

Chapter 2

Sea Ice Drift

Abstract In this chapter, sea ice drift is discussed in terms of statistical properties of individual Lagrangian trajectories. It is first shown that the Arctic sea ice velocity field can be objectively decomposed into a mean field and fluctuations. The mean field shows intra-annual (between winter and summer) as well as some interannual variability. The fluctuations, defined as the remaining part of the velocity field after removing the mean field, share similarities with fluid turbulence, such as a Kolmogorov-like scaling of the power spectral density, a ballistic regime within an inertial range of motion, or intermittency. However, significant differences are also observed: sea ice velocity distributions are clearly much more spread than Gaussian statistics, intermittency is more pronounced, and sea ice accelerations cannot be explained from wind stress statistics. These differences argue for a non-linear response of sea ice to forcing.

As mentioned in [Chap. 1](#), (Nansen 1902) reported that sea ice drifts an average of about 2 % of the wind speed, and 20–40° to the right of the wind direction. A first order justification of this 2 % rule of thumb can be obtained from the momentum equation (1.1) with several (rough) simplifying assumptions. We consider a one-dimensional model of the free drift of sea ice, i.e. rotational effects are not considered and the internal stress term $\nabla \cdot \sigma$ is neglected. We consider further that the ocean is at rest ($\mathbf{U}_w = 0$) and the wind forcing $\mathbf{U}_a$ constant. Then the equation of motion reads (Leppäranta 2005):

$$\rho_i h \frac{d\mathbf{U}_i}{dt} = \rho_a C_a |\mathbf{U}_a| \mathbf{U}_a - \rho_w C_w |\mathbf{U}_i| \mathbf{U}_i \quad (2.1)$$

Taking as an initial condition $\mathbf{U}_i(t = 0) = 0$, the solution of Eq. (2.1) is:

$$\mathbf{U}_i(t) = \sqrt{\frac{\rho_a C_a}{\rho_w C_w}} \mathbf{U}_a \tanh \left(\sqrt{\frac{\rho_a C_a \rho_w C_w}{\rho_i h}} \mathbf{U}_a t \right) \quad (2.2)$$

which gives a steady-state solution $\mathbf{U}_i = Na \mathbf{U}_a$, where $Na = \sqrt{\frac{\rho_a C_a}{\rho_w C_w}}$ is the so-called Nansen number. Taking $\rho_a = 1.3 \text{ kg/m}^3$, $\rho_w = 1025 \text{ kg/m}^3$ for sea water, $C_a \approx 1.5 \times 10^{-3}$ and $C_w \approx 5 \times 10^{-3}$ (Leppäranta 2005), we obtain $Na = 0.0195$, in excellent agreement with Nansen's rule of thumb.

However, owing to the simplifying assumptions, such derivation can only give a very approximate view of sea ice drift, when averaged over large spatial and temporal scales and for conditions associated with low internal stresses, such as a loose summer ice cover. Using data from the first automatic ice drifters (in 1979 and 1980), Thorndike and Colony (1982) analyzed in more detail the response of sea ice to geostrophic wind forcing, i.e. the wind that results theoretically from the balance between the Coriolis effect and the pressure gradient. More specifically, they considered the following linear relationship:

$$\mathbf{U} = \mathbf{A}\mathbf{G} + \mathbf{C} + \mathbf{E} \quad (2.3)$$

where $\mathbf{A} = Ae^{-i\theta}$ is a complex constant with θ the turning angle, $\mathbf{G}$ the geostrophic wind, $\mathbf{C}$ a constant mean velocity field assumed to be related to a mean ocean current, and $\mathbf{E}$ the residuals. As shown above, such a linear equation can be justified by a steady-state solution of the ice momentum equation in case of free-drift. Therefore, it ignores inertial effects, ice internal stresses, and non-geostrophic wind turbulence.

Using daily averages of sea ice velocity and surface pressure obtained from this limited dataset, Thorndike and Colony (1982) showed that typical values of the ratio $|\mathbf{A}|$ and the turning angle θ are smaller than Nansen's estimate and vary with the season and the region of the Arctic, both parameters decreasing with increasing ice thickness and compactness, such as in winter and north of Greenland and the Canadian archipelago (from $A \approx 0.8 \%$ and $\theta \approx 5^\circ$ in winter, to $A \approx 1 \%$ and $\theta \approx 20^\circ$ in summer). These results, which can be qualitatively explained by the damping effect of internal stresses, were confirmed by Thomas (1999) from a larger dataset covering 15 years (1979–1993).

On the other hand, only 65–70 % of the variance of the ice velocity in the central Arctic is explained by the geostrophic wind, and even less within 400 km of the coasts or in peripheral seas (Thomas 1999; Thorndike and Colony 1982). The remaining variance could come from measurement error, small time-scale processes (including inertia), ocean current variability, or from the mechanical behavior of sea ice. Therefore, at this stage, several questions arise:

- (i) Equation 2.3 considers a mean velocity field, which is generally calculated from simple averaging of velocity data at some arbitrary spatial scale (large enough to ensure a significant number of observations over each grid cell) and over the duration of the entire dataset. Are these averaging scales appropriate? Which physical processes are embodied in this mean circulation?
- (ii) What is the physical origin of the remaining variance? Can we see the fingerprint of oceanic or atmospheric turbulence, or of ice internal stresses, in these fluctuations?

As shown below, proper statistical and scaling analyses are needed to answer these questions.

2.1 Data

There are basically two sources of sea ice drift data: Lagrangian trajectories of buoys and drifting stations, and satellite-derived velocities. Both have advantages and disadvantages.

In the early days of Arctic exploration, positions of drifting stations or ships were obtained from astronomical measurements (star shots during nighttime, sun shots during daylight), with daily temporal sampling at best and an accuracy of ~ 1 km (Nansen 1902; Reed and Campbell 1962). Since the end of the 1970s, automatic ice drifters (buoys) have been launched every year within the Arctic basin, and more recently and irregularly around the Antarctic (e.g. Hutchings et al. 2012). In the Arctic, the corresponding database is maintained by the International Arctic Buoy Program (IABP), see Fig. 2.1. Until recently, the positioning system of the buoys was ARGOS, based on a Doppler shift effect, with an accuracy of 300–500 m (Thomas 1999; Thorndike 1986b). The raw IABP trajectories are then smoothed by cubic interpolation, a procedure that likely reduces the positioning standard error to 100–150 m (Lindsay and Stern 2003). The ice velocity $\mathbf{U}$ is then calculated from successive positions $\mathbf{X}$, $\mathbf{U}(\mathbf{X}, t) = \frac{\mathbf{X}(t+\tau) - \mathbf{X}(t)}{\tau}$, where τ is here the temporal sampling of the measure (the dependence of the estimate of $\mathbf{U}$ on time

