

Chapter 2

Transition to the 20th Century

Abstract Chapter Two deals with the transition period between circa 1880 and 1914, which prepares the way for the Twentieth century. It also advocates an attitude towards a development that is characteristic of a period when many engineering scientists believe in a then fixed paradigm and no further evolution is thought possible in spite of a contemporary revolution in theoretical and mathematical physics. Of course this corresponds to a period of natural consolidation with the general creation of efficient engineering schools all over Europe and the appearance of newborn ones in the USA. Of particular interest in this rather quiet landscape are queries concerning going beyond the most traditional behaviours (linear elasticity and Newtonian viscous fluids). Here are distinguished the emerging attempts at the description of more involved behaviours such as viscoelasticity (Voigt, Boltzmann, Volterra), and friction and plasticity (Tresca, Barré de Saint-Venant, Lévy, Huber, Mises). In spite of the relative quietness of the period, new interests of investigation are considered, mainly in the dynamic frame, the consideration of continua with internal degrees of freedom (Duhem, the Cosserat brothers), and elements of homogenization theory. Perhaps more attractive at the time were the discussions about the general principles of mechanics by people like Hertz, Mach, Duhem (with his general energetics), Poincaré, Hamel and Hellinger. This pondering will prove extremely useful in the second half of the Twentieth century.

2.1 Setting the Stage

We consider the period extending between circa 1880 and 1914. The French call it the “Belle époque”. For the Germans and the Austrians it was “der guten alten Zeit”. Parodying a well known English writer, we could also say that “it was the best of times”, but the “worst of times” was to come soon. Indeed, Queen Victoria was ruling over an Empire that never saw the sun going down; Britannia ruled the

world with the largest fleet ever. Victoria's son Edward was going to succeed her after enjoying life in Paris where the "French cancan" was being illustrated by the most fashionable painters. Oxford and Cambridge still were the best universities in the United Kingdom, if not in the world. Victoria's family was reigning in many countries in Europe. The Kaiser was taking care of a powerful industrial country where *Technische Hochschulen* had replaced the Polytechnic schools (with a much too French sounding name, according to Prussian philology). St Petersburg, also ruled by one of Victoria's family relation, had a strong Polytechnic Institute. The Austro-Hungarian Empire was ruled by a benevolent ageing emperor. Technical universities existed in all parts of this empire, whether in Austria (Vienna) itself but also in Hungary (Budapest) and Galicia (Krakow, Lvov).

Italy was a rather young kingdom but with old universities—among the oldest in the world (Ferrara, Bologna) -, but also with two Polytechnic schools in Torino and Milano and a *Scuola Normale Superiore* in Pisa, a remnant of French (Napoleonic) influence. Switzerland had his two polytechnic institutions in Zürich (German speaking) and Lausanne (French speaking), both created by alumni from French "grandes écoles". As to France, living in its third republic and thinking about a possible revenge against the Prussians who had taken over Alsace and Lorraine in a brief war in 1870–1871, it was extending its colonial empire by imitation of the British while having instituted a charge free education at almost all levels. But it kept the formation of its elite in the "grandes écoles" such as the *Ecole Normale Supérieure* and the *Ecole Polytechnique* and its engineering schools of application (example of so formed scientist: Henri Poincaré; See [Chap. 7](#)). The country was working towards an exemplary full laicity, separating the Church and the State. Marcelin Berthelot—a thermo-chemist—epitomized the hero of republican science opposite to the clerical "reaction" identified with Pierre Duhem, while Georges Clemenceau—the future French Prime Minister victor of WWI—was representing the radical atheist left on the political spectrum.

The Chinese Empire was soon to suffer mortal attacks from its own socialists. European countries had succeeded to unite against some of the Chinese in Peking. Japan was accommodating European and American culture and creating universities and schools as imitations of those in these two parts of the world. But its industry was growing, having transformed its shoguns into industrialists, and creating a network of universities to replace the former scarce teaching of "Hollandish studies" (as European science was denominated). Moreover, Japan could now defeat countries like Russia in a military confrontation.

