Early Mathematics Learning in Perspective: Eras and Forces of Change

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One of the endlessly alluring aspects of mathematics is that its thorniest paradoxes have a way of blooming into beautiful theories. Phillip J. Davis (1964)

As the opening chapter in this important volume that looks deeply at the changing and somewhat paradoxical nature of early mathematics learning, our goal is to position those shifting perspectives within a historical framework. By conceptualizing how views of early mathematics learning have taken shape over the past century through the pushes and pulls of both endogenous (internal) and exogenous (external) forces, one can better grasp the re-conceptualization of mathematics learning conveyed within the ensuing chapters. There are perhaps few who would argue with the underlying premise of this book; that the character of early mathematics education has changed dramatically over the last century not only in terms of the pedagogical approaches to teaching young children, but also in relation to the content and goals of that instruction. However, the progression of that change may be less evident and, consequently, worthy of scrutiny.

Changes in complex domains such as early childhood mathematics rarely happen abruptly or without inducement. Rather, such transformations seemingly unfold over the course of many years in response to internal and external conditions. Here we endeavor to unearth those inducements, some of which arise more directly from within the community of researchers and practitioners invested in early mathematics teaching and learning. Other of those inducements can be situated within the broader educational and psychological communities, reflecting varied if not conflicting theoretical orientations toward human learning and development (see Fig. 1). Thus, in this chapter, we attempt to identify six particular periods or eras associated with

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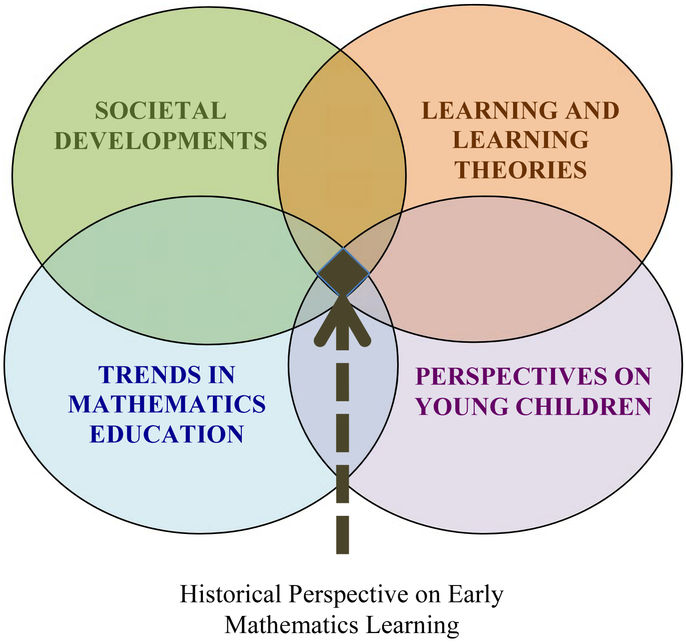

Fig. 1 The sources of evidence in constructing eras of change in early mathematics learning

mathematics learning in young children and seek to explore psychological, socio-cultural, and neurophysiological developments that may have helped to shape those eras.

Before we begin this historical overview of early mathematics learning, we want to state frankly that there is not exactness to the characterization we present. Historical analysis of this sort is characteristically inferential. Consequently, others who would engage in a comparative historical examination of the early mathematics literature or judge the endogenous and exogenous forces that were at work within each time period may reach different conclusions. Moreover, the boundaries between historical eras are neither rigid nor fixed. Here we looked at 20-year periods as meaningful, generational units for analysis, but the trends underway within the era do not simply begin or end at the preset time points. Further, our own interests and empirical foci, such as our investment in the study of learning theories and children's understanding of fractions and their analogical reasoning, will undoubtedly color the perspectives we forge herein from the existing evidence.

Nonetheless, with these caveats in mind, the eras we identify derive from the theoretical and empirical literature of that period, and consider personages whose writings and thinking were particularly influential. We also signify some important events within mathematics, education, or the broader society that are anchored to these time periods. Also, we consider the views of children and the teaching of mathematics to children that were prevailing within each era, as well as the presence of alternative or competing perspectives that may have signaled subsequent changes on the horizon.

