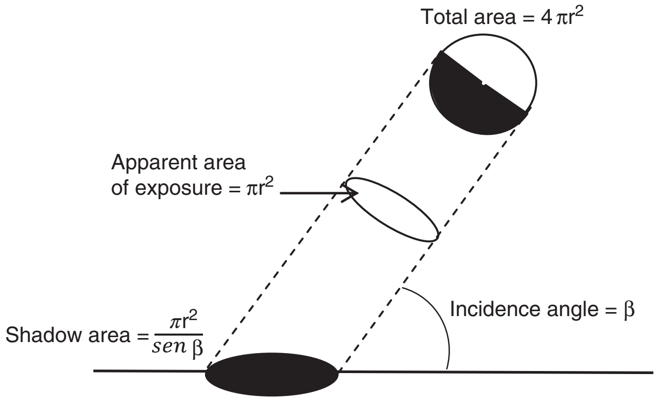

Fig. 2.3 Geometry of the shadow from a sphere exposed to radiation

$$A_s = \frac{\pi r^2}{\operatorname{sen}\beta}\ \mathrm{m}^2 \tag{2.14}$$

as it is shown in Fig. 2.3.

Irregular bodies as those of animals offer many difficulties for the evaluation of the area exposed to radiation; in such cases there were used equivalent bodies, as spheres or cylinders. These models are related to the real bodies by an analogy with the general body conformation. For example, a quadruped as a cow or a swine can be represented by a horizontal cylinder with the appropriate measures.

The proportion of the body area that receives radiation is determined by a *shape factor*, which depends on the geometry and on the radiation directional properties. For practical purposes, the shape factor is determined as the relation of the shadow area to the total area of the body or object:

$$F_c = \frac{A_s}{A} \tag{2.15}$$

Shape Factor for a Sphere

Let us have a sphere of radius r, as that in Fig. 2.3. The shadow projected by it has an area $A_s = \pi r^2/\sin\beta$, while the total area of the sphere is $A = 4\pi r^2$. The shape factor is then calculated as:

$$F_c = \frac{A_s}{A} = \frac{\pi r^2/\sin\beta}{4\pi r^2} = \frac{0.25}{\sin\beta} \tag{2.16}$$

Shape Factor for a Vertical Cylinder

Let us have a cylinder of radius r and length z, placed vertically on a horizontal surface. It receives radiation at an angle β with respect to the surface and casts a shadow on it. Total area of the cylinder and that of its shadow are given respectively, by

$$A = 2\pi r(\mathrm{r} + z)\ \mathrm{m}^2$$
$$A_\mathrm{s} = \pi r^2 + 2rz(\tan\beta)^{-1}\ \mathrm{m}^2$$

Then, the shape factor will be

$$F_\mathrm{c} = \frac{\pi r^2 + 2rz\,(\tan\beta)^{-1}}{2\pi r(r + z)} \tag{2.17}$$

Shape Factor for a Horizontal Cylinder

The shade area cast by a horizontal cylinder over a horizontal surface depends on the elevation angle of the radiation source (β) and the azimuth angle of the cylinder axis with respect to the radiation source (ω). See Fig. 2.4. Then the total area of the shade will be:

$$A_\mathrm{s} = 2rz(\sin\beta)^{-1}\left[1 - (\cos\beta)^2(\cos\omega)^2\right]^{1/2}\ \mathrm{m}^2 \tag{2.18}$$

As the total area of the cylinder is the same above given, the shape factor can be defined as

$$F_\mathrm{c} = \frac{2rz(\sin\beta)^{-1}\left[1 - (\cos\beta)^2(\cos\omega)^2\right]^{1/2} + \pi r^2(\tan\beta)^{-1}(\cos\omega)}{2\pi r(r + z)} \tag{2.19}$$

It must be stressed that, according to Monteith and Unsworth (2008), the shape factor of horizontal cylinders with relation $z/r = 4$ is independent from the azimuth angle, ω, if the elevation of the radiation source is high as $\beta > 40°$.

As it was suggested by Campbell and Norman (1998), a cylinder with hemispherical ends would be better than a simple cylinder as a model for quadruped animals. However, the respective equation given by those authors does not take into consideration the azimuth angle; this is an important aspect, as for the practical use in animals on range in the open field. Equation 2.19 can be modified in order to obtain F_c for a cylinder with hemispherical ends

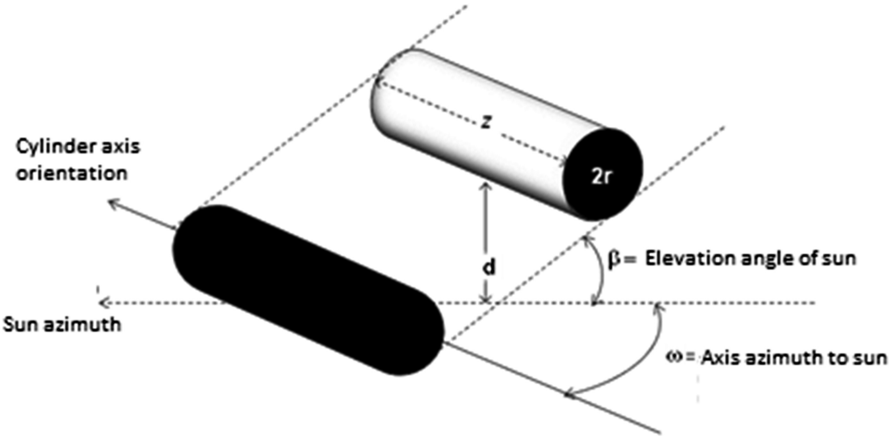

Fig. 2.4 Radiation geometry of a horizontal cylinder of radius r and length z, placed at a distance d from a horizontal surface. There are a solar elevation angle β and an azimuth angle ω

$$F_{\mathrm{c}} = \frac{(\sin\beta)^{-1}\left\{2rz\left[1-(\cos\beta)^2(\cos\omega)^2\right]^{1/2}+\pi r^2\right\}}{2\pi r(2r+z)} \tag{2.20}$$

Finally, as for the validity of the cylinder model for animals, Underwood and Ward (1966) measured 25 men and 25 women in swimsuits and photographed them at 19 different angles; in each case the shaded area was evaluated by planimetry methods. The average projected body area was close to that of a cylinder 1.65 m high and 0.12 m radius. A similar study was done by Clapperton et al. (1965) with sheep; it was found that the bodies of those animals (with a normal wool fleece) were equivalent to cylinders 0.91 m long with radius $r = 0.23$ m.

Shape Factor for a Horizontal Prolate Ellipsoid

As it is shown in Fig. 2.5, a prolate ellipsoid is a body with a circular transverse section and an elliptical longitudinal one which is often used as model for bird bodies. In most of those cases, there has a small difference between the greatest (R) and the smallest (r) radius, given in metres, in a way that the azimuth angle ω of the body axis can be neglected without much error. The surface area can be approximated by using Knud Thomsen's formula:

$$A = 4\pi\left(\frac{2(Rr)^x + r^{2x}}{3}\right)^{1/x}\ \mathrm{m}^2 \tag{2.21}$$

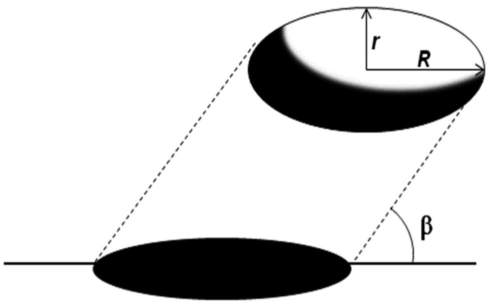

Fig. 2.5 Radiation geometry of a horizontal ellipsoid with greatest radius R and smallest radius r, exposed to a radiation source at an elevation angle β

where $x = 1.6075$.