Finally, the United States, recovering from a painful civil war but the beneficiaries of an important immigration from European countries (Ireland, Germany, Russia, Italy, Sweden, etc.), were building an enormous industrial potential where the automobile would soon become one of the main output. With Edison, Tesla, Pupin, Bell and others, electricity and telephone had left the laboratory to become essential elements in everyday life. Simultaneously, John D. Rockefeller was building his immense fortune with the exploitation of oil fields, while moguls from steel industry (Andrew Carnegie) and transcontinental railroads (Leland Stanford) made huge donations that contributed to the creation of new teaching institutions

(Carnegie Technical Schools, CALTECH, Stanford University)—that were going to play an important role in the development of mechanical engineering in the 20th century. Still the USA were not exactly part of the “concert” of the nations, although they were also involved in wars and occupations that now look quite colonial in character (Cuba, Mexico, Puerto Rico, Hawaii, the Philippines). Concerning higher education, while the Ivy League colleges (Harvard, Yale, Princeton, etc.) remained the most powerful institutions for the formation of the elite, the Massachusetts Institute of Technology, created in 1866, really opened engineering sections in the 1880s, and “Agricultural and Mechanical (A&M)” colleges were being founded in various states (Texas, Virginia, Ohio, etc.) for the training of technicians.

This sets the stage for new developments that are more international in nature, refer more than before to a real behaviour of materials in their mechanical response, and require a deeper thinking about the bases of the theory of continua, and mechanics in general. The following three sections are devoted to these aspects. Moreover, following the French physicist Léon Lecornu who writes in 1918, we can distinguish between “rational mechanics” (a pure construct of the mind), “physical mechanics” (based on observation and experiments) and “applied mechanics”, the later in fact meaning “engineering mechanics”. We shall use this denomination.

2.2 Describing More Real Mechanical Behaviours

According to Lecornu (1918), more realistic mechanical behaviours primarily come into the picture via observation and experiments. In agreement with this remark we single out the behaviours of friction, plasticity and visco-elasticity. All these have for main property to be related to dissipation.

2.2.1 Friction

It is Charles Augustin de Coulomb (1736–1806), a former student of the (military) “*Ecole du Génie*” of Mézières in France, a pioneer in geotechnical engineering, who created the *science of friction* in the 18th century. He did that at a time when the notion of vector did not exist so that we let the reader imagine the difficulties (still present with our students) met with questions of signs. This does not relate to continua, but still it may provide some constructive idea about the behaviour known as (perfect) plasticity. Furthermore, everyone experiences the production of heat wherever friction is in action. But Coulomb and his contemporaries did not have any knowledge about thermo-dynamics. As a matter of fact, we believe that a correct inclusion of the phenomenon of friction in irreversible thermodynamics had to await the second part of the 20th century to find a satisfactory formulation.

2.2.2 Plasticity

Here also it was soon realized that in testing elastic materials to a higher mechanical load a kind of limit—the elastic limit—appeared after which one could hardly control the deformation. In 1D one simply reaches a level, say σ_0 , of stress (force per unit area of the section of the sample), at which one loses the control of the elongation. We now say that we observe *plastic flow*, while mathematically we formalize this by saying that we lose the uniqueness in the response in deformation. But real materials are three-dimensional, the stress is a more complex object (tensor) than a scalar, and the datum of one single scalar to characterize the entry into the *plastic regime* is not always sufficient. One must think in terms of a convenient representation of a tensorial state of stress and deformation. Thanks to Cauchy who related this to the representation in terms of ellipsoids, we also know that the length of *principal axes* of the ellipsoids representing stresses and strains are convenient representations of the actual state.

Henri E. Tresca (1814–1885), a professor at a Paris institution known as the *Conservatoire National des Arts et Métiers* (for short, *CNAM*) conducted in the early 1870s a series of fine experiments on metals whereby he constructed in an appropriate representation of the principal stresses the elastic limit of the said metals (cf. Tresca 1872). Practically simultaneously, Adhémar J.C. Barré de Saint-Venant (1797–1886) gave the mathematical formulation of these results (1871). Three important remarks are in order: first, it is noticed that no change in volume (so called *isochoric* deformation in the modern jargon) is observed during plastic deformation; second, the directions of the principal stresses coincide with those of the principal stresses (this assumes an *isotropic* response); third, the maximum shearing (or tangential) stress at a point is equal to a specific constant. This can be written as $\tau_M = k$. In mathematical terms, we have