For example, early childhood educators have traditionally advocated for learning through play, but views of the nature and purpose of play have varied markedly among educational researchers and practitioners. In part, these variations exist as a function of the changing beliefs about children's cognitive capabilities and about the role of early educational experiences in enhancing the capabilities of young minds. They also mirror shifting psychological orientations toward learning and philosophical perspectives toward knowledge and knowing. An examination of the writings of such influential theorists as Dewey, Thorndike, Piaget, Vygotsky, Flavell, and Rogoff will serve to illuminate these shifting orientations toward young minds, mathematics, and the learning of mathematics. Drawing on these writings, we chart the course of early mathematics education in relation to these theoretical underpinnings, consider emerging trends, and address the implications for early mathematics research and practice.

Era of Experiential Learning (1900–1920)

Every historical analysis begins at some predetermined point in time. For our purposes, this analysis begins at the turn of the 20th century. So much of the world was undergoing change as the century dawned; civil conflicts in the United States and France had ended and the industrialization of much of Europe and the US was well underway. In terms of psychology, there was a growing interest in the nature of childhood itself—a concept that was not commonly considered prior to the industrialization of world powers—and an investment in developing mental faculties beginning at a young age (Elkind 1998). In the late 19th century, mathematics was considered important as a way to exercise the mental faculties in the developing mind (Sztajn 1995), and we see this notion carried forward into the early 20th century.

Those who were associated with early mathematics learning at the close of the 19th century, however, had not fully embraced the character of young children and their predisposition toward learning through free play that would become a feature of the next era. Rather, the available methods to teaching young children from that period were somewhat formal and structured in nature as perhaps best exemplified by the work of Friedrich Froebel. In particular, the techniques used by Froebel, who has been credited with introducing the concept of "kindergarten" into Western culture, involved concrete materials, such as geometric figures, that were deeply mathematical and which could be used to engage young children in the learning of mathematical concepts (Balfanz 1999). *Froebel Gifts*, as his educational materials were called, were very carefully devised and intended to be systematically used in the early childhood setting to foster particular ways of thinking and behaving. Froebel, as with Montessori who borrowed from his work, understood the role of "free work" or activities to teach the young. However, he saw these mathematical activities not as an end in and of themselves but as important ways of exercising the developing mind of the child. Thus, Froebel's approach could be described as rather formal and less spontaneous than we would see in the early 20th century in what we have labeled the *Era of Experiential Learning*.

By the early 1900s, the influence of Froebel was fading in favor of more holistic and less orchestrated conceptions of early childhood education (Balfanz 1999). Whereas Froebel's kindergarten was rather structured in its treatment of mathematics, the more child-centered orientations of this era focused on the child as a social being. "Free work" was still central to early mathematics learning in this era, but the child was given increased freedom to choose and to freely explore the mathematically associated objects and activities that populated the learning environment. Mathematics was not directly the focus of learning in this setting, but was rather understood as manifestations of young children's true interests that needed to be appropriately fed and actively nurtured through relevant and engaging experiences (Dewey 1903). Mathematical experiences existed but were informal in nature and embedded within children's exploratory activities. In other words, the learning of mathematics was somewhat more incidental than intentional and the consequence of learning *in* experience rather than learning *from* experience (Saracho and Spodek 2009).

Influential Personages

No name is more associated with this Era of Experiential Learning than that of John Dewey, the pragmatist and the father of progressive education. In his formulation of progressive education, Dewey was influenced by Montessori's idea of learning through activities and appreciated the efforts of Froebel to create learning environments for the young. Among the tenets of the progressive movement was Dewey's often-expressed idea that education is the process of living and not simply a preparation of later life (Dewey 1900/1990). Toward that end, Dewey argued that the content of learning should derive from the children's existing interests and draw meaningfully from the children's life in the broader social community. Mathematics was not to be dealt with as isolated content nor used as mental calisthenics, but was to be experienced fully and naturally by children through hands-on, project-based activities that built on children's existing interests and that pertained to activities (e.g., cooking, building) that were valued outside of school.

The focus on the teaching of mathematics to young children through engrossing experiences of value personally and socially coincided with the emergence of a new field, developmental psychology. Unlike the developmental psychology of today, this earliest manifestation of this field had more to do with systematic observation than experimental study. This focus can be clearly seen in the work of G. Stanley Hall (1907), considered the founder of developmental psychology, the first president of the American Educational Research Association, and the father of the child study movement. Perhaps best known for his fascination with *peculiar and exceptional* *children*, Hall devised methods for the detailed documentation of children's physical attributes and psychological behaviors.