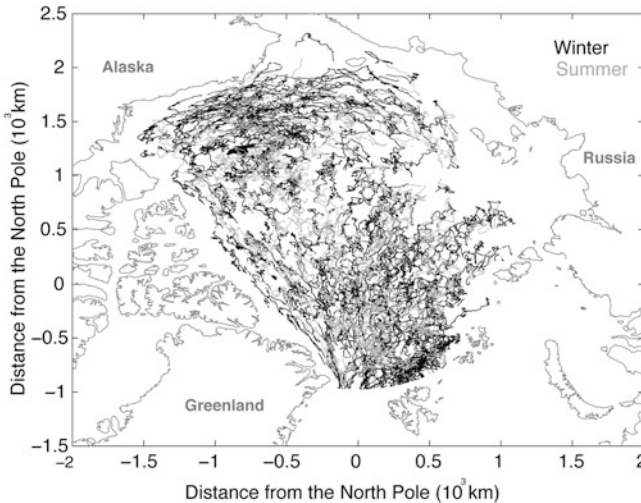


Fig. 2.1 IABP buoys trajectories recorded between December 1978 and December 2001 in the Arctic basin. The tortuous character of sea ice motion is obvious (adapted from Rampal et al. 2009b)

scale τ will be discussed below). Using a positioning error in the range 150–500 m and $\tau = 3$ h for the IABP data, one finds an accuracy of $\delta U = \frac{\sqrt{2}\delta x}{\tau} \approx 2\text{--}6$ cm/s for the ice speed. In the mid-1990s, GPS started to replace ARGOS as the positioning system for some buoys, with an improved accuracy (down to 20–30 m Heil et al. 2008).

Satellite estimates of ice velocity are based on several kinds of sensors, such as synthetic aperture radar (SAR), radar scatterometers, or microwave radiometers. The basic tool to estimate displacements, and so velocities, over the ice cover is based on area correlation techniques between successive images of the same field (e.g. Ezraty et al. 2006; Fily and Rothrock 1987). The principal limitation of such analysis is that it measures spatial lags, and therefore very poorly retrieves the rotational component of ice motion. More sophisticated algorithms based on a combination of area-correlation techniques and feature-matching techniques have been developed and applied to SAR imagery (Kwok et al. 1990), furnishing the RGPS dataset (Kwok 1998). This dataset gives Lagrangian tracking of more than 40000 points over the Arctic basin, separated on average by 10 km.

The velocity accuracy obtained from classical correlation techniques applied to radiometers or SAR images ranges from several mm/s to several cm/s, for a temporal sampling of 2–3 days and a spatial resolution varying from 5 to 60 km, depending on the sensor/satellite (Rozman et al. 2011). The RADARSAT dataset shows better results, with a position accuracy of ~ 300 m (Lindsay and Stern 2003). This gives a velocity error of ~ 2 mm/s, but still for a temporal sampling of 3 days on average.

From these numbers, one realizes that satellites are unrivaled for the analysis of sea ice drift patterns with regular spatial and temporal sampling, but are strongly limited in terms of time resolution, i.e. they cannot be used to explore velocity fluctuations at “small” time scales. In this case, the analysis of buoy trajectories is required, at the cost of irregular spatial sampling.

2.2 How to Extract a Mean Field

We start here with a quotation from Thorndike (1986a): “The motion of a particle of sea ice can be partitioned into a predictable component, associated with the long-term average wind and ocean currents, and a random part associated with the short-term fluctuations in the wind and current”. This suggests (i) the existence of an Arctic general circulation (AGC), and (ii) that the remaining fluctuating part of the motion simply results from oceanic and atmospheric turbulence.

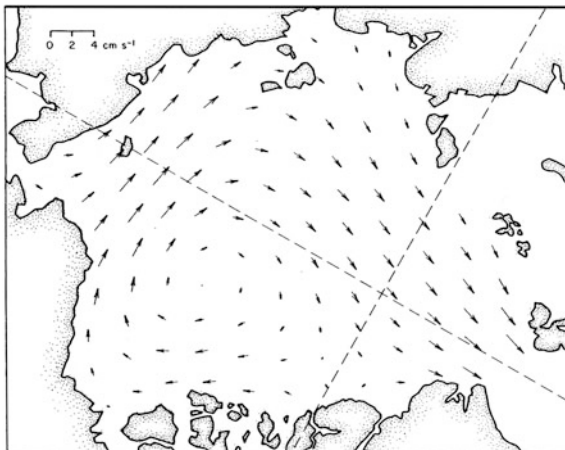
Such a general circulation was already postulated by Nansen, and the Fram’s journey confirmed his guess about a transpolar current. Almost one century later, Colony and Thorndike (1984) published the first estimate of the AGC from a compilation of trajectories, from the Fram’s cruise to the first trajectories of automatic ice drifters (1979–1982). The main drawback of this dataset was its very

large spatial and temporal sampling variability. Therefore, Colony and Thorndike (1984) used an optimal linear interpolation procedure to spatially smooth the sparse data, and calculated a mean field for a time interval spanning 90 years. The resulting field shows two distinct features (Fig. 2.2): an anti-cyclonic circulation in the western Arctic, the so-called Beaufort Gyre, and the transpolar drift in the Eurasian basin. This mean field was then considered as the AGC, linked to first order to the large-scale atmospheric circulation (Colony and Thorndike 1984). The role of this circulation on the mass balance of Arctic sea ice, and so on its long-term evolution, is essential, especially through the sea ice export across Fram Strait (e.g. Kwok and Rothrock 1999; Kwok et al. 2004; Rampal et al. 2011). Consequently, many authors discussed a possible correlation between the inter-annual variability of atmospheric circulation and the evolution of sea ice concentration over the Arctic (e.g. Maslanik et al. 2007; Rigor et al. 2002). In so doing, they implicitly assumed that the AGC depicted in Fig. 2.2 might be affected by large-scale, long-term variability. In other words, are the averaging scales used by Colony and Thorndike (1984) to extract their mean field (90 years and ~ 200 km) appropriate? Indeed, if these averaging scales are too large, homogenization will be too strong and small-scale/short-term variability of the mean circulation will be lost. Conversely, if they are too small, the AGC will include a stochastic component, biasing the analysis of velocity fluctuations as well as their causes.

To define averaging scales on a physical basis, Rampal et al. (2009b) developed an approach inspired from the study of fluid turbulence and its associated diffusion properties. Such an analogy was already suggested by Thorndike (1986a), and is based on a decomposition of the velocity field into a predictable (in a deterministic sense) mean “flow” and its fluctuations—the so-called “Reynolds decomposition”.