$$\text{Sup}_{\alpha,\beta} |\sigma_\alpha - \sigma_\beta| = 2k, \quad \alpha, \beta = 1, 2, 3, \quad (2.1)$$

where the Greek indices label the principal stresses. Introducing the tangential stresses, this can also be expressed by the following set of three inequalities:

$$2|\tau_1| \equiv |\sigma_2 - \sigma_3| \leq k, \quad \text{etc}, \quad (2.2)$$

by circular permutation. In an astute plane representation this is represented by a hexagon (see Maugin 1992, Fig. 1.18). The interior domain (a convex domain with angular corners) is the domain of elasticity. Although the criterion provided by (2.2) gives good results in the case of metals, this definition of the elastic limit by pieces of intersecting straight lines offers some difficulties in analytic treatment of problems. Nonetheless, Barré de Saint-Venant was able to give the solution of exemplary problems such as: the torsion of a circular shaft, the plane deformation of a hollow circular cylinder under the action of an internal pressure, etc. These are problems that we still give students to solve without the help of a computer (see, e.g., Maugin 1992, Appendix). It was a simple but formidable idea of Huber

(1903–1904, in Poland; see Chap. 8) to replace the hexagon of Tresca by a circumscribed circle (obviously a convex domain; see Maugin 1992, Fig. 1.18) of radius k and equation

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2k. \quad (2.3)$$

This elastic limit is said to provide a *maximum-distortion-energy theory* of yielding. This shows that a plasticity criterion must involve the notion of *energy*. But it will take almost a 100 years to include such criteria within a good thermo-mechanical description of plasticity (cf. Maugin 1992). We note that other criteria were proposed before to measure the energy of deformation, including by the famed William J.M. Rankine (1820–1872; a precursor of Pierre Duhem with his “science of energetics”; he also worked on the fatigue of metals)—maximum-stress theory—in Scotland and Beltrami (maximum-strain-energy theory) in Italy. Richard von Mises (1913) proposed the same criterion as Huber in 1913 on a pure mathematical basis in order to facilitate calculations. In the future other criteria would be proposed for anisotropic media, the plasticity of soils, the case of porous media, etc.

Maurice Lévy (1838–1910), proposed to discard the elastic behaviour—as negligible for some materials—and to consider only the plastic one, thus in so-called *rigid-plastic* bodies (cf. Lévy 1871). This is a rather highly singular behaviour since nothing happens to the strain, not even an elastic one, in so far as the plasticity threshold is not reached and then we have an uncontrolled plastic flow occurring along a plateau in stress.

Prandtl and von Kármán solved other problems of elasto-plasticity in the early 20th century. Other well known scientists who worked in plasticity in that period were Bauschinger (1886), and Mohr (1900).

We do not know if the French are that much conservative but we can say here that they do not throw things away: according to my friend James Casey from Berkeley, you can still find the specimens used by Tresca in his (1870) experiments kept in a box in the basement of the actual *CNAM*. In early times *CNAM*, created during the French revolution, was an institution somewhat similar to the *Royal Institution* in London where both Humphry Davy and Michael Faraday gave public lectures and conducted experiments. It is complemented by a rich museum of science and technology.

Although we cannot claim that this belongs to “physical mechanics” (on the contrary) we mention here the remarkable mathematical model of dislocation obtained in a pure ideal construct—or a thought experiment—by the Italian mathematician Vito Volterra (1860–1940) in 1907, at a time when no dislocation had really been observed (this had to await 1956 with the use of electronic microscopy). This concerns lines along which discontinuities in the elastic displacement field occur. This is mentioned here because of its timeliness and the fact that it would later on play a role in the study of ductile materials (i.e., essentially plastic materials). [Volterra’s astute thought-experiment consists in cutting a cylinder (devoid of its central axial part—because this “core” corresponds to a singularity-) displacing the two faces of the cut in a certain way and welding them

back so as to create a special type of discontinuous material displacement (cf. Volterra 1907, 1909, 1939)].