The shaded area will be

$$A_s = \pi R^2 \left[1 + \frac{r^2}{R^2 tan^2 \beta}\right]^{1/2} \text{m}^2$$

according to Monteith and Unsworth (2008). Finally:

$$F_\text{c} = \frac{A_\text{s}}{A} \tag{2.22}$$

2.1.3.2 Shape Factor and Irradiance

By means of the shape factor, it is possible to estimate the amount of thermal radiant energy reaching the surface of an animal or object. The flux of energy leaving the radiation source (radiosity) as evaluated at a horizontal plane is symbolised by $\mathbf{J}_\text{h}$. If it is assumed that this energy attains its target surface without losses, the irradiance of that surface can be considered as equal to the radiosity of the radiant source. Thus, the radiant flux that effectively reaches the target surface is defined by

$$\mathbf{R} = F_\text{c}\,\mathbf{J}_\text{h}\ \text{W m}^{-2} \tag{2.23}$$

In other words, the shadow projected by a body over a horizontal surface allows the determination of the average irradiance of the body with respect to a radiation source, for example, the sun, since the values of the direct solar radiation, the surface area of the body, the solar elevation angle and the azimuth angle of the object with respect to sun.

2.1.3.3 Radiation Exchange Between Surfaces

By considering the simplest case, let us have two parallel planes, S_1 and S_2, each one at a known surface temperature T_1 and T_2, respectively. The respective emissivity coefficients are ε_1 and ε_2. Then

$$\mathbf{R}_1 = \varepsilon_1 \sigma T_1^4$$
$$\mathbf{R}_2 = \varepsilon_2 \sigma T_2^4$$

The amount of radiant energy leaving S_1 in direction to S_2 is given by

$$\mathbf{R}_1 = \mathbf{J}_1 - \mathbf{G}_1$$

where $\mathbf{J}_1$ is the radiosity and $\mathbf{G}_1$ the irradiance of S_1. However, we know that $\mathbf{J} = \mathbf{R} + (1 - \varepsilon)\mathbf{G}$ and also that $\mathbf{R} = \varepsilon\sigma T^4$. Thus, we have for the two surfaces

$$\mathbf{J}_1 = \varepsilon_1 \sigma T_1^4 + (1 - \varepsilon_1)\mathbf{G}_1$$
$$\mathbf{J}_2 = \varepsilon_2 \sigma T_2^4 + (1 - \varepsilon_2)\mathbf{G}_2$$

Now, suppose that all the radiant energy leaving S_1 attains S_2 without losses, in a way that any fraction reflected by a surface reaches another, it follows that

$$\mathbf{R}_1 = -\mathbf{R}_2$$
$$\mathbf{G}_1 = \mathbf{J}_2$$
$$\mathbf{G}_2 = \mathbf{J}_1$$

By rearranging those equations and expressing them in terms of matrices, we have

$$\begin{bmatrix} \varepsilon_1 \sigma T_1^4 \\ \varepsilon_2 \sigma T_2^4 \end{bmatrix} = \begin{bmatrix} 1 & -(1-\varepsilon_1) \\ -(1-\varepsilon_2) & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{G}_1 \end{bmatrix}$$

whose solution is

$$\begin{aligned}\begin{bmatrix} \mathbf{J}_1 \\ \mathbf{G}_1 \end{bmatrix} &= \begin{bmatrix} 1 & -(1-\varepsilon_1) \\ -(1-\varepsilon_2) & 1 \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_1 \sigma T_1^4 \\ \varepsilon_2 \sigma T_2^4 \end{bmatrix} \\ &= [1 - (1-\varepsilon_1)(1-\varepsilon_2)]^{-1} \begin{bmatrix} \varepsilon_1 \sigma T_1^4 + (1-\varepsilon_1)\varepsilon_2 \sigma T_2^4 \\ (1-\varepsilon_2)\varepsilon_1 \sigma T_1^4 + \varepsilon_2 \sigma T_2^4 \end{bmatrix}\end{aligned}$$

As the thermal radiant flux is given by $\mathbf{R}_1 = \mathbf{J}_1 - \mathbf{G}_1 = -\mathbf{R}_2$, then

$$\mathbf{R} = \frac{\varepsilon_1\sigma T_1^4 + (1-\varepsilon_1)\varepsilon_2\sigma T_2^4 - (1-\varepsilon_2)\varepsilon_1\sigma T_1^4 - \varepsilon_2\sigma T_2^4}{1-(1-\varepsilon_1)(1-\varepsilon_2)}$$
$$= \frac{\varepsilon_1\varepsilon_2\sigma\left(T_1^4 - T_2^4\right)}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\ \mathrm{W\,m^{-2}} \tag{2.24}$$

2.2 Conduction

2.2.1 Definitions

Thermal conduction is the transfer of heat among parts of a body by means of the kinetic energy displacement of the molecules, or by the transportation of free electrons as in the case of metals. Such a flux passes from the highly energised molecules to those less energised ones, in other words, from a zone of high temperature to another at lower temperature.

A fundamental aspect of the conduction is the need of direct contact among the molecules of the bodies or surfaces involved. Thus, the transfer of thermal energy by such a process can occur only within the mass of a body, or between two bodies in mutual direct contact.

Thermal diffusivity is the physical property of the substance from which a body is constituted; it expresses the ability of the body to transfer thermal energy, in relation to its ability to store energy (*specific* heat). Then

$$D = \frac{k}{\rho c_\mathrm{p}}\ \mathrm{m^2\,s^{-1}} \tag{2.25}$$

where k is the *thermal conductivity*, ρ is the *density* and c_p is the specific heat of the material the body is constituted from. As greater or smaller is the thermal diffusivity of the substance, the greater or smaller is the speed in which the heat can spread through the body. Table 2.2 shows the values of thermal conductivity for some materials.

The ability of some substances to permit the passing of the energy flux is greater than that of other substances, as it occurs with respect to the electrical energy. Such an analogy is not incidental and is enough to justify the use of the electrical model to explain thermal conduction.

Let us have a board made of any material, with a surface area A and a thickness Δx. One of the faces of the board is at a temperature T_1, while the other face is at temperature T_2. The heat flux through the board will be proportional to its area and to the absolute value of the temperature differential, $|T_1 - T_2|$, but inversely

Table 2.2 Coefficients of thermal conductivity (k) for some materials

Material	Conductivity, $W \cdot m^{-1} \cdot {}^{\circ}C^{-1}$
Diverses	
Water (0°C)	0.562
Cotton	0.06
Aluminium	220
Sand (20 cm layer)	0.027
Asbestos cement board	0.63
Copper	386
Concrete	1.4
Cork	0.039
Stucco	0.72
Glass fibre	0.037
Polystyrene	0.025
Common brick	0.72
Glass	0.76
Wood	
Agglomerate	0.087
Plywood	0.12
Softwood	0.12
Hardwood	0.16
Animal tissues	
Subcutaneous fat (blubber)	0.18–0.25
Muscle	0.41
Skin (human)	0.037

From Rohsenow et al. (1998), Monteith and Unsworth (2008) and others

proportional to the thickness Δx. The thermal flux by conduction between both faces of the board will be given by

$$\mathbf{K} = K\,A(T_1 - T_2)\ \mathrm{W} \tag{2.26}$$

where K is the *thermal conductance* (W m^{-2} °C^{-1}) of the board. By taking into account the electric model, we must have the *thermal resistance*, r, of the board material; it is the inverse of the conductance, or $r = 1/K$. Then, after eliminating the term A from the formula 2.26 we have the thermal flux by conductance

$$\mathbf{K} = \frac{T_1 - T_2}{r}\ \mathrm{W\,m^{-2}} \tag{2.27}$$

2.2.2 Conduction in Multiple Layers

Suppose a board constituted into many layers of different materials and different thickness. Figure 2.6 shows the three-layer case.

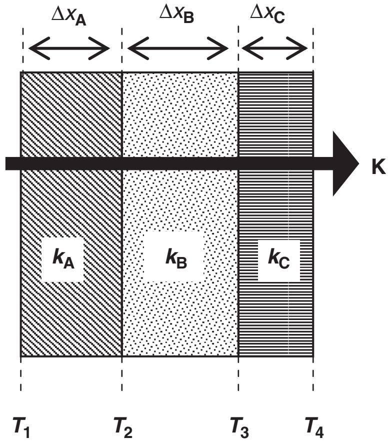

Fig. 2.6 Heat flux by conduction, **K**, through three layers of different thickness made from different material. Values T_1 to T_4 are temperatures of the surfaces limiting the layers, Δx_A to Δx_C are the respective layer thicknesses and k_A and k_C are the conductivity coefficients of the layers

By considering Eq. 2.27 we have:

$$\mathbf{K} = \frac{T_1 - T_2}{r_A} + \frac{T_2 - T_3}{r_B} + \frac{T_3 - T_4}{r_C}$$

where r_i (i = A, B, C) is the thermal resistance of the ith layer

$$r_i = \frac{\Delta xi}{k_i}$$

and Δx_i and k_i are the thickness and the thermal conductivity of the constituting material, respectively.