The starting point is the turbulent diffusion theory of (Taylor 1921), who showed that in the case of steady and homogeneous turbulence without mean flow, the diffusion of single particles through time, measured by the variance of the distance from the origin $\langle (X'(t) - X'(0))^2 \rangle = \langle r'^2(t) \rangle$, is linked to the variance of

Fig. 2.2 An estimate of the mean field of Arctic sea ice motion obtained by Colony and Thorndike (1984) from an interpolation of various manned research stations or buoy trajectories over the period 1893–1983



the speed fluctuation $\langle U'^2(t) \rangle$ and the normalized turbulent autocorrelation function $C(\tau)$ through:

$$\langle r'^2(t) \rangle = 2\Gamma \langle U'^2(t) \rangle \int_0^t C(\tau) d\tau \quad (2.4)$$

where Γ is the so-called integral time (Taylor 1921). $C(\tau)$ is defined as:

$$C(\tau) = \frac{1}{\langle U'^2 \rangle t_{\max}} \int_0^{t_{\max}} \mathbf{U}'(t) \mathbf{U}'(t + \tau) dt \quad (2.5)$$

where $U' = U - \langle U \rangle$ is the velocity fluctuation and $t_{\max}$ the duration of data coverage. $C(\tau)$ decreases theoretically with increasing time lag, e.g. following an exponential decay, as the memory of previous displacements is lost beyond the characteristic memory time $\Gamma = \int_0^\infty C(\tau) d\tau$. This fundamental property can be used to check whether the mean flow has been correctly removed. Indeed, if the mean flow is not properly removed, residual autocorrelation will remain for very large time scales, a signature of its predictable character in the deterministic sense. Conversely, if the scales of averaging of the mean flow are too small, $C(\tau)$ becomes negative for intermediate time lags, as a random part of the motion is artificially included in the mean flow, and therefore removed from the velocity fluctuation field.

This methodology was applied to Arctic sea ice velocity by (Rampal et al. 2009b) from the IABP ice drifters dataset (Fig. 2.3). These authors calculated a velocity fluctuation field using variable time (T) and space (L) averaging scales for the mean circulation. To estimate the integral time $\Gamma^{L,T}$ corresponding to each pair of averaging scales, they integrated the autocorrelation function, averaged over all the buoy trajectories, up to the time of first zero crossing, which gives an upper bound on the true scale (Poulain and Niiler 1990) (see Rampal et al. 2009b for more technical details). Figure 2.3, corresponding to winter (November to May), shows that, as expected, when no mean field is removed, some autocorrelation remains after 20 days. A similar result was obtained previously by Thorndike (1986b) from a limited dataset. For $L < 400$ km and $T < 160$ days, the autocorrelation function shows negative values until approximately 20 days, and consequently $\Gamma^{L,T}$ underestimates the genuine integral time. $\Gamma^{L,T}$ increases with increasing L and T and stabilizes around 1.5 days for $L = 400$ km and $T = 160$ days, for which the autocorrelation function follows the theoretical “turbulent” behavior (Fig. 2.3). This integral time scale is similar to that found for isopycnal oceanic turbulence (Zhang et al. 2001).

We can therefore conclude that 400 km and 160 days are the appropriate averaging scales to define an AGC for sea ice in winter. A similar analysis performed for summer trajectories gives averaging scales of 200 km and 80 days, for

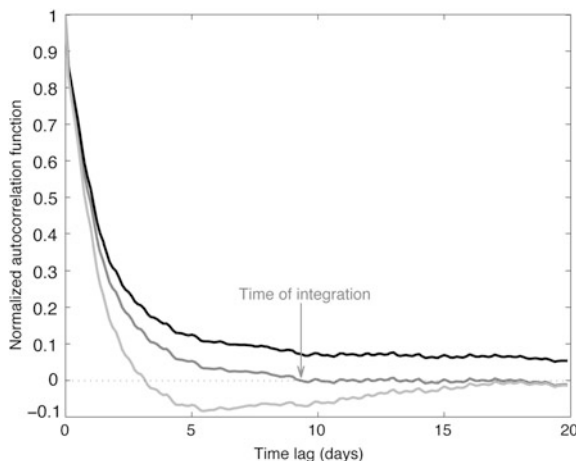


Fig. 2.3 Normalized autocorrelation functions of sea ice velocity fluctuations in winter. These velocities are obtained after removing a mean velocity field estimated from different spatial and temporal averaging scales. *Black curve*: no mean field removed. *Light gray*: mean field calculated for $L = 50$ km and $T = 40$ days. *Dark gray*: mean field calculated for $L = 400$ km and $T = 160$ days. The “time of integration” t_i indicates the time over which, in practice, the autocorrelation function $C(\tau)$ is integrated to estimate the integral time: $\Gamma = \int_0^{t_i} C(\tau) d\tau$ (from Rampal et al. 2009b)

an integral time of 1.3 days, slightly below the winter value (Rampal et al. 2009b). It is probably not fortuitous that the appropriate time scales correspond roughly in both cases to—actually are slightly below—the duration of one season, highlighting a seasonal variability of sea ice general circulation. These averaging scales also indicate an interannual variability of the AGC (Fig. 2.4), meaning that Colony and Thorndike’s map of Fig. 2.2 represents an oversimplification. As an example, cyclonic circulation can be observed instead of the Beaufort Gyre (Fig. 2.4e). The only apparently persistent feature is the transpolar drift, although its magnitude and precise direction relative to Fram Strait may change from one year or season to another (compare Fig. 2.4a and b), with probable consequences in terms of sea ice export.

This variability of the AGC is likely associated with a change of forcing, as atmospheric and/or oceanic conditions exhibit a similar seasonal and interannual variability (e.g. Hurrell 1995; Thompson and Wallace 1998). It may have a strong impact on sea ice export and mass balance (see Conclusion and Perspectives, and Rampal et al. 2011). As an example, it has been argued that atmospheric circulation patterns favoring sea ice export to the northern Atlantic may have triggered the recent record lows of summer sea ice extent (Wang et al. 2009). However, the analysis presented above demonstrates that any correlation analysis between atmospheric circulation indexes, the sea ice AGC, and sea ice mass balance, should be performed using a sea ice circulation determined from the appropriate temporal and spatial scales, in order to remove any random fluctuation from the analysis.