2.2.3 Visco-Elasticity

Two models of visco-elasticity were mentioned in Chap. 1: the Maxwell and Kelvin-Voigt models that both involve a relaxation time, but with relaxation in stress and strain, respectively. Ludwig Boltzmann (1844–1906), renowned for his seminal work in the kinetic theory of gases and for his statistical definition of entropy, also proposed a model of visco-elasticity for solids, but in an original form. The idea is to take account in a good mathematical way of what happened in the past to the material: the past history of the strain should be involved, with an obvious more important influence of the recent past. In modern terms, this can be exemplified by a 1D stress–strain *functional* relation over time of the type (cf. Boltzmann 1874).

$$\sigma(x, t) = E_{\text{relax}}e(x, t) + \int_{-\infty}^t K(t - t')\dot{e}(x, t')dt', \quad (2.4)$$

where E_{relax} is the instantaneous modulus for relaxation, and K is a relaxation function. The later must be such as to favour the influence of the recent past, thus decreasing sufficiently fast in its argument. Using a modern illustrative jargon, we can say that the material so described possesses a *fading memory* of the past. In substituting (2.4) in the dynamical equation of linear momentum, we would be led to a new kind of equation, an *integro-differential equation*. It happens that functionals over time such as (2.4) and integro-differential equations were one of the fields to which Vito Volterra (already cited but also Volterra and Pérès 1936) contributed much with applications, not to mechanics, but to a kind of population dynamics [competition between species yielding a celebrated equation obtained independently by Alfred Lotka (1880–1949)]. The generalization of equations such as (2.4) will have a blossoming heritage in the 1960s (see Chaps. 5 and 11).

All three behaviours highlighted in the present section were still missing a good, if any, thermodynamic basis although some authors, e.g., Pierre Duhem, were pondering this matter, but as mere wishful thinking at the time.

2.3 New Interests of Investigation

2.3.1 Dynamics

With the pioneering work of Georg Bernhard Riemann (1826–1866) and those of William J.M. Rankine (1820–1872), Pierre H. Hugoniot (1815–1887), and Jacques C. E. Jouguet—known as Emile Jouguet (1871–1943)—one attacks the field of the

nonlinear dynamics in continua. Having established the required equations governing the discontinuities of fields, these scientists could prove the existence and propagation of shock waves and also detonation waves (Jouguet 1906). Duhem also studied such waves in nonlinear elasticity and his friend Jacques Hadamard (1865–1963) provided a useful classification of propagating discontinuities depending on what is the order of the derivative of the basic field that is discontinuous (cf. Hadamard 1903). Jouguet was much influenced by Duhem in adopting a thermo-mechanical viewpoint. These studies in various schemes of deformable-solid mechanics will be taken over in the 1950–1970s, in particular by Truesdell, Ericksen and others (e.g., Peter J. Chen for a long time at Sandia National Laboratories; see his book, Chen 1976). Ernst Mach, in his experimental study of shock waves in fluids, was led to introducing the “Mach” number (as a measure of relative velocity compared to the sound speed) and the “Mach” angle (in the reflection of such shock waves). No need to mention the role of these studies in the future developments of aerodynamics. In that field, a seminal work was that of Prandtl with the notion of *boundary layer* and its elegant mathematical formulation using asymptotics for a mathematically singular problem.

2.3.2 Internal Degrees of Freedom

The period 1880–1910 saw the introduction of the idea that, perhaps, a material point in a continuum would be characterized by more than a simple translation (displacement) in space. It seems that Duhem (1893) was responsible for the idea to consider a triad of rigid vectors (so-called “directors”) at each material point in order to describe the orientational changes in some kind of internal rotation. But this was more an idea than a true complete development. The Cosserat brothers were also led to consider the possible existence of internal couples (1909). They more or less were forced to do that by imposing an invariance (so-called *Euclidean invariance*) in a Lagrangian-Hamiltonian formulation, which invariance treats on an equal footing translations and rotations. This gave rise to the possible existence of a new type of internal force, the *couple stress* along with that of stress, and the possibility to have *non-symmetric* stresses. This was a first application of an argument of elementary group theory in continuum mechanics. As such, it was applauded by Elie Cartan (1869–1951), the famous geometer and specialist of the theory of Lie groups. Hellinger (1914) acknowledged the possible enrichments provided by Duhem and the Cosserats but without further elaboration. The argument of Euclidean invariance was exploited in papers by Sudria (1926, 1935). More recently it was applied by Toupin (1964) and Maugin (1970) for Cosserat continua and micromorphic ones, respectively.