Considering the electrical model, the total resistance to heat flux through the three layers can be estimated by adding the three series partial resistances, $r_T = r_A + r_B + r_C$:

$$\mathbf{K} = \frac{(T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4)}{r_A + r_B + r_C} = \frac{T_1 - T_4}{r_T}$$

Generalising for n layers, we can have a more practical formula:

$$\mathbf{K} = \frac{\rho c_p (T_1 - T_{n+1})}{\sum_{i=1}^{n} r_i} \text{ W m}^{-2} \tag{2.28}$$

where ρ is the density (g m^{-3}) and c_p is the specific heat (J g^{-1} °C^{-1}) of air at ambient temperature. Then the thermal resistance can be defined as

$$r_i = \frac{\rho c_p \, \Delta xi}{k_i} \text{ s m}^{-1} \tag{2.29}$$

2.3 Convection

2.3.1 Definition

Convection is the transfer of thermal energy by displacement of the air or any other fluid. If such a displacement is caused by a density differential − a consequence of the temperature difference − then the process is named as *free convection* or *passive convection*. If the displacement is caused by active forces, as pumps, ventilators or any wind source, we have a *forced* or *active* convection.

Let us have an object immersed into a fluid at a temperature that is lower than that of the object. The fluid in direct contact with the surface of the object will be heated to a temperature close to that of the surface. Even if the fluid is moving, there is always a layer of some thickness attached to the object's surface − the *boundary layer* − which is at a temperature higher than that of the fluid around it. As more turbulent is the fluid displacement, the less tick is this boundary layer.

Thermal energy is transferred by conduction from the object's surface into the boundary layer, which becomes less dense than the rest of the fluid involving it; there has a tendency for it to ascend against the gravity force and push away the surface. As it happens, the mass of heated fluid is displaced carrying itself the absorbed heat and is substituted by an equivalent volume of colder, denser fluid; this results into an increased temperature differential at the object's surface again, and the process is repeated.

Heat flux by convection depends on the following: (a) the temperature differential between the surface and the fluid in contact to it and (b) the *convection coefficient* of the fluid relative to the physical structure of the surface. Then

$$\mathbf{C} = h_c(T_s - T_a)\ \mathrm{W\,m^{-2}} \tag{2.30}$$

In the above equation, h_c is the *convection coefficient*, given by

$$h_c = \frac{k}{d}\frac{d}{z}\ \mathrm{W\,m^{-1}\,^\circ C^{-1}} \tag{2.30a}$$

where d is the *characteristic dimension* (m) of the surface, object or body; z is the thickness of the boundary layer (m). In practical terms, Eq. 2.30 can be re-written as:

$$\mathbf{C} = \frac{\rho c_p(T_s - T_a)}{r_H}\ \mathrm{W\,m^{-2}} \tag{2.31}$$

where ρ is the density ($\mathrm{g{\cdot}m^{-3}}$) and c_p the specific heat ($\mathrm{J{\cdot}g^{-1}{\cdot}{}^\circ C^{-1}}$) of the air at the given temperature; r_H is the resistance of the fluid to the heat transfer by convection,

$$r_H = \frac{\rho c_p d}{k N_u}\ \mathrm{s\,m^{-1}} \tag{2.32}$$

The value N_u in Eq. 2.32 is the *Nusselt number*, whose definition and calculation will be discussed as follows.

2.3.2 Free Convection

2.3.2.1 Nondimensional Numbers

The Nusselt number for free convection is defined in its general form as

$$N_u = a\,G_r^m P_r^n \tag{2.33}$$

where a, m and n are constants whose values vary according to the specific case; G_r is the *Grashof number*,

$$G_r = \frac{g\,d^3(T_\mathrm{s} - T_\mathrm{a})}{\nu^2(T_\mathrm{a} + 273.15)} \tag{2.34}$$

where g is the acceleration of gravity, d is the characteristic dimension of the surface, ν is the kinematic viscosity of the air at temperature T_a (°C) and T_s is the surface temperature (°C).

Another nondimensional number to be considered is P_r or the *Prandtl number*, which is associated to the relation between actual thickness of the boundary layer and its thickness needed to perform the desired heat transfer. Then

$$P_r = \frac{\rho c_\mathrm{p} \nu}{k} \tag{2.35}$$

where ρ, ν, k and c_p are thermo-physical properties of the air at temperature T_a, as described in Chap. 1.

Some equations for the calculation of the Nusselt number will be given as follows, according to the geometrical conformation and the position of a specific surface or object.

Vertical Planes (Churchill and Chu 1975b)

$$N_u = \left\{ 0.825 + \frac{0.387(G_r P_r)^{1/6}}{\left[1 + (0.492/P_r)^{9/16}\right]^{8/27}} \right\}^2 \tag{2.36}$$

For the present case, the thermo-physical properties of the air must be calculates for the temperature $T_\mathrm{m} = 0.5\ (T_\mathrm{s} + T_\mathrm{a})$.

Vertical Cylinders (Monteith and Unsworth 2008)

$$N_u = 0.65\ G_r^{1/4}\,P_r^{1/3} \quad \text{for } 10^4 < G_r < 10^9 \tag{2.37a}$$

$$N_u = 0.123\ G_r^{1/3}\,P_r^{1/3} \quad \text{for } 10^9 < G_r < 10^{12} \tag{2.37b}$$

Equation 2.36 can be used also for this case if the cylinder has a relation diameter/length $\geq 35/G_r^{1/4}$.

Horizontal or Inclined Planes (Incropera et al. 2007)
When the plane has an inclination angle θ in relation to the vertical that is greater than or equal to 0, the g value in Eq. 2.34 must be substituted by $g.\cos\theta$; then use the N_u in Eq. 2.36. On the other hand, if the plane is almost horizontal, its characteristic dimension must be calculated as

$$d = (\text{Surface area})/\text{Perimeter m}$$

and N_u is calculated by using one of the equations:

$$N_u = 0.54\ G_r^{1/4}\,P_r^{1/4} \quad \text{for}\ \ 10^4 \leq G_r\,P_r \leq 10^7 \tag{2.38a}$$

$$N_u = 0.15\ G_r^{1/3}\,P_r^{1/3} \quad \text{for}\ \ G_r\,P_r > 10^7 \tag{2.38b}$$

Horizontal Cylinders (Churchill and Chu 1975a)

$$N_u = \left\{ 0.6 + \frac{0.387(G_r P_r)^{1/6}}{\left[1 + (0.559/P_r)^{9/16}\right]^{8/27}} \right\}^2 \tag{2.39}$$

Spheres (Churchill 2002)

$$N_u = 2 + \frac{0.589(G_r\,P_r)^{1/4}}{\left[1 + (0.469/P_r)^{9/16}\right]^{4/9}}. \tag{2.40}$$

2.3.3 *Forced Convection*

2.3.3.1 Nondimensional Numbers

Nusselt number for the forced convection is defined by the general equation

$$N_u = b\,R_e^p P_r^q \tag{2.41}$$

where R_e is the *Reynolds number*, given by

$$R_e = \frac{Ud}{v} \tag{2.42}$$

where U is the speed of the wind or the air displacement (m s^{-1}), d is the characteristic dimension of the surface or object (m) and ν is the kinematic viscosity of the air at the given temperature (m^2 s^{-1}).

When a plane, smooth surface is exposed to a nonturbulent air stream, the boundary layer will become turbulent only in the case the Reynolds number exceeds the value $R_e = 10^5$; however, if the air stream is a turbulent one, this limit decreases to $R_e = 4{,}000$ (Monteith and Unsworth 2008). More details at this respect can be found in Gates (1980) and Rubesin et al. (1998).