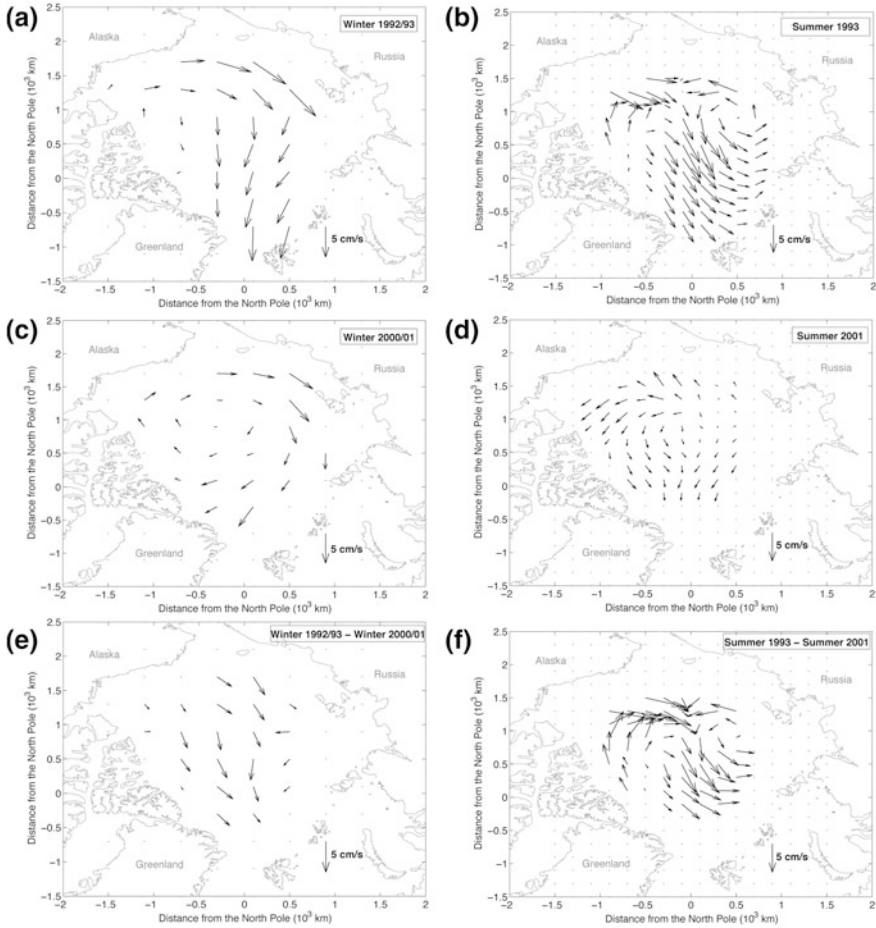


Fig. 2.4 Arctic general circulation (AGC) calculated from a non-arbitrary decomposition of the sea ice velocity field (see text and Rampal et al. 2009b for details) for different years and seasons. Compare with Colony and Thorndike’s map of Fig. 2.2 (from Rampal et al. 2009b)

2.3 Diffusion Regimes

Once a general circulation has been determined at the appropriate scales, it can be removed from the velocity field in order to explore the fluctuating part of sea ice motion. The aim of this section is to analyze the statistical properties of these fluctuations in order to understand their physical origin, and particularly to check whether we can retrieve the fingerprint of oceanic or atmospheric turbulence, as suggested by Thorndike (1986a), and/or of another mechanism such as the intrinsic mechanical behavior of sea ice through the effect of internal stresses. To do so, it is interesting to build analogies with the Lagrangian statistics of fluid turbulence (e.g. Arneodo et al. 2008; Mordant et al. 2003).

The preliminary step, however, is to answer a question already formulated by Thorndike (1986b): Can a sea ice velocity be defined? The answer is not as trivial as it appears. Obviously, depending on the sampling interval of the trajectories, τ , it is always possible to define a velocity measured at this time scale, $\mathbf{U}'(\mathbf{X}, \tau) = \frac{\mathbf{X}(t+\tau) - \mathbf{X}(t)}{\tau} - \langle \mathbf{U} \rangle$ (we consider velocity fluctuations $\mathbf{U}'$ in this section). The pertinent question is to check whether a limit $\mathbf{U}'(\mathbf{X}, \tau \rightarrow 0) = \mathbf{U}'(\mathbf{X})$ exists and is approached for available time intervals, i.e. those longer than the sampling interval (3 h for the IABP data). The answer is related to the diffusion regimes of sea ice with regard to the turbulent diffusion theory of Taylor (1921) introduced in the previous section. From Eq. (2.4) and the definition of the integral time $\Gamma = \int_0^\infty C(\tau) d\tau$, it can be shown that particle diffusion $\langle r^2(t) \rangle$ has two asymptotic regimes. For $t \ll \Gamma$, particle diffusion scales as $\langle r^2(t) \rangle \sim t^2$: this is the so-called “ballistic regime” for which particles keep memory of previous velocity characteristics (magnitude as well as direction). For $t \gg \Gamma$, a regime corresponding to vanishing velocity correlations and “random walk” motion takes place, giving $\langle r^2(t) \rangle \sim t$, as for molecular diffusion.

A condition for the convergence of $\mathbf{U}'(\mathbf{X}, \tau \rightarrow 0)$ is that the associated variance $\langle (\mathbf{U}'(\mathbf{X}, \tau \rightarrow 0))^2 \rangle$ converges as well (Thorndike 1986b). As $\langle \mathbf{U}'^2 \rangle \sim \langle r^2(t) \rangle / t^2$, this condition consists merely of the existence of a ballistic regime for sea ice—in simpler words, a velocity is well defined for a ballistic trajectory. Figure 2.5, extracted from (Rampal et al. 2009b), shows sea ice diffusive displacements r'_x and r'_y (along the x and y directions of a Cartesian coordinate system centered on the North Pole with the y axis following the Greenwich meridian) caused by the velocity fluctuation field, in winter and summer. The symmetry about the zero line indicates that the mean circulation was correctly determined and removed (see Sect. 2.2). The shape of the envelope of the buoy trajectories qualitatively follows Taylor’s diffusion predictions, with a rapid growth during the first few days followed by a slowing down. This is quantitatively confirmed in Fig. 2.6 where the two diffusion regimes are clearly identified, with a crossover at the integral time scale $\Gamma \approx 1.5$ days. This implies that a true velocity can be defined for sea ice, and is correctly estimated at the IABP time resolution of 3 h, as already suggested by Thorndike (1986b).