The significance of developmental psychology, in general, and the child movement in particular was the now commonly accepted premise that young children are much more than miniature adults. Rather, they live and learn differently and those differences undergo systematic change over time. In terms of mathematics, this translated into critical questions about what the mind of the child was able to grasp mathematically and how best to harness the burgeoning knowledge about the physical and psychological development of children to teach them mathematical concepts and procedures appropriately (Alexander et al. 1989). Questions of developmental appropriateness and about how best to bring children and mathematics together reappear throughout the ensuing eras. For those of the Experiential Learning Era, that question was best answered by allowing young children to shape the educational agenda through the enactment of their interests and choices and by positioning the particulars of mathematical concepts and processes as background to the wants and desires of those children. Learning by doing remained the rule of the day, even as the uniqueness of young children was embraced.

Views of Children and the Teaching of Mathematics

In order to address growing concerns in the early part of the century about appropriate instruction in kindergarten, and in particular the appropriateness of Froebelian kindergarten, the International Kindergarten Union formed the Committee of Nineteen. A lack of consensus within the Committee meant that three reports on the content and goals of kindergarten were eventually issued, ranging from support of the Froebel method to endorsement of a more child-centered progressive orientation. Patty Smith Hill wrote the report supporting a child-centered approach, which ultimately won favor in academia (Beatty 1995). Following this report, play became a legitimate part of kindergarten programs and it was recommended that learning be guided by activities of interest to the child (Saracho and Spodek 2009).

Around this time, Margaret MacMillan established the first nursery school in England. Play was also an important part of this approach, but there was little concern for academic subjects. When academic subjects were introduced to older children, no particular approach to teaching was prescribed (Saracho and Spodek 2008). What was important instead to McMillan was the child's health and hygiene, with a particular concern for poor and working class children. As a result, outdoor play and good air ventilation were incorporated into the program, to the extent that her early buildings were only partially enclosed and the children sometimes even napped outside (Beatty 1995). As nursery schools gained popularity over the next few decades, their focus would broaden to include the child's general well-being and readiness for more formal learning in school.

Inspired by Dewey, Kilpatrick (1926) forwarded the Project Method, which engaged children in learning activities that were purposeful and practical. According to Kilpatrick, children would be naturally stimulated to learn if they were provided with interesting experiences that involved them in the community. This followed the principles of progressive education wherein it was expected that children are naturally curious and interested in the world around them and that those curiosities and true interests included mathematical concepts and processes. There was also the presumption that the mind of the child was highly capable of dealing with mathematical concepts and procedures if those concepts and procedures were embedded in activities that the children valued and that they could reasonable pursue. Thus, when engaged in the Project Method, children may count or measure objects as they worked toward a larger goal, such as raising chickens (Saracho and Spodek 2008, 2009).

Mathematics was not emphasized in its own right, but it was expected that children would learn some mathematical ideas as they participated in the projects. We see similar orientations to mathematics learning in contemporary sociocultural theories of learning and development, such as Rogoff's (1990) concept of legitimate peripheral participation. As with the aforementioned discussion, this overview of the Experiential Learning Era of early mathematics education introduces several themes about children and mathematics learning that will periodically reappear in our analysis. The first has to do with the perceived capacities of the young mind and whether the mind of the child is conceived as fertile ground for grasping basic mathematical concepts and procedures or not. The second has to do with the need to foreground the mathematical concepts or procedures or whether the mathematics should be embedded in socioculturally valued experiences or activities. For those in the Experiential Learning Era, children were perceived as highly self-directed and inquisitive learners who were able to acquire mathematical understanding if they were allowed to explore those ideas within the context of self-chosen, self-directed and socially valued activities.

Competing Views

Perhaps the most evident contrast to arise during this Experiential Learning Era was the all too familiar theme of traditional or basic skills education that has remained the counterpoint to progressive movements throughout the century. Specifically, Deweyan approaches to early childhood education were not the only ones that conflicted with the ideas forwarded by Froebel and his notion of kindergarten (Balfanz 1999). Another critic was Thorndike (1913), a behaviorist in terms of this theoretical orientation toward learning and development, whose work can be seen as the backbone for basic skills training and development. Thorndike purported that formal instruction in arithmetic was fruitless before second grade, and that even when mathematics was introduced, understanding was not a pre-requisite for acquiring mathematical skill (Baroody 2000). Rather than believing that mathematics could be learned incidentally through purposeful activities, Thorndike, who equated learning with behavioral change and manifestations, believed that mathematics must be systematically structured and practiced and, thus, had no place in early childhood education. Thorndike, along with many other critics, thought that the early years should be focused instead on social development and health (Balfanz 1999).