Equations for the Nusselt number in the case of forced convection will be given according to the surface type.

Horizontal or Vertical Planes, Flux Parallel to the Surface (Incropera et al. 2007)

Assuming the direction of the flux as the same as that of the characteristic dimension d (m), we have

$$N_u = 0.332\, R_e^{1/2} P_r^{1/3} \tag{2.43}$$

Cylinders, Flux Perpendicular to the Axis (Churchill and Bernstein 1977)

$$N_u = 0.3 + \frac{0.62\, R_e^{1/2} P_r^{1/3}}{\left[1 + (0.4/P_r)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{R_e}{282{,}000}\right)^{5/8}\right]^{4/5} \tag{2.44}$$

In this case, the characteristic dimension is $d =$ cylinder diameter (m), and the thermo-physical properties of the air must be calculated for the temperature $T_m = 0.5(T_s + T_a)$.

Cylinders, Flux Parallel to the Axis (Incropera et al. 2007)

$$N_u = 0.0296\, R_e^{4/5}\, P_r^{1/3} \tag{2.45}$$

where $d =$ cylinder length (m). According to Monteith and Unsworth (2008), for the case of body of a quadruped animal, it would be better to use the body volume V (m^3) to estimate the characteristic dimension as $d = V^{1/3}$, even for flux parallel to the axis or perpendicular to the axis.

Spheres (Incropera et al. 2007)

$$N_u = 2 + \left(0.4\, R_e^{1/2} + 0.06\, R_e^{2/3}\right) P_r^{0,4} \left(\frac{\mu}{\mu_s}\right)^{1/4} \tag{2.46}$$

where μ and μ_s are the coefficients of dynamic viscosity of the air at temperatures T_a and T_s, respectively. This equation can be eventually simplified by eliminating μ and μ_s.

2.3.4 Mixed Convection

In many circumstances, free convection occurs together with the forced one. A way that can be followed in order to solve this problem is to determine the Grashof and Reynolds numbers first and then calculate

$$\xi = \frac{G_r}{R_e^2} \tag{2.47a}$$

Lloyd and Sparrow (1970) studied the association of ξ value with that $N_u/R_e{}^{0.5}$ and observed that for P_r values about 0.71 (often determined for atmosphere under normal conditions) the critical value of ξ which eliminated the occurrence of forced convection was 0.08. On the other hand, the results of the cited paper showed also that convection was purely free when $\xi > 3$. According to Chapman (1987), those results involve an error less than 5 %. Then, we can apply the following criterion to decide what convection type is occurring:

If $\xi \leq 0.08$ there is forced convection;
If $\xi \geq 3$ there is free convection.

If $0.08 < \xi < 3$, the Nusselt number must be calculated for both convection types and then it is calculated the weighted value:

$$N_u = \left[\left(N_u^N\right)^p + \left(N_u^F\right)^p\right]^{1/p} \tag{2.47b}$$

according to Churchill (1977), and where:

N_u^N = Nusselt number for free convection.
N_u^F = Nusselt number for forced convection.
$p = 3$ for the general case, $p = 4$ for spheres, and $p = 3.5$ for horizontal cylinders.

2.3.5 Forced Convection in Tubes

In biological studies, it is eventually needed to determine heat transfer by convection within the respiratory ways. Thus, the knowledge of the principles involved in that mode of heat transfer can be of great interest.

Forced convection is generally predominant in such cases, in which there are considered tubes of small internal dimensions (diameter d and length z). The equation proposed by Sieder and Tate (1936) is considered yet as efficient enough to estimate the Nusselt number for those cases:

$$N_u = 1.86\left[\left(\frac{d}{z}\right)R_e P_r\right]^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \tag{2.48}$$

where $\mu = \nu\rho$ is the dynamic viscosity of the air at the temperature of the flux within the tube and μ_s is the same property, for the temperature of the tube walls (in general, this temperature is the same as that of the body or rectal temperature).

Equation 2.48 refers to the case of a laminar flux. For a turbulent flux, which occurs in most of the cases, one can use the equations

$$N_u = 0.0395\ R_e^{0.75}\ P_r^{1/3} \quad \text{for } 10^4 < R_e < 5 \times 10^4 \tag{2.48a}$$

$$N_u = 0.023\, R_e^{0.8}\, P_r^{1/3} \quad \text{for } 3 \times 10^4 < R_e < 10^6 \tag{2.48b}$$

However, when the tube is very short with a relation of length to diameter as that $400 > z/d > 10$, it must be used the following equation proposed by Nusselt (1931):

$$N_u = 0.036\ R_e^{0.8}\ P_r^{1/3}\left(\frac{d}{z}\right)^{1/18} \tag{2.48c}$$

in which the atmospheric properties must be calculated for the temperature within the tube – which is that of the expired air, in animals. Practical application of those principles to animals will be discussed in details later in Chap. 4.

2.4 Mass Transfer

2.4.1 *Definitions*

Among organisms and the atmosphere there are constant transfers of mass, especially gases as O_2, CO_2 and water vapour. Those transfers are processed within the stomatal cavities of the plant leaves, while in animals, it occurs through the tissues of the respiratory system and the boundary layer at the body surface. Such transfers are generally turbulent ones; the interfaces between liquids and gases are zones in which the water molecules pass down from a physical state to another, and latent heat is exchanged along that process.

In order to understand the mass transfer process, let us suppose two points in the space, x_1 and x_2, each of them with a different concentration of water vapour, C_1 e C_2, respectively. In general terms, it is known that the concentration of a gas is given by

$$C = \frac{M\rho}{M_a\rho_a}$$

where M and ρ are the molecular mass and the density of that gas, respectively, while M_a and ρ_a refer to the air. Those two points are separated one from another by a layer with thickness dz that is permeable to the gas molecules, whose displacement is assumed to be at random. In those circumstances, there has equal probability of any molecule in x_1 be transferred to x_2 and *vice versa*.

Now, by applying Fick's law the mass flux can be estimated as

$$\Phi_m = -D\frac{\mathrm{dC}}{\mathrm{d}z} = -\frac{D}{\mathrm{d}z}(C_1 - C_2) \tag{2.49}$$

where D is the gas diffusivity in the air and dC/dz is the concentration gradient.

2.4.1.1 Nondimensional Numbers

For the present purposes, the term mass transfer refers to the water vapour transfer, that is, to the evaporation process. The vapour transfer from surfaces exposed to air flux of a given intensity is analogous to the process of thermal convection, which was discussed before in this chapter (Sect. 2.3). Thus, the Nusselt number is used here also, but as a modified form – the *Sherwood number*, S_h – which is nondimensional also.

Now, by taking Eq. 2.49 again and eliminating its negative sign, we have

$$\mathbf{E} = \frac{D_v}{d}S_h(C_1 - C_2) \tag{2.50}$$

where $\mathbf{E}$ is the flux density of thermal energy by evaporation, D_v is the diffusivity of water vapour in the air, d is the characteristic dimension of the surface, C_s is the concentration of water vapour at the surface and C_a is the concentration of water vapour in the atmosphere.

On the other hand, the Sherwood number can be defined as

$$S_h = \frac{\mathbf{E}}{(D_v/d)(C_s - C_a)} \tag{2.51}$$

and it represents the rate of vapour transfer that can occur if the same concentration differential would exist through an air layer of thickness d. In practical terms, it is possible to determine an equation for S_h in specific cases by merely calculating Grashof, Reynolds and Prandtl numbers for each case, in the same way as to determine the Nusselt number for convection. Then, it is found the N_u equation needed for the actual case and the Prandtl number, P_r, is substituted in that equation by the following nondimensional value:

$$S_c = \frac{v}{D_v} \tag{2.52}$$

which is the *Schmidt number*, v is the kinematic viscosity of the air and D_v is the vapour diffusivity in the air at temperature T_a. Such a change of P_r by S_c is due to the differences in the effective thickness of the boundary layer, with respect to heat and mass transfer.