In their global analysis, Rampal et al. (2009b) did not distinguish between zonal and meridional components of motion. Such distinction was made by Lukovich et al. (2011) in a regional study of sea ice drift on the seasonal sea ice zone of the southern Beaufort Sea, north of Alaska, in winter. From this limited dataset (20 buoys), these authors reported two distinct diffusion regimes, a ballistic regime in t^2 for zonal motion, and a possible “intermediate” regime in $t^{5/4}$ for meridional motion along the so-called circumpolar flaw lead, both observed over a time range of 2–200 days. From an autocorrelation analysis of sea ice speed, they obtained an integral time scale Γ slightly larger for zonal motion (1.2 days) than for meridional motion (0.7 days). They interpreted the ballistic zonal motion observed at

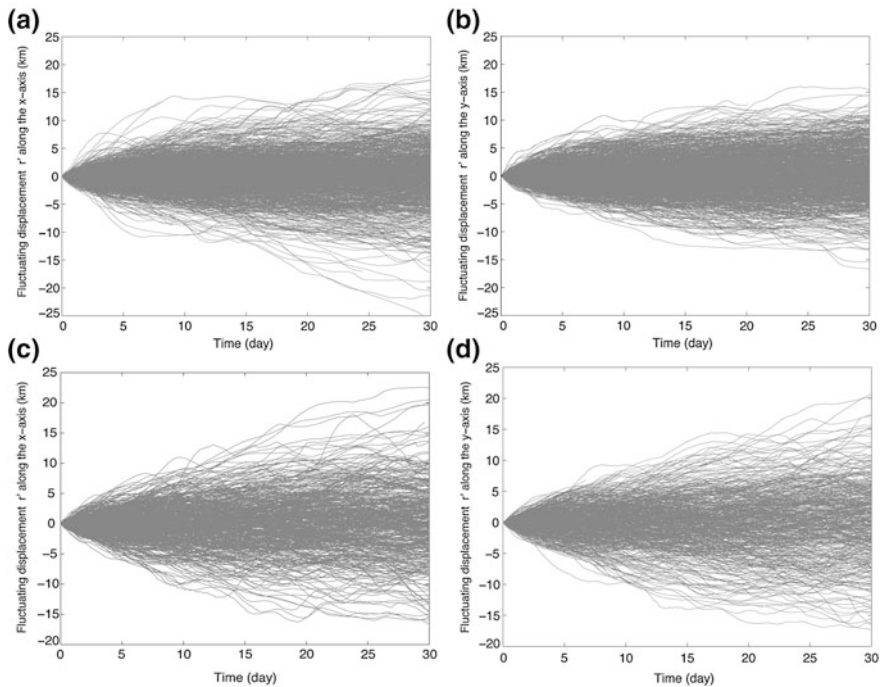
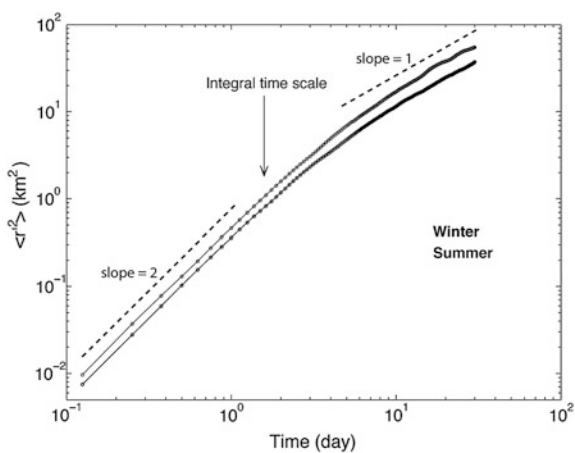


Fig. 2.5 Buoy displacements r'_x (a and c) and r'_y (b and d) caused by the sea ice velocity fluctuation field for winter (a and b) and summer (c and d). This illustrates the diffusion regimes of sea ice: a “rapid” initial ballistic regime ($\langle r'^2(t) \rangle^{1/2} \sim t$) followed by a molecular diffusion-like regime ($\langle r'^2(t) \rangle^{1/2} \sim t^{1/2}$) (from Rampal et al. 2009b)

Fig. 2.6 Diffusion regimes of sea ice motion fluctuations obtained from the time scaling of the variance $\langle r'^2(t) \rangle$ of displacement fluctuations r' . The initial ballistic regime $\langle r'^2(t) \rangle \sim t^2$ is obtained for $t < \Gamma = 1.5$ days, whereas a random walk regime is obtained for $t > \Gamma$. (from Rampal et al. 2009b)



timescales much longer than the corresponding integral timescale as a result of advection-dominated flow, but one may question whether the mean field was correctly removed (see Sect. 2.2). Following Elhmaidi et al. (1993), they argued that the $\langle r^2(t) \rangle \sim t^{5/4}$ anomalous meridional diffusion could be a signature of a deformation-dominated (shear and divergence) regime. Although the distinction between a $t^{5/4}$ and a classical linear regime is probably difficult to detect with a limited dataset, the possibility of such anomalous diffusion for sea ice would be worth pursuing in the future for a better understanding of the underlying processes.

2.4 Turbulent-Like Fluctuations

Once we know that the velocity fluctuations are correctly measured from ice drifter trajectories, and the diffusion regimes determined, further statistical analyses can be performed, and analogies with fluid turbulence pursued. The classical turbulent diffusion theory assumes a Gaussian distribution of velocities (Batchelor 1960; Frisch 1995; Taylor 1921). Eulerian wind velocities (i.e. the velocities “felt” by an ice parcel moving at a much lower speed than the wind speed) are Gaussian (Fig. 2.7). Lagrangian oceanic velocities are also Gaussian, once the mean flow has been correctly removed (Swenson and Niiler 1996; Zhang et al. 2001), although some non-Gaussian deviations have been claimed (Bracco et al. 2000). If one assumes that sea ice velocity fluctuation statistics are a direct inheritance of oceanic and atmospheric turbulence (see the beginning of this chapter and Thorndike 1986a), one would expect Gaussian statistics for sea ice as well. The reality is significantly different (Rampal et al. 2009b): sea ice velocity PDFs are exponential instead of Gaussian, with a few outliers above 60 cm/s (Fig. 2.8). This means that sea ice velocities fluctuate more (are more spread out),

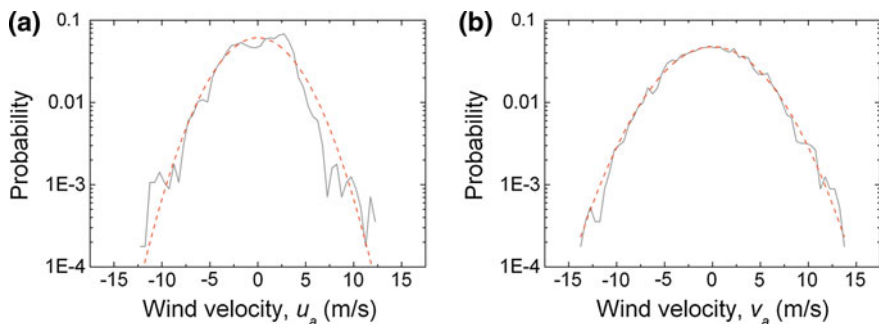


Fig. 2.7 Probability density functions (PDF) of the u_a (left) and v_a (right) components of the wind velocity vector $\mathbf{U}_a$ measured from November 1997 to September 1998 at an altitude of ~ 10 m on the meteorological tower of the SHEBA camp in the Beaufort Sea (see Persson et al. (2002) for more details on the measurements and Perovich et al. (1999) for the SHEBA program). The corresponding Gaussian distributions (red dashed lines) are given for comparison

in relative value, than wind or currents, and suggests that another term, namely sea ice mechanics and the internal stress, is playing a significant role.

