

Preface

As stated in the Preface to the first edition, this book intends to provide material for the practical side of standard courses on analysis and algebra on differentiable manifolds at a middle level, corresponding to advanced undergraduate and graduate years. The exercises focus on Lie groups, fibre bundles, and Riemannian geometry. Aims, approach and structure of the book remain largely the same as in the first edition. In the present edition, the number of figures is 68.

The prerequisites are linear and multilinear algebra, calculus of several variables, various concepts of point-set topology, and some familiarity with linear algebraic groups, the topology of fibre bundles, and manifold theory.

We would like to express our appreciation to the authors of some excellent books as those which appear in the references in chapters. These books have served us as a source of ideas, inspiration, statements and sometimes of results. We strongly recommend these books to the reader.

We introduce now a brief overview of the contents.

Chapters 1 to 6 contain 412 solved problems, sorted according to the aforementioned topics and in almost the same vein, notations, etc., as in the first edition, but 39 problems of the first edition have been deleted and 76 new problems have been added in the present edition. The first section of each chapter gives a selection of those definitions and theorems whose terminology, with ample use throughout the book, could be misleading due to the lack of universal acceptance. However, *we should like to insist on the fact that we do not claim that this is any kind or part of a book on the theory of differentiable manifolds.*

We now underline some of the changes in this edition.

Unlike the first edition of the book, in the present edition the Einstein summation convention is not used.

We consider in Sect. 1.3 (and only there) differentiable structures defined on sets, analysing what happens when one of the properties of being Hausdorff or second countable fails to hold. We thus try to elicit in the reader a better understanding of the meaning and importance of these two properties.

In Chap. 1 of the present edition, we have added, as an instructive example, a problem where we prove in detail that the manifold of affine straight lines of the

plane, the 2-dimensional real projective space \mathbb{RP}^2 minus a point, and the infinite Möbius strip are diffeomorphic.

In Chap. 4, two new problems have been added in the section concerning the exponential map, where the simply connected Lie group corresponding to a given Lie algebra is obtained. The section devoted to the adjoint representation, contains six new problems concerning topics such as Weyl group, Cartan matrix, Dynkin diagrams, etc. Similarly, the section devoted to Lie groups of transformations has been increased in ten new application problems in symplectic geometry, Hamiltonian mechanics, and other related topics. Finally, we have added in the section concerning homogeneous spaces two problems on homogeneous spaces related to the exceptional Lie group G_2 .

The section on characteristic classes in Chap. 5 includes two new problems on the Godbillon–Vey class in the present edition. Moreover, the last section, devoted to almost symplectic manifolds, Hamilton’s equations, and the relation with principal $U(1)$ -bundles, contains five new problems, including topics as Hamiltonian vector fields.

In the present edition, the section of Chap. 6 concerning Riemannian connections has been enlarged, including six new problems on almost complex structures. The section on Riemannian geodesics also includes four new problems on special metrics. Moreover, a completely new section is devoted to a generalisation of Gauss’ Lemma. The section on homogeneous Riemannian and Riemannian symmetric spaces contains two new problems about general properties of homogeneous Riemannian manifolds and two new problems on specific three-dimensional Riemannian spaces. Furthermore, a short novel section deals with some properties of the energy of Hopf vector fields. The section on left-invariant metrics on Lie groups contains in particular two new problems: One gives in a detailed way the structure of the Kodaira–Thurston manifold; and the other furnishes the de Rham cohomology of a specific nilmanifold.

Chapter 7 offers an expanded 56-page long collection of formulae and tables concerning frequent spaces and groups in differential geometry. *Many of them do not actually appear in the problems, but having them collected together may prove useful as an aide-mémoire, even to teachers and researchers.*

At the end of the references to each chapter, several books (or papers) appear that have not been explicitly cited, but such that they have inspired several ideas of the chapter and/or are very useful references.

All in all, we hope that this new edition of the book will again render a good service to practitioners of differential geometry and related topics.

We acknowledge the anonymous referees for their thorough, enlightening and suggestive reports; their invaluable suggestions and corrections have contributed to improve several aspects of contents as well as presentation of the book.

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