Following the Cosserats, it is tempting to use a Lagrangian-Hamiltonian formulation to generalize the theory of classical elasticity, still in the absence of any dissipative process. Since small-strain elasticity corresponds to a theory that involves only the first gradient of the displacement in the energy, the next step would be to

consider a better approximation of the displacement function at each material point, hence to envisage an energy density that depends also on the second-gradient of this displacement (and higher-order gradients if needed). This step was taken by J. Le Roux in 1911 and 1913 in his doctoral thesis published as two memoirs in the Scientific Annals of the *Ecole Normale Supérieure* in Paris. The second gradient of the displacement will be felt only in problems where a sufficiently spatially non-uniform state of strains exists. This is the case of torsion, a case duly examined by Le Roux in his pioneering work. But this type of considerations was left dormant for practically 50 years. We shall return to this matter and the Cosserats' media in detail with the developments of the 1960–1970s (Chap. 13).

2.3.3 Elements of Homogenisation

Just for the sake of completeness we mention the first, naïve, technique of homogenization for inhomogeneous bodies made of grains, e.g., for dielectrics by Maxwell. This technique is essentially one that is called the *rule of mixtures*, according to which effective properties are defined by an average accounting for the relative proportion of different components in the material. Precise mathematical techniques of homogenization will be proposed in the 1970–1980s only.

2.4 Pondering the Principles

When we carefully scrutinize the scientific atmosphere of the finishing 19th century and the dawn of the 20th century, we get the feeling that most scientists have agreed on a view that concludes that *everything has settled*. There is nothing in view such as an *epistemological rupture* (in the words of Gaston Bachelard) or a radical *change of paradigm* (in the words of Thomas Kuhn), although Albert Einstein is formulating his theory of special relativity (1905), and Max Planck is introducing his quantum (taken over by Einstein in his theory of the photo-electric effect; also 1905). This kind of attitude that marks an accomplishment, favours the reflection on the bases of the theory (Newton's one for mechanics) and the formulation of an axiomatic approach.

Concerning mechanics *per se*, even Heinrich Hertz (1857–1894) published a successful book on the principles of mechanics (Hertz 1899). Of course, Henri Poincaré (1854–1912), with his truly aerial view of science in its totality wrote beautiful books (e.g., *Science and Method*) that are still spot on, even though we may not share his epistemological views. Being himself an active participant in the field, he knows well the problem posed by the relativity of motion. He is the one who identified the group structure of Lorentz transformations between frames. Ernst Mach (1838–1916), an Austrian physicist and philosopher whose name is definitively attached to aerodynamics with the *Mach number*, wrote an articulated

criticism of Newton's views in his book on the *Science of Mechanics* (cf. Mach 1911). The reading of this book is said to have deeply influenced the young Einstein. Mach pays special attention to the notion of inertia and its physical origin. Lecornu, writing in 1918, is more conscious of the forthcoming developments as he already knows elements of Einstein's gravitation theory.

In a different class we note the axiomatization of classical mechanics by Georg Hamel (1877–1954) in Hamel (1908). This was to have a long standing influence especially in Germany. The mathematician Ernst Hellinger (1883–1950), in an encyclopaedia article for mathematicians, provided in a nut shell a remarkable synthesis of the bases of continuum mechanics as of the 1910s. He captured all essential recent developments such as those due to Boltzmann (visco-elasticity), Duhem, and the Cosserat brothers. Finally, we shall consider the view of Duhem in greater detail.