Era of Childhood Readiness (1920–1940)

Two trends that appeared within the Era of Experiential Learning—learning through play and the focus on early childhood as a particular period of development—carried forward into the Era of Childhood Readiness. What distinguished this new era from the previous, as we will discuss, was the acceptance of mathematics as not solely as means to an end, but as a curricular end in and of itself. These trends combined together positioned the early educational years as a time to prepare the child for the more formal study of the domain of mathematics—to ensure that they were "ready" to think and perform mathematically in subsequent years.

As more students began to attend schools during the early twentieth century, an increased focus was placed on mathematics that was considered to be practical for the average person. This was especially true during the Great Depression, since limited availability of jobs kept many students in school for longer. The resulting increase in high school enrollment meant that more students were focused on training for jobs rather than college. Mathematics was de-emphasized, and in some cases, the mathematics requirements for graduation were removed altogether (Walmsley 2007). Instead, courses such as home economics, art, and physical education increased in popularity at the secondary level. Partly in response to the decreased focus on mathematics in schools, the National Council of Teachers of Mathematics (NCTM) was founded in 1920 (Austin 1921). Meanwhile, a parallel trend was occurring in early childhood education. In particular, mathematics in the early years was extremely limited. The focus shifted to play, imagination, physical movement, and social skills in part because this is what many educators felt young minds were able to cognitively and physically address (Saracho and Spodek 2009).

Several reasons contributed to this reality, including dominant theoretical perspectives during this era coupled with the changes occurring in the later grades. Mathematics was included as part of the first and second grades, but the amount of time spent on mathematics instruction was a fraction of the time spent on reading, language, and even recess (Balfanz 1999). Likely influenced by the continuing arguments by Thorndike and his adherents, it was determined that to focus any more specifically or directly on mathematics in these young years would not prove fruitful. Rather, drawing on the work in child study of the prior decades, it was held that educators needed to ascertain whether young children showed signs in their play and interactions with others that they were cognitively predisposed for formal instruction, including formal instruction in mathematics in the years to come.

Personages

Two contemporaries and colleagues warrant particular recognition for their role in shaping this Era of Readiness, Arnold Gesell and Frances Ilg. As has often been the case in the history, especially in these early years, Arnold Gesell came to education from a different field. He had trained to be a physician, but become enamored with questions of nature versus nurture and the role that each played in the development of children with disabilities. From decades of systematic research with Frances Ilg, he argued that nurture had a significant role to play in children's developmental trajectory. Gesell and Ilg did not discount the power of nature but felt that there was much that could be done within the early years of life to stimulate cognitive capacity—to build on what nature had provided.

Gesell's writing on *The Mental Growth of the Preschool Child* (1925), and *The Preschool Child from the Standpoint of Public Hygiene and Education* (1923), as well as his work with Ilg on the development of early childhood assessments served to justify the time as one of nurturing the young child—of readying them for the more formalized instruction in mathematics and other contents that would follow. His influence also extended beyond the educational community to parents concerned with child development and child rearing. This influence was largely due to his highly cited volume that documented early childhood milestones, *An Atlas of Infant Behavior* (1934) and to the two guides for child rearing that he coauthored with Ilg, *Infant and Child in the Culture of Today* (1943), and *The Child from Five to Ten* (1946).

By the close of this era, many regarded Gesell as the foremost authority on child rearing and child development. Not only did he argue strongly for the influence of early nurturance at home, but he also became an advocate for a nationwide nursery school system that could provide the early stimulation and support that he promoted in his writings to educators and to parents. And it was these strongly held beliefs in the importance of readiness to later development that mark this era, particularly when coupled with the Thorndikian perspective that training in mathematics should be reserved for later elementary and not attempted within the early grades.

Views of Children and the Teaching of Mathematics

Views of young children and the teaching of mathematics during this era were shaped by an emerging interest into the inner workings of the human mind (cognition) in relation to the behavioral indicators of capability (behaviorism). In line with Thorndike's work, some theorized that formal instruction in mathematics was unnecessary—and perhaps even harmful—in the early years and should be delayed until the child was in a formal school setting (Balfanz 1999). As a result, the mathematics curriculum was limited in the early years. Nursery schools, which were popularized during this time, held little regard for academic subjects in general and even less for mathematics in particular. They instead encouraged dramatic play, physical movement, and even caring for animals (Saracho and Spodek 2009).