For example, if the equation chosen for the Nusselt number is

$$N_u = 0.332\, R_e^{1/2}\, P_r^{1/3}$$

then the corresponding equation for the Sherwood number will be

$$S_h = 0.332 R_e^{1/2} S_c^{1/3}$$

2.4.2 Evaporation from Wet Surfaces

Suppose a wet surface, that is, one covered with a water film. The surface is at a temperature T_s (just that at which the water will evaporate) and the vapour pressure in the boundary layer over the surface is the saturation vapour pressure of the air at the same T_s temperature. On the other hand, the atmosphere is at a temperature T_a and partial vapour pressure P_v. If this system is adiabatic and there has no energy transfer by radiation or conduction, then at equilibrium, the sensible heat exchange by convection will be equal to the latent heat dissipation by evaporation, that is, $\mathbf{E} = \mathbf{C}$. The convective exchange is given by Eq. 2.31,

$$\mathbf{C} = \frac{\rho c_p (T_s - T_a)}{r_H}\ \mathrm{W\,m^{-2}}$$

where $r_H = \frac{\rho c_p d}{k N_u}$ $\mathrm{s\,m^{-1}}$.

Similarly, the flux of heat by evaporation from a wet surface is given by

$$\mathbf{E} = \frac{\rho c_p [P_s(T_s) - P_v]}{P_a \gamma r_V}\ \mathrm{W\,m^{-2}} \tag{2.53}$$

where ρ, c_p and γ are thermo-physical properties of the air at temperature T_a; P_a is the actual atmospheric pressure (kPa) and r_V is the resistance to mass transfer, given by

$$r_V = \frac{d}{D_V\, S_h}\ \mathrm{s\,m^{-1}} \tag{2.53a}$$

It must be stressed that the saturation vapour pressure at the surface, $p_S(T_S)$, must be calculated for the surface temperature, T_S.

However, once evaporation and convection are functions of the wind velocity, Monteith and Unsworth (2008) suggested that Eq. 2.53 can be conveniently changed into

$$\mathbf{E} = \frac{\rho c_p [P_s(T_s) - P_v]}{\gamma^* r_H} \tag{2.54}$$

where

$$\gamma^* = P_a \gamma \left(\frac{r_V}{r_H} \right) \tag{2.54a}$$

Strictly speaking, Eqs. 2.53 and 2.54 are valid only when the surface is completely wetted and covered by a water film. As for the cutaneous surface of animals, such a condition would imply in sweating rates of 1,000 $g \cdot m^{-2} \cdot h^{-1}$ or greater; this is true for humans and equines, but other animal species (as ruminants in general) lose heat by sweat evaporation without having a fully wet skin surface.

Then, if the surface is humid, but without a visible water film, it is needed to modify Eq. 2.54 as

$$\mathbf{E} = \frac{w \, \rho \, c_p [P_s(T_s) - P_v]}{\gamma^* r_H} \tag{2.55}$$

according to McArthur (1987) and in which the value w is the proportion of the actual evaporation rate to the rate that would be observed if the surface were wet (Gagge 1981).

The amount of thermal energy needed to convert 1 g of water at temperature T_s (surface) into 1 g of vapour at temperature T_a (atmosphere) is given by

$$\lambda - c_{pv}(T_s - T_a) \text{ Joules}$$

after Monteith (1972), in which λ is the latent heat of vaporisation of water at temperature T_s (Eq. 1.46) and c_{pv} is the specific heat of the water vapour at temperature T_a, estimated by Eq. 1.49. By relating the amount of effectively dissipated energy to that lost in the case that the surface was covered with a water film, it is possible to determine the correction factor shown in Eq. 2.53a and which is

$$w = \frac{\gamma^* r_H \mathrm{S} \left[\lambda - c_{pv}(T_s - T_a) \right]}{\rho \, c_p [P_s(T_s) - P_v]} \tag{2.56}$$

according to Silva (2000) and where S is the sweating rate ($g \cdot m^{-2} \cdot s^{-1}$).

2.5 Problems

Problem 2.1. The surface of a black body was at a temperature of 760 K. Calculate: (a) emissive power, (b) the wavelength at which the maximum emissive power occurs and (c) the power emitted at 1.1 μm and 6.5 μm wavelengths, respectively.

(a) *The Stefan-Boltzmann law is applied*:

$$\mathbf{R}_b = \sigma T^4 = 5.67 \times 10^{-8}(760)^4 = 18.9164 \ \mathrm{W\,m^{-2}}$$

(b) *By the Wien's Law*:

$$\lambda_{max} = \frac{2,897.8}{T} = \frac{2,897.8}{760} = 3.81 \ \mu\mathrm{m}$$

(c) *According to the Planck's law*:

$$\mathbf{R}_{\lambda,T} = \frac{a}{\lambda^5\left[e^{b/(\lambda T)} - 1\right]} = \frac{3.741775 \times 10^8}{\lambda^5\left[exp\left\{\dfrac{14,387.8}{\lambda T}\right\} - 1\right]}$$

$$\mathbf{R}_{1.1;760} = \frac{3.741775 \times 10^8}{(1.1)^5\left[exp\left\{\dfrac{14,387.8}{(1.1)(760)}\right\} - 1\right]} = 7.79 \ \mathrm{W\,m^{-2}}$$

$$\mathbf{R}_{6.5;760} = \frac{3.741775 \times 10^8}{(6.5)^5\left[exp\left\{\dfrac{14,387.8}{(6.5)(760)}\right\} - 1\right]} = 1,853 \ \mathrm{W\,m^{-2}}$$

Problem 2.2. A hog's body can be represented as a horizontal cylinder with hemispherical ends, 0.90 m length and 0.35 m average diameter. This animal is standing in a pen exposed to sun at an azimuth angle of 75°. The zenith angle of the sun is 7°, and the solar radiation was measured as S = 614 W m^{-2}. Calculate the direct solar radiation effectively received by the animal.

Data:

r = cylinder radius = 0.35/2 = 0.175 m
z = cylinder length = 0.9 m
β = sun elevation angle = $90 - \theta = 90 - 7 = 83°$
ω = azimuth angle of cylinder axis to sun = 75°
S = direct solar radiation = 614 W m^{-2}

By applying Eq. 2.20

$$F_c = \frac{(\sin\beta)^{-1}\left\{2rz\left[1-(\mathbf{cos}\beta)^2(\mathbf{cos}\omega)^2\right]^{1/2}+\pi r^2\right\}}{\mathbf{2\pi r(2r+z)}}$$
$$= \frac{1.00751\left\{2(0.175)(0.9)[1-(0.014852)(0.066987)]^{1/2}+\pi(0.175)^2\right\}}{2\pi(0.175)(0.35+0.9)} = 0.297$$

Animal's irradiance at the given conditions will be

$$\mathbf{S}_{\text{animal}} = F_c\,\mathbf{S}_{\text{dir}} = (0.297)(614) = 182.4\,\text{W m}^{-2}$$

Problem 2.3. There has a concrete wall 15 cm thick with external revetment of a 2.5-cm stucco layer and an internal one of 2-cm stucco plus 1-cm wood agglomerate. The external surface is exposed to 35°C temperature, while the internal one at 20°C. The atmospheric pressure is 98.5 kPa. Calculate the rate of thermal transfer by conduction through the wall.