Pierre Duhem (1861–1916) is a remarkable character who combines in one person a brilliant and sharp mind, a prolific writer and contributor to phenomenological physics, the champion of energetics, a philosopher of science, and the true creator of the history of medieval science. He made a big “mistake” early in his career. Aged only 24, as a student at the *Ecole Normale Supérieure* in Paris, he definitely criticized the work of Marcelin Berthelot in thermo-chemistry (he blatantly asserted that some principle regarding thermodynamic potentials and proposed by that hero of French republican science, was wrong—but Duhem was later proved to be absolutely right). This hindered the whole university career of Duhem who nonetheless produced a lot of good science, philosophy and epistemology, but in Bordeaux and not in Paris. In centralized France this was as bad as the original sin.

In the philosophy and methodology of science Duhem wrote two remarkable books, one on the *Aim and Structure of Physical Theory* (original French in 1906) and the other with a title repeating Plato's motto “*To save the phenomena*” (original French in 1908 with Greek title). In the first of these he exposes at length the under determination of theory by fact, the rejection of metaphysics and models (as used by, e.g., Kelvin and Maxwell in the UK and Boussinesq in France), and natural classification, rather than explanation, as the very object of physical theory (this can be discussed). The contents of the second book are clearly explained by its title. His view on the unifying role of thermodynamics in all of physical sciences (mechanics, electricity and magnetism, heat, etc.) is masterly but quite lengthily expended in his treatise on “energetics” or “general thermodynamics” (Duhem 1911). This has a flavour of axiomatic nature that will influence C.A. Truesdell in the 1950–60s. Duhem was a good friend of mathematicians Henri Poincaré (1854–1912) and Jacques Hadamard (1865–1963).

Personal touch. Both Duhem and Poincaré died untimely while Hadamard reached almost a hundred, using the facility of the library at *Institut Poincaré* in Paris until the end. According to the librarian—a certain Paul Belgodère—of this institute that the author knew as a student, Hadamard used to come to the library every afternoon in the late 1950s, asked to consult one of Poincaré's or Duhem's works of bygone days (say, from 1890 to 1905), and systematically fell asleep, being wakened up by the librarian at the closing time. The same scenario was repeated from day to day.

For our main concern in this book and forthcoming chapters, the most relevant writing of Duhem is the one on the “evolution of mechanics” (Duhem 1903). In one section of this opus, Duhem examined what, at the time, he called the “*nonsensical*” *branches of mechanics*. What he means by this somewhat eccentric expression are the fields of physics, mechanics and electromagnetism that do not fit yet in his general framework of thermodynamics. It is interesting to note the list of these fields: so-called false equilibria, hysteresis phenomena, and electro-magnetic theory in materials. These are precisely dissipative phenomena such as thermodynamically irreversible reactions, friction, plasticity, etc. Now looked upon with our present knowledge, this sounds like a tentative proposal of research programme for the next generation, something quite equivalent in its own field to the Erlangen program (1872) of Felix Klein in geometry and the list (1900–1902) of unsolved—at the time—problems proposed by David Hilbert in pure mathematics—that in fact included the axiomatization of the whole of physics as Problem no six.

The flame of Duhem’s approach to general thermodynamics was successfully carried over by Th. De Donder (1872–1957) and other physicists from the Netherlands and Belgium between 1930 and 1970, resulting in the now commonly admitted *theory of irreversible processes* (S. De Groot, P. Mazur, I. Prigogine). However, both Duhem and these scientists did not possess the mathematical tools—such as convex analysis and nonlinear optimization—to deal with some of the properties (plasticity, hysteresis), so that they could deal only with *linear* irreversible processes. The solution would come in the 1970–1980s for *nonlinear* irreversible processes.

2.5 Concluding Remark

The considered interval of time was the last period during which the same scientists worked in so many different fields, but still within phenomenological physics. This marks the end of an era that was typically that of the 19th century. Examples of exceptions in more recent times will be Lev D. Landau and P. G. de Gennes. From now on, some specialization will be necessary, and this will be the case in practically all forthcoming chapters.

2.6 Further Reading

On the principles of mechanics first-rank contributions are by Barré de Saint-Venant (1851), Hertz (1899), Duhem (1906, 1908, 1911), Hamel (1908, 1927), Mach (1911) and Hellinger (1914). On Pierre Duhem see Ariew (2007) and Manville (1927). On Boussinesq we recommend Bois (2007). On the general history of the strength of materials, Timoshenko (1953) remains an unavoidable reference.

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