To go further, the intermittency of the motion should be analyzed. This can be done through the analysis of the statistics of velocity fluctuation increments $\Delta_\tau \mathbf{U}' = \mathbf{U}'(t + \tau) - \mathbf{U}'(t)$ for different time scales τ (e.g. Mordant et al. 2003). In fluid turbulence, the PDFs of Lagrangian velocity increments continuously change as τ decreases within the inertial range of motion towards the Kolmogorov dissipative time scale τ_η . They are Gaussian at large time scales, above the integral time Γ , in agreement with Gaussian statistics of velocities and the absence of correlation between successive velocity values at these time scales. As τ decreases, the PDFs widen towards a stretched exponential form, $P(\mathcal{X}) \sim \exp\left(-\left(\frac{\mathcal{X}}{\mathcal{X}_0}\right)^\alpha\right)$ with $\alpha < 1$ (Mordant et al. 2003), in agreement with stretched exponential statistics for Lagrangian accelerations below τ_η (La Porta et al. 2001; Volk et al. 2008). Indeed, the acceleration estimated at time scale τ is given by $\left(\frac{\Delta \mathbf{U}'}{\tau}\right)$, and stabilizes below the dissipative time scale τ_η , i.e. a genuine acceleration is correctly approached for $\tau \ll \tau_\eta$. A similar analysis has been performed by Rampal et al. (2009b) for IABP velocity increments (Fig. 2.9). As for fluid turbulence, the PDF continuously changes as the time scale τ decreases, but the shapes of the distributions are significantly different. For $\tau > \Gamma$, the PDF is exponential and independent of τ , in agreement with exponential velocity statistics (see Fig. 2.8) and the absence of correlation between successive velocity values in this case. The variability of velocity increments increases towards small time scales ($\tau \ll \Gamma$), as the PDF develops power law tails that depart significantly from a stretched exponential form (Rampal et al. 2009b). In other words, the variability of sea ice velocity increments is even more extreme than that of fluid turbulence. This is another indication that the fluctuating part of sea ice motion is not (only) a direct consequence of wind or oceanic turbulence.

The widening of the velocity increment PDFs is an indication of a temporal “localization” of velocity fluctuations at short time scales, i.e. of intermittency.

Fig. 2.8 PDFs of the u' (circles) and v' (squares) components of the sea ice velocity fluctuation vector $\mathbf{U}'$. The PDFs are symmetrical and show an exponential form that departs significantly from the corresponding Gaussian distribution (plotted for winter as the gray dashed line). (from Rampal et al. 2009b)

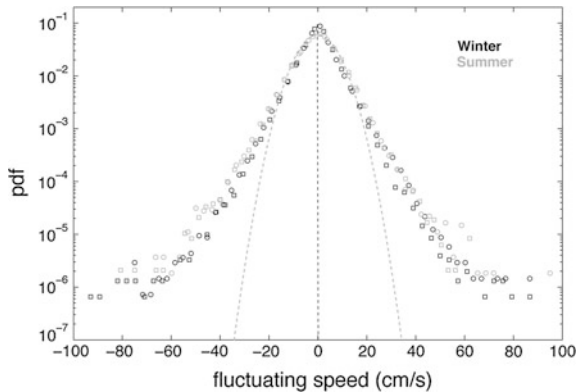
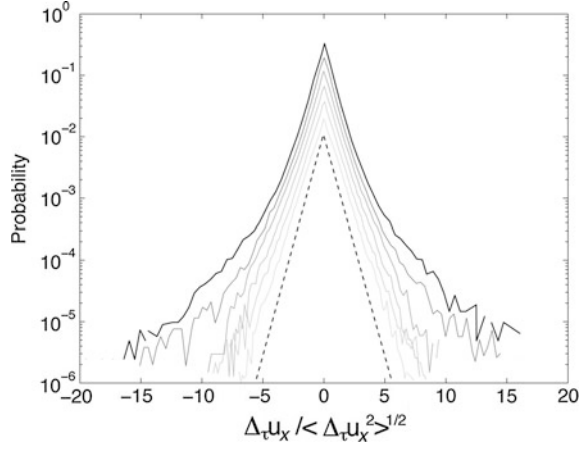


Fig. 2.9 PDFs of the sea ice velocity fluctuation increment $\Delta u'$ (x -component, normalized by its standard deviation) calculated over different time intervals τ . From top to bottom: $\tau = 0.125, 0.25, 0.5, 1, 2, 5$ days. The *dashed line* shows an exponential decay and is for reference only (from Rampal et al. 2009b)

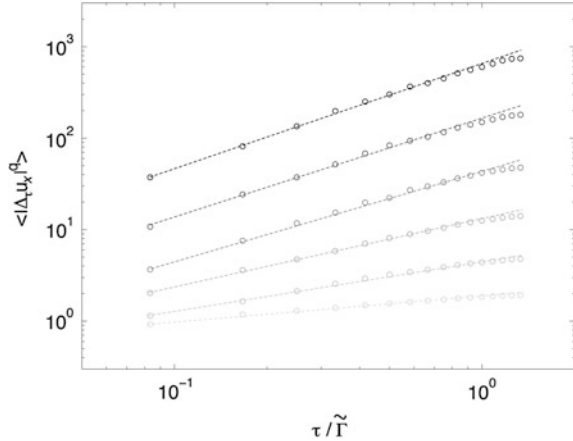


A quantification of this intermittency can be done from the scaling of the structure functions:

$$\langle \Delta_\tau U'^q \rangle \sim \tau^{\zeta(q)} \quad (2.6)$$

Figure 2.10 shows the structure functions $\langle |\Delta_\tau u'|^q \rangle$ of the y -component of Arctic sea ice fluctuating velocity, u' (similar results are obtained for the y -component, v'), for $0.5 \leq q \leq 3$. Scaling (Eq. 2.6) is observed from the time resolution of the IABP dataset (3 h) to $\tau \approx \Gamma$, which defines the upper bound of the so-called inertial range for turbulent flows. The slopes of Fig. 2.10 define the function $\zeta(q)$, which is non-linear (Rampal et al. 2009b). This nonlinearity is a signature of multifractality in the time domain, i.e. of intermittency, in agreement with the functional change in the PDFs (Fig. 2.9), and is remarkably well approximated by a quadratic fit $\zeta(q) = aq^2 + bq$ with $a = -0.09$ and $b = 0.65$.