Data:

n = *number of layers* = 4
Δx_1 = *layer 1 thickness* = 0.025 m
Δx_2 = *layer 2 thickness* = 0.15 m
Δx_3 = *layer 3 thickness* = 0.02 m
Δx_4 = *layer 4 thickness* = 0.01 m
Concrete conductivity = 1.4 $\text{W·m}^{-1}\,°\text{C}^{-1}$
Stucco conductivity = 0.72 $\text{W·m}^{-1}\,°\text{C}^{-1}$
Wood agglomerate conductivity = 0.087 $\text{W·m}^{-1}\,°\text{C}^{-1}$
T_1 = *external temperature* = 35°C
T_5 = *internal temperature* = 20°C

Atmospheric properties at T_a = 35°C = 308.15 K:

$$P_a = \text{atmospheric pressure} = 98.5\ \text{kPa}$$

$$\rho = \text{density} = \frac{3,484.358P_a}{T_a} = \frac{3,484.358(98.5)}{308.15} = 1,113.77\ \text{g m}^{-3}$$

$$c_p = \text{specific heat}$$
$$= 1.00522 + 0.0004577\ \exp\left\{\frac{T_a}{32.07733}\right\}\ \text{J g}^{-1}\,°\text{C}^{-1}$$
$$= 1.00522 + 0.0004577\ \exp\left\{\frac{35}{32.07733}\right\} = 1.006583\ \text{J g}^{-1}\,\text{C}^{-1}$$

Thermal resistances:

$$r_i = \frac{\rho c_p \Delta xi}{k_i} \ \mathrm{s\,m^{-1}}$$

$$\text{Concrete } (0.15 \text{ m}) : r_1 = \frac{1{,}113.77(1.006583)(0.15)}{1.4} = 120.1181 \ \mathrm{s\,m^{-1}}$$

$$\text{Stucco } (0.025 \text{ m}) : r_2 = \frac{1{,}113.77(1.006583)(0.025)}{0.72} = 38.9272 \ \mathrm{s\,m^{-1}}$$

$$\text{Stucco } (0.02 \text{ m}) : r_3 = \frac{1{,}113.77(1.006583)(0.02)}{0.72} = 31.1417 \ \mathrm{s\,m^{-1}}$$

$$\text{Agglomerate } (0.01 \text{ m}) : r_4 = \frac{1{,}113.77(1.006583)(0.01)}{0.087} = 128.8623 \ \mathrm{s\,m^{-1}}$$

$$\mathbf{K} = \frac{\rho c_p (T_1 - T_{n+1})}{\sum_{i=1}^{n} r_i} \ \mathrm{W\,m^{-2}}$$

$$\sum_{i=1}^{4} r_i = 120.1181 + 38.9272 + 31.1417 + 128.8623 = 319.0493$$

$$\mathbf{K} = \frac{\rho c_p (T_1 - T_{n+1})}{\sum_{i=1}^{n} r_i} = \frac{1,113.77(1.006583)(35 - 20)}{319.0493} = 52.71 \ \mathrm{W\,m^{-2}}$$

Problem 2.4. In a location at 23°45′ south latitude and 189 m altitude there has a chamber which must be maintained at the temperature of 15°C, while in the outer, the temperature is as high as 38°C. The walls of the chamber were made of wood compensate 2.5 cm thick, and its inner surface is covered by a layer of polystyrene 5 cm thick. Calculate the thermal flux through that wall.

Data:

T_a = ambient temperature = 38°C = 311.15 K
T_{int} = chamber temperature = 15°C
L_t = Latitude = 23°45′ = 23.75°
z = Altitude = 189 m
Δx_1 = wood layer thickness = 0.025 m
Δx_2 = polystyrene layer thickness = 0.05 m
k_1 = thermal conductivity of the wood compensate = 0.12 s m^{-1}
k_2 = thermal conductivity of polystyrene = 0.025 s m^{-1}

Atmospheric properties at 38°C:

$$g = 9.78013 + 8.18 \times 10^{-5} L_t + 1.168 \times 10^{-5} L_t^2 - 3.1 \times 10^{-6} z$$
$$= 9.78013 + 8.18 \times 10^{-5}(23.75) + 1.168 \times 10^{-5}(23.75)^2 - 3.1 \times 10^{-6}(189)$$
$$= 9.788075 \ \mathrm{m\,s^{-2}}$$

$$P_a = 101.325 \exp\left\{\frac{-zg}{287.04 T_a}\right\}$$
$$= 101.325 \exp\left\{\frac{-189(9.788075)}{287.04(311.15)}\right\} = 96.65 \ \mathrm{kPa}$$

$$c_p = 1.00522 + 0.0004577 \exp\left\{\frac{T_a}{32.07733}\right\}$$
$$= 1.00522 + 0.0004577 \exp\left\{\frac{38}{32.07733}\right\} = 1.006716 \ \mathrm{J\,g^{-1}\,{}^{\circ}C^{-1}}$$

$$\rho = \frac{3{,}484.358\, P_a}{T_a} = \frac{3{,}484.358 \ (96.65)}{311.15} = 1{,}082.32 \ \mathrm{g\,m^{-3}}$$

We have two layers, then
Compensate (0.025 m):

$$r_1 = \frac{1{,}082.32(1.006716)(0.025)}{0.12} = 226.9972 \ \mathrm{s\,m^{-1}}$$

Polystyrene (0.05 m):

$$r_2 = \frac{1{,}082.32(1.006716)(0.05)}{0.025} = 2{,}179.1777 \ \mathrm{s\,m^{-1}}$$

$$\sum_{i=1}^{4} r_i = 226.9972 + 2{,}179.1777 = 2{,}406.1749 \ \mathrm{s\,m^{-1}}$$

$$\mathbf{K} = \frac{\rho\, c_p(T_a - T_{int})}{\sum_{i=1}^{n} r_i} = \frac{1{,}082.32(1.006716)(38 - 15)}{2{,}406.1749} = 10.42 \ \mathrm{W\,m^{-2}}$$

If a more precise temperature control is desired within the chamber, the above calculations must be done using thicker polystyrene layers, until it is practical or economically available. For example, it is desired a thermal flux rate which is 10 % of that above calculated or 1.04 W m^{-2}:

$$\mathbf{K} = \frac{\rho c_p(T_a - T_{int})}{\sum_{i=1}^{n} r_i} = \frac{1{,}082.32(1.006716)(38 - 15)}{226.9972 + r_2} = 1.04$$

$$1.04(226.9972 + r_2) = 1{,}082.32\ (1.006716)(38 - 15)$$
$$\therefore r_2 = 10.6688698$$
$$r_2 = \frac{\rho c_p \Delta x}{k_2} = \frac{1{,}082.32(1.006716)\Delta x}{0.025} = 23{,}869.6795$$

$$43{,}583.468084\ \Delta x = 23{,}869.6795 \quad \therefore \Delta x = 0.548\ \text{m} = 54.8\ \text{cm}$$

In order to have a thermal flux as small as 10 W m^{-2} it would be needed a 55-cm polystyrene layer! Of course, it is not practical. Then a solution of compromise must be searched for, by establishing an acceptable thermal flux.

Problem 2.5. Suppose a horizontal plane with dimensions 0.4 × 0.95 m, whose surface is at 35°C. The ambient temperature is 30°C, and a 0.5 m s^{-1} wind blows perpendicularly to the shortest side of the plane. The respective location is at 21° latitude and 630 m altitude. Calculate the thermal exchange by convection.

Data:

T_a = air temperature = 30°C = 303.15 K
T_s = plane surface temperature = 35°C
U = wind velocity = 0.5 m s^{-1}
L_t = latitude = 21°
z = altitude = 630 m

$$\begin{aligned} g &= 9.78013 + 8.18 \times 10^{-5} L_t + 1.168 \times 10^{-5} L_t^2 - 3.1 \times 10^{-6} z \\ &= 9.78013 + 8.18 \times 10^{-5}(21) + 1.168 \times 10^{-5}(21)^2 - 3.1 \times 10^{-6}(630) \\ &= 9.785046\ \text{m s}^{-2} \end{aligned}$$

$$\begin{aligned} P_a &= 101.325 \exp\left\{\frac{-zg}{287.04\, T_a}\right\} = 101.325 \exp\left\{\frac{-630(9.785046)}{287.04(305.65)}\right\} \\ &= 94.449809\ \text{kPa} \end{aligned}$$

Thermal properties of atmosphere at temperature $T_m = 0.5(T_a + T_s) = 32.5°C$:

$$\begin{aligned} c_p &= 1.00522 + 0.0004577\ exp\left\{\frac{T_m}{32.07733}\right\} \\ &= 1.00522 + 0.0004577\ exp\left\{\frac{32.5}{32.07733}\right\} = 1.006481\ \text{J} \cdot \text{g}^{-1} \cdot {}^{\circ}\text{C}^{-1} \end{aligned}$$

$$\rho = \frac{3{,}484.358\, P_a}{T_a} = \frac{3{,}484.358(94.3951)}{305.65} = 1{,}076.0881\ \text{g m}^{-3}$$

$$\begin{aligned}k &= \rho c_{\mathrm{p}}(1.888 \times 10^{-5} + 1.324 \times 10^{-7} T_{\mathrm{a}}) \\ &= 1,076.0881(1.006481)[1.888 \times 10^{-5} + 1.324 \times 10^{-7}(32.5)] \\ &= 0.025109\end{aligned}$$

$$\begin{aligned}v &= 1.32743 \times 10^{-5} + 9.22286 \times 10^{-8} T_{\mathrm{a}} \\ &= 1.32743 \times 10^{-5} + 9.22286 \times 10^{-8}(32.5) = 1.6272 \times 10^{-5}\ \mathrm{m^2 s^{-1}}\end{aligned}$$