With more and more children attending kindergarten, it was gradually becoming linked to the public school system. As this happened, mathematics became even more de-emphasized in early childhood. One contributing factor was that textbooks were written with the assumption that kindergarten students had no prior knowledge of mathematics; arithmetic was limited or absent until first grade (Balfanz 1999). Given the emphasis in later grades on mathematics that would be useful to the average person, mathematics for young learners was perhaps meaningless. Instead, the purpose of kindergarten was readiness for more formal learning; for example, following directions and complying with rules were emphasized (Saracho and Spodek 2009).

Competing Views

The nature versus nurture discussion that Gesell and Ilg brought to the public attention in this period can be contrasted with that of another developmentalist, Jean Piaget (1926/1930, 1952, 1955). Like Gesell, Piaget came to his interests in education and human learning and development from an alternative profession. In Piaget's case, this profession was science and biology. Piaget, like G.S. Hall and Arnold Gesell, was an acute observer of nature. In fact, even as a child, it was apparent that Piaget had remarkable capacity to build upon direct, detailed observations; having published a book about birds that were found around his home in Switzerland before he was 10. What Piaget's observations of animal life and later human life led him to was a more stage-like perspective on human cognition and a stronger appreciation for the "nature" side of human development than his predecessors.

During the 1920s and 1930s, Piaget began publishing books based on observations and clinical interviews of young children which presented his ideas about the child's emerging ability to think logically (Piaget 1926/1930)—what became known as genetic epistemology. While many praised his use of naturalistic settings, some also criticized his research for not being "scientific" enough (Beatty 2009). Likewise, his ideas about the child's egocentric nature were initially met with mixed results among early childhood educators. His work enjoyed a brief reception in the United States during this time but did not fully take hold until a few decades later. Meanwhile in Geneva, he continued to conduct research with children and to develop a more fully-articulated theory of child development that would eventually gain widespread support and make a profound contribution to the mathematics education of young children.

The most influential aspect of Piaget's prolific work was the stages of development he conceptualized and the more focused attention on cognition rather than on behavioral manifestations. Although Piaget presented these as general stages and not fixed timeframes of development, their consequences, which carried over into the next era, was to think of the capacities of young mind's including mathematical capabilities as rather rigid or set.

Era of Cognitive Development (1940–1960)

The prioritizing of social skills over academic rigor that began early in the twentieth century led to growing concerns by mid-century that students were ill-prepared for technical jobs. This idea that was especially highlighted by War World II, when it became clear that soldiers often did not have the mathematical skills needed for their jobs in the military. Moreover, many critics felt that "practical" mathematics was no longer enough even for the average student and that, in order to flourish in an increasingly technological world, higher levels of mathematics needed to be required at the secondary level. This view was coupled with a shifting focus from memorization to understanding, most notably reflected in the work of Brownell (Kilpatrick 1992; Lambdin and Walcott 2007). By the end of this era, the New Math movement, which emphasized mathematical ideas and structures and included more rigorous mathematics than prior decades, had taken shape (Herrera and Owens 2001; Jones and Coxford 1970; Walmsley 2007). Launched in 1957, Sputnik further created a sense of urgency to update mathematics education at all levels and secured the momentum of the New Math movement in the decade to come.

With the external forces in play, fascination with the "black box" of the human mind moved into prominence within the educational community and strict behavioral theories of learning faded into the background (Newell et al. 1957, 1958). How the mind works and how the understanding of mental processing could be harnessed into better educational outcomes became the approach *de rigueur*. Increased attention was also being paid to the cognitive and mathematical skills of young children during this period, as evidenced by the significant amount of research that emerged in this area over the next several decades. This research supported the idea that young children can learn mathematical ideas if they are grounded in the child's experiences, and that children begin school with significant informal experiences on which to build. Moreover, building on these experiences can help students make sense of the new information they are encountering in school settings (Baroody and Ginsburg 1990).