Fig. 2.10 Lagrangian structure functions of the x -component of the sea ice fluctuating velocity for $q = 0.5, 1, 1.5, 2, 2.5$ and 3 for winter. The *dashed lines* are the least squares power law fits $\langle \Delta_\tau u'^q \rangle \sim \tau^{\zeta(q)}$, yielding the exponents $\zeta(q)$ (from Rampal et al. 2009b)



Multifractality is a well-known property of turbulent flows at high Reynolds numbers (Frisch 1995; Mordant et al. 2003; Schmitt et al. 1992). The results of Rampal et al. (2009b) show that it also occurs for a solid, the Arctic sea ice cover, although the distribution of velocity increments is even more spread out in the case of sea ice.

2.5 Sea Ice Acceleration and the Dynamical Origin of Intermittency

We have seen above that sea ice motion shares several properties with fluid turbulence, such as diffusion regimes and intermittency, i.e. sea ice velocity fluctuations are “turbulent-like”. On the other hand, we also noted important differences, particularly stronger variability and intermittency in the case of sea ice. As already stressed, this precludes a simple inheritance from atmospheric and/or oceanic turbulence, contrary to what Thorndike (1986a) suggested. This raises the question of a possible dynamical origin of this intermittency, through the sea ice momentum equation (Sect. 1.2) that links sea ice acceleration to wind forcing, ocean drag, and the ice internal stress field.

We are faced here with a question similar to the one asked at the beginning of Sect. 2.3 for velocity: Can a sea ice acceleration be defined from available measurements? Once again, Thorndike (1986b) already asked this question 25 years ago. As already mentioned above, ice acceleration is obtained from $\left(\frac{\Delta_\tau U}{\tau}\right)_{\tau \rightarrow 0}$, and

therefore the associated variance $\left\langle \left(\frac{\Delta_\tau U}{\tau}\right)^2 \right\rangle_{\tau \rightarrow 0}$ should stabilize above the time

resolution of the measurements in order to correctly estimate the acceleration. The analysis of the structure functions in Fig. 2.10 shows that

$\left\langle \left(\frac{\Delta_\tau U}{\tau}\right)^2 \right\rangle \sim \frac{1}{\tau^2} \langle \Delta_\tau U^2 \rangle \sim \tau^{\zeta(2)-2} = \tau^{-1.06}$, i.e. the variance of acceleration

increases towards short time scales down to the time resolution of the IABP data. In other words, the equivalent of the Kolmogorov dissipative time scale of fluid turbulence, τ_η , for sea ice motion is well below 3 h, with the consequence that ice acceleration is not correctly approached at this time resolution (note however that such acceleration is necessarily defined at sufficiently small time scales).

Nevertheless, from the analysis of velocity increment PDFs detailed in the preceding section, we expect a very strong intermittency of the true sea ice acceleration, with a PDF characterized by heavy, power law tails. Indeed, such heavy tails are already present at $\tau = 3$ h (Fig. 2.9) and should even widen below 3 h. This is in agreement with estimated accelerations obtained by Chmel et al. (2007) from the GPS trajectory of the North-Pole Station 32 sampled at a time resolution of 10 min. At such small time scale, these authors reported a power law distribution of accelerations, $P(A_i) \sim A_i^{-\alpha}$ with $\alpha \approx 4$.

Can such power law statistics be expected from simple arguments based on the momentum equation of sea ice (Eq. 1.1)? As discussed in Sect. 1.2, the main driving force of sea ice motion is the wind stress, which scales as $\tau_a \sim |\mathbf{U}_a| \mathbf{U}_a$. Wind speeds are normally distributed (Fig. 2.7 and Frisch 1995), and if a random variable $\mathcal{X}$ follows a standard normal distribution, the variable $\mathcal{Y} = |\mathcal{X}| \mathcal{X}$ will follow a χ^2 distribution with one degree of freedom, with a PDF given by:

$$P(\mathcal{Y}) = \frac{1}{\sqrt{2\pi}|\mathcal{Y}|} \exp\left(-\frac{|\mathcal{Y}|}{2}\right) \quad (2.7)$$

This distribution exhibits exponential tails: while more spread out than Gaussian statistics, the distribution of wind stresses cannot explain the extreme variability of sea ice accelerations. Once again, another source of variability and intermittency should be sought.

As shown above, the intermittency of sea ice kinematics implies that an estimate of acceleration, $\left(\frac{\Delta \mathbf{U}'}{\tau}\right)$, increases with decreasing time scale τ , and that the true value cannot be approached from buoy position measurements with a sampling frequency of a few hours. This raises the interesting question of the importance of the inertial forces associated with acceleration in the momentum balance of the ice (Thorndike 1986b). It is generally believed that these forces are very small compared to the other terms of the force balance (e.g. Coon 1980; Hibler 1984; Leppäranta 2005), thus suggesting the use of a “quasi-static” approximation of (Eq. 1.1) to estimate the amplitude of the internal stress term as a residual of the force balance (e.g. Rothrock et al. 1980). This assumption is valid when considering large time scales (>1 day), i.e. it is relevant for most applications and modeling issues. However, assuming that the scaling properties of sea ice velocity increments detailed in the previous section can be extrapolated below $\tau = 3$ h, at which time scale will the inertial forces become comparable to the other terms? To estimate this, the statistical and scaling properties of the ice velocity fluctuation field can be used. Using a mean speed $\langle U'_i \rangle = 7.8 \text{ cm}\cdot\text{s}^{-1}$ (Rampal et al. 2009b), $f \approx 1.4 \times 10^{-4} \text{ s}^{-1}$, a thickness $h = 1 \text{ m}$ and $\rho_i = 910 \text{ kg}\cdot\text{m}^{-3}$ (Timco and Frederking 1996), one finds a Coriolis “force” (per unit area) $\rho h f \langle U'_i \rangle \approx 10^{-2} \text{ Pa}$. A typical average surface wind speed $\langle U_a \rangle$ is $5 \text{ m}\cdot\text{s}^{-1}$, although values larger than $10 \text{ m}\cdot\text{s}^{-1}$ are observed (see Fig. 2.7). This leads to a wind stress $\tau_a = \rho_a C_a \langle U_a \rangle^2 \times \approx 5 \cdot 10^{-2} \text{ Pa}$, taking $\rho_a = 1.3 \text{ kg}\cdot\text{m}^{-3}$ and $C_a = 1.5 \times 10^{-3}$ (Leppäranta 2005). For the IABP time resolution $\tau = 3 \text{ h}$, a typical incremental velocity $\left\langle (\Delta U')^2 \right\rangle_{\tau=3h}^{1/2}$ is about $2 \text{ cm}\cdot\text{s}^{-1}$, giving an inertial “force” term of $\rho h \frac{\langle (\Delta U')^2 \rangle_{\tau}^{1/2}}{\tau} \approx 1.7 \times 10^{-3} \text{ Pa}$. This is still well below the wind stress or the Coriolis term, in agreement with the quasi-static assumption noted above. However, if one extrapolates the scaling $\frac{1}{\tau} \langle \Delta U'^2 \rangle_{\tau}^{1/2} \sim \tau^{-0.53}$ below 3 h, one finds that the inertial force becomes comparable to the Coriolis term for a time scale $\tau \approx 400 \text{ s}$, and to the wind stress for $\tau \approx 20 \text{ s}$. The question that arises is