Non-dimensional numbers:

For forced convection in the present case $d = 0.95$; for free convection:

$$\mathrm{d} = \frac{0.4(0.95)}{2(0.4 + 0.95)} = 0.14074$$

By using Eqs. 2.35, 2.42 and 2.34, we calculate, respectively:

$$P_r = \frac{\rho\, c_{\mathrm{p}}\, v}{k} = \frac{1076.0881(1.006481)(1.6272 \times 10^{-5})}{0.025109} = 0.701883$$

$$R_e = \frac{U\ d}{v} = \frac{0.5\ (0.95)\ V4}{1.6272 \times 10^{-5}} = 2,191.25$$

$$G_r = \frac{g\ d^3(T_{\mathrm{s}} - T_{\mathrm{a}})}{v^2\, T_{\mathrm{a}}} = \frac{9.785046(0.14074)^3(35 - 30)}{(1.6272 \times 10^{-5})^2(303.15)} = 1,699.206.2$$

But $G_r/R_e^{\,2} < 0.08$, then it is a case of forced convection. By using Eq. 2.43, we have

$$N_u = 0.332\ R_e^{1/2} P_r^{1/3} = 0.332(29.191.25)^{1/2}(0.701883)^{1/3} = 50.4103$$

$$r_{\mathrm{H}} = \frac{\rho\, c_{\mathrm{p}}\, d}{k N_u} = \frac{1076.0881(1.006481)(0.95)}{0.025109(50.4103)} = 812.8834$$

$$\mathbf{C} = \frac{\rho\, c_{\mathrm{p}}(T_{\mathrm{s}} - T_{\mathrm{a}})}{r_{\mathrm{H}}} = \frac{1076.0881(1.006481)(35 - 30)}{812.8834} = 6.7\ \mathrm{W\, m^{-2}}$$

Problem 2.6. A hog whose skin is at a 34°C temperature has the trunk as a horizontal cylinder with dimensions 0.90 × 0.35 m and is standing in an environment where temperature is 28.5°C, the atmospheric pressure is 94.5 kPa and there has a 1.1 m s^{-1} wind blowing perpendicularly at the body axis of the animal. Calculate the thermal exchange by convection.

Data:

T_a = ambient temperature = 28.5°C = 301.65 K
T_s = skin surface temperature = 34°C
U = wind velocity = 1.1 m·s^{-1}
P_a = atmospheric pressure = 94.5 kPa

As the wind blows perpendicularly at the animal's body axis, then the characteristic dimension of the body is $d = 0.35$. On the other hand, the wind velocity is $U > 1$ m·s^{-1}; thus we have a forced convection.

Properties of atmosphere at temperature $T_m = 0.5(T_a + T_s) = 31.25$°C:

$$c_p = 1.00522 + 0.0004577\ exp\left\{\frac{T_m}{32.07733}\right\}$$
$$= 1.00522 + 0.0004577\ exp\left\{\frac{31.25}{32.07733}\right\} = 1.006432\ \mathrm{J \cdot g^{-1} \cdot ^\circ C^{-1}}$$

$$\rho = \frac{3{,}484.358 P_a}{T_a} = \frac{3{,}484.358\,(94.5)}{301.65} = 1{,}091.57\ \mathrm{g\ m^{-3}}$$

$$v = 1.32743 \times 10^{-5} + 9.22286 \times 10^{-8}\ \mathrm{m}$$
$$= 1.32743 \times 10^{-5} + 9.22286 \times 10^{-8}(31.25) = 1.6156 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}$$

$$k = \rho c_p\left(1.888 \times 10^{-5} + 1.324 \times 10^{-7} T_m\right)$$
$$= \rho c_p\left[1.888 \times 10^{-5} + 1.324 \times 10^{-7}(31.25)\right] = 0.025287\ \mathrm{W\,m^{-2}\,^\circ C^{-1}}$$

$$P_r = \frac{\rho\, c_p\, v}{k} = \frac{1091.57(1.006432)\left(1.6156 \times 10^{-5}\right)}{0.025287} = 0.701901$$

$$R_e = \frac{U\,d}{v} = \frac{1.1(0.35)}{1.6156 \times 10^{-5}} = 23{,}830.156$$

$$N_u = 0.3 + \frac{0.62\, R_e^{1/2} P_r^{1/3}}{\left[1 + (0.4/P_r)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{R_e}{282.000}\right)^{5/8}\right]^{4/5}$$
$$= 0.3 + \frac{0.62(23.830.156)^{1/2}(0.701901)^{1/3}}{\left[1 + (0.4/0.701901)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{23.830.156}{282.000}\right)^{5/8}\right]^{4/5}$$
$$= 87.422$$

$$r_H = \frac{\rho\, c_p\, d}{k\, N_u} = \frac{1{,}091.57(1.006432)\,(0.35)}{0.025287(87.422)} = 173.9341$$

$$\mathbf{C} = \frac{\rho\, c_p (T_s - T_a)}{r_H} = \frac{1091.57(1.006432)(34 - 28.5)}{173.9341} = 34.7 \text{ W m}^{-2}$$

Problem 2.7. Let us have a horizontal cylinder with 0.72 m diameter and surface temperature of 36.5°C, which is suspended in a place where the acceleration of gravity is 9.7861 m s^{-2}, the atmospheric pressure is 99.641 kPa, dry bulb temperature is 32°C and a 0.9 m·s^{-1} wind blows perpendicularly at the cylinder axis. Determine the thermal exchange by convection.

Data:

g = acceleration of gravity = 9.7861 m·s^{-2}
P_a = atmospheric pressure = 99.641 kPa
T_a = dry bulb temperature = 32°C = 305.15 K
U = wind velocity = 0.9 m·s^{-1}
T_s = surface temperature of the cylinder = 36.5°C
d = diameter of the cylinder = 0.72 m

Properties of the air at temperature $T_m = 0.5\ (T_a + T_s) = 34.25$°C:

$$\begin{aligned} c_p &= 1.00522 + 0.0004577\, exp\left\{\frac{T_m}{32.07733}\right\} \\ &= 1.00522 + 0.0004577\, exp\left\{\frac{34.25}{32.07733}\right\} = 1.006553 \text{ J g}^{-1}\,°\text{C}^{-1} \end{aligned}$$

$$\rho = \frac{3{,}484.358\, P_a}{T_a} = \frac{3{,}484.358(98.7)}{307.4} = 1{,}129.5631 \text{ g m}^{-3}$$

$$\begin{aligned} v &= 1.32743 \times 10^{-5} + 9.22286 \times 10^{-8} T_m \\ &= 1.32743 \times 10^{-5} + 9.22286 \times 10^{-8}(34.25) = 1.643 \times 10^{-5} \text{ m}^2\text{ s}^{-1} \end{aligned}$$

$$\begin{aligned} k &= \rho c_p \left(1.888 \times 10^{-5} + 1.324 \times 10^{-7} T_m\right) \\ &= \rho c_p \left[1.888 \times 10^{-5} + 1.324 \times 10^{-7}(34.25)\right] = 0.02667 \text{ W m}^{-2}\,°\text{C}^{-1} \end{aligned}$$

$$P_r = \frac{\rho\, c_p\, v}{k} = \frac{1{,}129.5631(1.006553)\left(1.643 \times 10^{-5}\right)}{0.02667} = 0.700425$$

$$R_e = \frac{Ud}{v} = \frac{0.9(0.72)}{1.643 \times 10^{-5}} = 39{,}440.0487$$

$$G_r = \frac{g\,d^3(T_s - T_a)}{v^2 T_a} = \frac{9.7861(0.72)^3(36.5 - 32)}{(1.643 \times 10^{-5})^2(305.15)} = 327,845,113.3$$