Personages

As noted, no individual was more contributory to shaping the Era of Cognitive Development than Jean Piaget. While other children were playing with toys and enjoying their childhoods, Piaget was already immersed in scientific study of animals and fossils. More than his developmental predecessors, Piaget (1952, 1955) showed the world of living organisms—humans included—as growing and changing in systematic ways. But the changes that Piaget documented through his clinical methods were stage based, meaning that they were conceived as transformative, distinct, and dramatic rather than gradual and continuous. Those cursorily familiar with Piaget's theoretical and empirical writings are likely aware of his four developmental stages: sensorimotor, preoperational, concrete operational, and formal operational.

Even those who have moved away from the initial conceptualization of these stages or who are more liberal in their age-related characterizations (i.e., neo-Piagetians) still bow to Piaget's notion of stage-like development in young children (Case 1985, 1992; Flavell 1985; Flavell et al. 1993). A detailed discussion of these stages is beyond the scope and intention of this chapter. Yet, because of their relevance to this examination of the history of early mathematics education, we will briefly address the characterizations of the sensorimotor and preoperational stages and consider the implications for early mathematics education.

From the moment of their birth, children are thrust into a strange, new world that they must come to know through their senses—their primary tools for growth and development. Given the primary role of the senses in the first months after birth, it is understandable that Piaget would refer to this initial period of cognitive development that was conceived to run to about age two as the sensorimotor stage. Even in their rudimentary formation of mathematical conceptions and procedures, children at this young age remain dependent on direct, physical examination and exploration to survive and to understand their world and their place in that world (Alexander 2006). This sensory-physical exploration is aided by the fact that these young children are maturing neurologically and motorically, and they are acquiring the ability to express their wants and needs to others around them.

Toward the end of this initial developmental stage, according to Piaget, young children begin to realize that the things they see and do can be represented symbolically with words or numbers. They begin to think symbolically by language, in problem solving, and through imaginative play (Piaget 1955). Just as increased mobility and linguistic facility are keys to change, this acquisition of symbolic understanding becomes a catalyst for moving young children into a new realm of development—preoperational thinking. Thus, while there may be a genetic predisposition in the preoperational child to sense quantity or spatial orientation, critical to later mathematical development, this grasping of symbolic representation makes more conceptual growth possible. Along with the ability to use symbols and signs critical to mathematical thinking and learning, there are certain defining attributes of the preoperational mind—a period that runs from around ages 2 to 7. For one, young children become more skilled at engaging in conventional rather than idiosyncratic communication (Piaget 1955). Another cognitive achievement of the preoperational stage has to do with young children's conception of time and space (Piaget 1952).

Still, as noted, there were clear limitations within Piagetian theory to the nature of the thinking and reasoning of which preoperational children were presumed capable. Those limitations were associated with such processes of egocentrism (self-centeredness), conservation (maintenance of quantity or mass), and reversibility (reverse thinking). For example, according to Piagetian theory, young children must abandon their propensity toward idiosyncratic speech and become more facile in conventional language as they develop (Piaget 1955). To Piaget, idiosyncratic speech was evidence of egocentrism or children's tendency to view the world through their own knowledge and experiences without regard to other perspectives. Although egocentrism is often used pejoratively when applied to more mature individuals, this was not Piaget's intention. Rather, Piaget (1926/1930, 1952) sought to demonstrate that preoperational thinkers had an understandably limited sphere of existence that resulted in their interpretation events solely through their personal lens of experience.

Interestingly, for all the remarkable cognitive and mathematical accomplishments that young children realize in these initial two stages of development, there were seemingly negative consequences that arose from the adaptation of a Piagetian perspective—consequences not intended by Piaget. For one, many within the educational realm did not recognize that Piaget was describing typical behavior of children—what most children would be expected to do under most conditions. Instead, the idea of individual differences or variability got lost in the curricular programs that were devised. Activities were excluded or not introduced under the notion that children of a given age would not be able to benefit from them (White and Alexander 1986). Further, the adaptation of a stage model of development led to an all-or-nothing conception of learning. Children were either preoperational or concrete operational or not. There was less room for cognitive variability based on task or context.

Views of Children and the Teaching of Mathematics

Although Piaget believed that very young children were not capable of developing true number concepts, he insisted that they were curious and naturally interested in patterns. He also believed that developing understanding, rather than rote memorization, was most important for learning mathematics (Baroody 2000). Whereas his earlier work stressed the relation between language and logical thought, his later work supported the idea that logical thought is developed through children's activity (Beatty 2009).