whether the multifractality of sea ice motion indeed holds down to such small time scales, or if a lower bound to the inertial range of sea ice motion, corresponding to some equivalent of the Kolmogorov time scale of dissipation, is attained before. Owing to the propagation of errors when estimating accelerations from buoy positions, it is questionable whether one could explore this issue from such data. Other kinds of field instrumentation, such as seismometers (e.g. Marsan et al. 2011) or accelerometers, might be an interesting alternative in the future.

2.6 Spectral Analysis

So far, we have focused on sea ice kinematics in the time domain. A complementary approach is a spectral analysis in the frequency domain. From a linear theoretical analysis of free drift, i.e. using a linearized drag coefficient, Leppäranta et al. (2012) recently showed that the Lagrangian power spectrum of sea ice velocity $p_i(\omega)$ is expressed in this ideal case as the combination of the power spectrum of the forcing $p_F(\omega)$ and of a modulation factor $a(\omega, f)$ that can be understood as the sea ice spectral response to white noise, $p_i(\omega) = a(\omega, f)p_F(\omega)$, where ω is the frequency. This model exhibits a singularity at the Coriolis frequency $\omega = -f$, a signature of inertial oscillations. The modulation factor tends towards a constant as $\omega \rightarrow 0$ and is essentially flat for $\omega < 0.38 \text{ cycles} \cdot \text{day}^{-1}$, meaning that the sea ice velocity spectrum simply follows the forcing spectrum in this range. Towards high frequencies, and typically for $\omega > 1 \text{ cycle} \cdot \text{h}^{-1}$, $a(\omega, f) \sim \omega^{-2}$ (Leppäranta et al. 2012).

Rampal et al. (2009b) calculated the Lagrangian power spectrum of the IABP velocity fluctuations $\mathbf{U}_i'$ from a Fourier transform of the autocorrelation function (see Sect. 2.2). Obviously, the time resolution of the IABP dataset does not allow one to explore the high frequency range. Rampal et al. (2009b) reported a Kolmogorov-like scaling $p_i(\omega) \sim \omega^{-2}$ over the frequency range $0.1\text{--}2 \text{ cycle} \cdot \text{day}^{-1}$. For lower frequencies corresponding to time scales significantly larger than the integral time scale Γ , the scaling falls off, in agreement with the white noise character of the sea ice velocity in this case (see Sect. 2.3), whereas a clear inertial peak is observed at $\omega = -f \approx 2 \text{ cycles} \cdot \text{day}^{-1}$. Sea ice inertial oscillations have been reported and studied by many authors (e.g. Colony and Thorndike 1980; Heil and Hibler 2002; Hunkins 1967; McPhee 1978; Thorndike 1986b). The associated inertial frequency peak is less pronounced in winter and/or within the central pack, compared to summer or peripheral zones of the Arctic basin, a signature of damping of these oscillations by ice internal stresses (Gimbert et al. 2012b).

The $p_i(\omega) \sim \omega^{-2}$ scaling reported by Rampal et al. (2009b) cannot be explained entirely by the high frequency asymptotic behavior of the free drift model of Leppäranta et al. (2012), as it extends over much lower frequencies. It might be instead a signature of the forcing spectrum. Indeed, such ω^{-2} Lagrangian scaling has been observed for oceanic (Lien et al. 1998) and atmospheric turbulence

(Gifford 1956; Hanna 1981). In this respect, the free drift model that predicts $p_i(\omega) = a \times p_F(\omega)$ for $\omega < 0.38 \text{ cycle-day}^{-1}$, where a is a constant, gives a correct description of sea ice drift in the spectral domain, and Thorndike's proposition of a fluctuating part of ice motion simply resulting from oceanic and atmospheric turbulence would appear reasonable. Such spectral analysis furnishes however only a limited view of the complexity of sea ice motion. Indeed, from the $p_i(\omega) \sim \omega^{-2}$ scaling, we expect that the second order structure function scales as $\langle \Delta_\tau U^2 \rangle \sim \tau$ (Biferale et al. 2006; Mordant et al. 2003), in agreement with the value $\zeta(2) = 0.95$ (see Rampal et al. 2009b) and Sect. 2.4), but the power spectrum scaling tells nothing about the multifractality of velocity fluctuations that fully characterizes the statistics.

Finally, we note that the Lagrangian spectrum of IABP sea ice velocity increments calculated at $\tau = 3 \text{ h}$ is flat, whereas Chmel and Smirnov (2007) reported a correlation time scale for their NP-32 acceleration time series of a few tens of minutes at most. This means that the fluctuating part of sea ice motion can be considered as a Markov process down to a (still unresolved) dissipative time scale that is much shorter than 1 h.

2.7 Concluding Remarks

In this chapter, we have explored the statistical and scaling properties of sea ice drift from buoy trajectories (Lagrangian tracers). The first important conclusion is that the velocity field of sea ice motion can be decomposed non-arbitrarily into a mean field and turbulent-like fluctuations. The properties of the remaining velocity fluctuation field have been explored in the time and frequency domains, in which they share similarities with fluid turbulence, such as a Kolmogorov-like scaling of the power spectral density, the presence of a ballistic regime within an inertial range of motion (whose lower bound is however not yet resolved), and the presence of intermittency. This may suggest a rather direct inheritance from the properties of atmospheric and oceanic turbulence, i.e. a linear response of sea ice to the forcing fields. However, a closer look indicates significant differences between geophysical fluid turbulence and sea ice motion: velocity PDFs are clearly not Gaussian, with much more spread; intermittency is more pronounced; and the distributions of sea ice accelerations cannot be explained by wind stress statistics. These differences are likely the signature of a non-linear response of sea ice to the forcing fields.

The kinematic response of a solid to forces and stresses occurs through a rheology F linking internal stress σ to strain ε and/or strain-rate $\dot{\varepsilon}$, $\sigma = F(\varepsilon, \dot{\varepsilon})$. If the properties detailed in this chapter suggest a non-linear character for F , an analysis of sea ice strains and strain-rates, i.e. velocity fluctuations and gradients in the spatial domain, is essential to go further.

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