As $x = 0.08 < G_r/R_e{}^2 < 3$, then it is a mixed convection. By applying the principles previously studied, we calculate the Nusselt number for both convection types in horizontal cylinders as follows:

Free convection (Eq. 2.39):

$$N_u^N = \left\{0.6 + \frac{0.387(G_r P_r)^{1/6}}{\left[1 + (0.559/P_r)^{9/16}\right]^{8/27}}\right\}^2$$

$$= \left\{0.6 + \frac{0.387[(327,845,113.3)(0.701828)]^{1/6}}{\left[1 + (0.559/0.701828)^{9/16}\right]^{8/27}}\right\}^2 = 73.030933$$

Forced convection (Eq. 2.44):

$$N_u^F = 0.3 + \frac{0.62\,R_e^{1/2} P_r^{1/3}}{\left[1 + (0.4/P_r)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{R_e}{282,000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(39,440.0487)^{1/2}(0.701828)^{1/3}}{\left[1 + (0.4/0.701828)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{39,440.048}{282,000}\right)^{5/8}\right]^{4/5}$$

$$= 72.187827$$

As the convection is a mixed one, we have

$$N_u = \left[\left(N_u^N\right)^{3.5} + \left(N_u^F\right)^{3.5}\right]^{\frac{1}{3.5}}$$

$$= \left[(73.030933)^{3.5} + (72.187827)^{3.5}\right]^{\frac{1}{3.5}} = 88.5156$$

$$r_H = \frac{\rho c_p d}{k N_u} = \frac{(1,129.5631)(1.006553)(0.72)}{(0.02667)(88.5156)} = 346.7664\ \mathrm{s\,m^{-1}}$$

$$\mathbf{C} = \frac{\rho\, c_p(T_s - T_a)}{r_H} = \frac{1,129.5631(1.006553)(36.5 - 32)}{346.7664} = 14.754\ \mathrm{W\,m^{-2}}$$

Problem 2.8. Consider the cylinder described in Problem 2.7 and the same environmental data. Suppose again that the surface of the cylinder is wetted and the atmospheric wet bulb temperature is 26°C. Determine the loss of latent heat from the cylinder surface.

Data:

g = *acceleration of gravity* = 9.78611 m·s^{-2}
P_a = *atmospheric pressure* = 99.641 kPa
T_a = *dry bulb temperature* = 32°C = 305.15 K
T_u = *wet bulb temperature* = 26°C
U = wind velocity = 0.9 m·s^{-1}
T_s = surface temperature of the cylinder = 36.5°C
d = diameter of the cylinder = 0.72 m
r_H = 346.7664 s m^{-1}

Properties of the air at T_a = 32°C:

$$c_p = 1.00522 + 0.0004577\, exp\left\{\frac{T_a}{32.07733}\right\}$$
$$= 1.00522 + 0.0004577\, exp\left\{\frac{32}{32.07733}\right\} = 1.006463 \text{ J g}^{-1}\,{}^\circ\text{C}^{-1}$$

$$\rho = \frac{3484.358 P_a}{T_a} = \frac{3484.358(98.7)}{305.15} = 1,137.7523 \text{ g m}^{-3}$$

$$v = 1.32743 \times 10^{-5} + 9.22286 \times 10^{-8} T_m$$
$$= 1.32743 \times 10^{-5} + 9.22286 \times 10^{-8}(32) = 1.623 \times 10^{-5} \text{ m}^2\text{ s}^{-1}$$

$$k = \rho\, c_p\left(1.888 \times 10^{-5} + 1.324 \times 10^{-7} T_m\right)$$
$$= \rho\, c_p\left[1.888 \times 10^{-5} + 1.324 \times 10^{-7}(32)\right] = 0.0265 \text{ W m}^{-2\circ}\text{C}^{-1}$$

$$\lambda = \textit{latent heat of vaporization}\ (\text{Eq. } 1.46)$$
$$= 2{,}500.788 - 2.37374(32) = 2,424.8284 \text{ J g}^{-1}$$

$$\gamma = \textit{psychrometric constant}\ (\text{Eq. } 1.47)$$
$$= \frac{c_p}{0.6223\,\lambda} = \frac{1.006463}{0.6223(2424.8284)} = 6.6852 \times 10^{-4}\,{}^\circ\text{C}^{-1}$$

$$D_v = \textit{water vapour diffusivity}\ (\text{Eq. } 1.48)$$
$$= 2.12138 \times 10^{-5} + 1.4955 \times 10^{-7}(32) = 2.6 \times 10^{-5} \text{ m}^2\text{ s}^{-1}$$

$$\begin{aligned} P_{\mathrm{S}}(T_{\mathrm{u}}) &= \textit{air saturation pressure at temperature } T_{\mathrm{u}} \text{ (Eq. 1.50)} \\ &= 0.61078 \times 10^{7.5T_{\mathrm{u}}/(T_{\mathrm{u}}+237.5)} \\ &= 0.61078 \times 10^{7.5(26)/(26+237.5)} = 3.356779 \text{ kPa} \end{aligned}$$

$$\begin{aligned} P_{\mathrm{S}}(T_{\mathrm{s}}) &= \textit{air saturation pressure at temperature } T_{\mathrm{s}} \text{ (Eq. 1.50)} \\ &= 0.61078 \times 10^{7.5T_{\mathrm{s}}/(T_{\mathrm{s}}+237.5)} \\ &= 0.61078 \times 10^{7.5(36.5)/(36.5+237.5)} = 6.094987 \text{ kPa} \end{aligned}$$

$$\begin{aligned} P_{\mathrm{v}} &= \textit{partial vapour pressure} \text{ (Eq. 1.51)}: \\ &= P_{\mathrm{s}}(T_{\mathrm{u}}) - P_{\mathrm{a}}\gamma(T_{\mathrm{a}} - T_{\mathrm{u}}) \\ &= 3.356779 - (99.641)\left(6.6852 \times 10^{-4}\right)(32 - 26) = 2.957107 \text{ kPa} \end{aligned}$$

Non-dimensional numbers:

$$R_e = \textit{Reynolds number} = \frac{U\,d}{v} = \frac{0.9(0.72)}{1.623 \times 10^{-5}} = 39,926.0629$$

$$S_c = \text{Schmidt number} = \frac{v}{D_{\mathrm{v}}} = \frac{1.623 \times 10^{-5}}{2.6 \times 10^{-5}} = 0.624231$$

Sherwood number (the same Eq. 2.44 is used as for N_u, but substituting in it the value P_r by S_c):

$$\begin{aligned} \mathrm{S}_h &= 0.3 + \frac{0.62\,R_e^{1/2} S_c^{1/3}}{\left[1 + (0.4/S_c)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{R_e}{282,000}\right)^{5/8}\right]^{4/5} \\ &= 0.3 + \frac{0.62(39,926.0629)^{1/2}(0.624231)^{1/3}}{\left[1 + (0.4/0.624231)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{39,926.0629}{282,000}\right)^{5/8}\right]^{4/5} \\ &= 113.82 \end{aligned}$$

$$r_{\mathrm{V}} = \frac{d}{D_{\mathrm{v}} S_h} = \frac{0.72}{(2.6 \times 10^{-5})(113.82)} = 243.3 \text{ s m}^{-1}$$

$$\gamma^* = P_{\mathrm{a}}\gamma\left(\frac{r_{\mathrm{V}}}{r_{\mathrm{H}}}\right) = 99.641\ (6.6674 \times 10^{-4})\left(\frac{243.3}{346.7664}\right) = 0.046612$$

$$\begin{aligned} \mathbf{E} &= \frac{\rho\, c_{\mathrm{p}}[P_{\mathrm{s}}(T_{\mathrm{s}}) - P_{\mathrm{v}}]}{\gamma^*\, r_{\mathrm{H}}} \\ &= \frac{1137.7523(1.006463)(6.094987 - 2.957107)}{0.046612(346.7664)} = 222.3 \text{ W m}^{-2} \end{aligned}$$

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