Hence, activities that involved concrete materials were thought to help support the eventual development of number concepts. Learning activities or exercises were derived based on classical Piagetian tasks or the concepts and processes that those tasks exemplified. Through the manipulation of concrete materials, mathematical ideas could be discovered and later formalized through instruction (Lambdin and Walcott 2007). Unlike Froebel's materials, which were inherently mathematical, these concrete materials could include activities with everyday objects (e.g., sorting or counting cookies). Such an approach is based in the idea that rote memorization of information is not adequate for developing either deep understandings of or positive attitudes toward mathematics (Baroody and Ginsburg 1990).

Competing Views

At this point in the history of mathematical learning and development, there was no argument that there was a systematic nature to growth in young children. The controversies that existed had more to do with whether that change was continuous or discontinuous (stage-like), and, relatedly, whether the mind of the young child could benefit from early exposure to mathematical concepts and procedures or not. The rumblings for more continuous development had always been present over the decades but those rumblings became to grow louder toward to end of this era. Regrettably, some of the strongest and more compelling voices for continuous change (i.e., those of Vygotsky and Luria) were not heard until years later as a result of political circumstances.

Some approaches to early mathematics education during this time represented a merging of several perspectives. For example, the Reggio Emilia approach involved children in the sort of pre-number activities such as classifying and sorting that were consistent with Piaget's notions of developmental appropriateness. At the same time, mathematics was embedded in project work, or in-depth explorations of familiar contexts, an aspect inspired by Dewey. The approach was also inspired by the work of Bruner (1960, 1961), Bruner et al. (1956), who argued strongly for the power of scaffolding by others, and Vygotsky, who held that children were co-creators of knowledge and they use a variety of cultural tools to aid in this process (Dodd-Nufrio 2011; Linder et al. 2011). This latter perspective would take a strong foothold in the decades to come.

Era of Socially-Scaffolded Development (1960–1980)

During the next era in the history of mathematical teaching and learning, the debates over continuous or discontinuous models of development became coupled with an augmented awareness of international competitiveness and growing concerns over students' preparedness. For instance, the New Math movement continued through the 1960s, but dissatisfaction with students' computational skills spurred a public push for a shift "back to basics" during the 1970s. Coupled with this trend was an increased focus on standardized testing and teacher accountability (Walmsley 2007). However, some argued that early childhood educators could promote fluency with basic skills while still supporting children's thinking in a way that is consistent with theories of cognitive development. In other words, academic learning of skills was not necessarily viewed as incompatible with play (Havis and Yawkey 1977).

During this time, there was also heightened attention on poverty and its impact on academic learning; raising awareness of the power of nurture or the social context to impact young children's subsequent growth and development. As a result, Head Start was founded in 1965 to target poor children and their parents. The program was meant to be comprehensive, including services that ranged from health and nutrition to parent education and involvement. Although it was not without its critics in the beginning, Head Start is considered a beneficial initiative in part because of longitudinal studies that illustrated a variety of positive, long-term outcomes (Beatty 1995). As with much of the readiness work in the prior eras, the focus within Head Start in these early years was on preparing the child, socially and cognitively, for the demands of formal instruction in mathematics and other domains that would soon follow.

Personages

Two individuals merit recognition for the shaping of this Era of Socially-Scaffolded Development—one a contemporary of Piaget whose writings did not become widely distributed until decades after his death (i.e., Vygotsky) and another whose writings still influence educational research and practice (i.e., Bruner). While we would still regard the theory and research of these two giants within the field of education as cognitive—that is both were invested in understanding the workings of the individual mind so as to promote learning and development—they manifest more emphasis on the role of the social context and those who populate that social environment in fostering young children's growth than did Piaget and his adherents. For this reason, while Piaget has been called a cognitive constructivist, Vygotsky and Bruner would be more suitably regard as social constructivists (Murphy et al. 2012).

Volumes have been dedicated to the research and influence of Lev Vygotsky and we will not be able to do justice to that legacy here. However, as it pertains to early mathematics learning and teaching several conclusions are especially noteworthy. First, while Vygotsky admired Piaget and found much within his writings with which he agreed, he differed strongly with Piaget on several critical accounts (Vygotsky 1934/1986). Specifically, Vygotsky (1978) did not hold to a stage or discontinuous view of development and put much more weight on the influence of more knowledgeable others to guide and support development. In this way, Vygotsky (1978, 1934/1987) was more invested in understanding optimal rather than typical development. It was not a question of what children generally could do mathematically without guidance, but rather what a given child could potentially demonstrate when functioning within a rich and supportive environment.

Similarly, Jerome Bruner (1960, 1966, 1974) argued for guided discovery for young children and was credited with introducing the now often-used term "scaffolding" into the educational vernacular. Bruner felt that an interplay of various forms of representation, most notably symbolic, is what defined children's development; not the stage-like progression that Piaget contended. Further, through guided discovery, children have the opportunity to explore mathematical concepts and procedures but under the watchful eye of teachers who could help to orchestrate events in such a way as to maximize the child's learning. Consistent with the New Math movement, Bruner believed that mathematical concepts could be taught with integrity to young children, provided that tasks were carefully chosen to illustrate important ideas at an age-appropriate level (Herrera and Owens 2001; Lambdin and Walcott 2007).

Views of Children and the Teaching of Mathematics

Researchers during this time were beginning to question some of Piaget's ideas related to number development (Baroody 2000). According to Piaget, instruction for young children who cannot yet conserve number should be restricted to pre-number activities such as ordering and classifying (Clements 1984). In a training study comparing this approach to one that focused explicitly on number concepts, Clements (1984) found that the group trained in number concepts outperformed the other group on number skills yet performed as well on a test of the pre-number skills. In other words, it was thought that number skills cannot only be taught at this age, but doing so can also reinforce pre-number skills.

Other evidence for learning number skills can be found with Sesame Street, a children's show that was popularized during this time. This show was designed not just for entertainment but to actually teach social behaviors and early academic skills to young children, particularly to those with an economic disadvantage. Across a multitude of studies from several countries, positive effects have been found for both. For example, children who viewed Sesame Street generally entered kindergarten with a greater range of number skills than those who did not, with effects lasting for years (Fisch et al. 1999).

Competing Views

While the influence of the social context, especially in the form of more knowledgeable others was gaining prominence, there were two groups of theorists and researchers who offered contrasting perspectives on young children and their learning of mathematics. On the one hand, there were the neo-Piagetian's like Robbie Case (1985) and John Flavell (1985) who retained a more cognitive orientation toward young children's development. While differentially or more liberally interpreting Piaget's work, they still held to a discontinuous view of learning and development and effectively documented both the capabilities but limitations of the young mind in terms of dealing with symbolic representations that mathematics learning demanded. Further, Flavell (with Miller and Miller 1993), who studied with Piaget, argued convincingly that attempts to document what typically occurs in the course of development for most children under most circumstances did not preclude efforts to appreciate individual variability. In this way, the homogeneity and heterogeneity of young children's development could rightfully co-exist.

At the other end of the spectrum, there was an escalation in the number of scholars trained in social anthropology and cultural anthropology who began to focus their expertise and interests onto questions of education and learning (e.g., Lave and Wenger 1991; Rogoff 1990). With this escalation, a harbinger for the era that would follow, less concern existed for the operations or development of the individual mind. Rather, the attention was on society or communities and on the activities of the collective as they engaged in socially-valued and socially-supported practices that served as evidence of mathematical learning. The mathematical learning of young children was not constrained to schooled versions of mathematics but was opened to everyday cognitions that occurred within the course of living and functioning within sociocultural communities.

Era of Culturally-Nested Learning (1980–2000)

As the Era of Culturally-Nested Learning began to take shape, there appeared to be rather strong and contrasting perspectives on mathematics learning and teaching in juxtaposition. While contrasting or competing views have always defined any period in the history of mathematics education, this difference was somewhat unique. For one, there were quite varied theoretical and empirical orientations among educational researchers generally, including those holding to more cognitive constructivist and social constructivist perspectives. Moreover, there were clearly distinct and seemingly conflicting orientations toward learning within the community of mathematics education, as we will discuss. Further, the conceptualization and operationalization of mathematics teaching and learning espoused among educational researchers stood in sharp contrast to the conceptualizations and operationalizations of mathematics teaching and learning operating within the educational system.

For instance, within the mathematics education community, there were those who continued to give primacy to the individual mind (e.g., cognitive constructivists or radical constructivists), whereas others held more steadfastly to a sociocultural or sociocontextual frame (e.g., socioculturalists or situated cognitivists). These contrasting views were artfully captured in Sfard's (1998) provocative article on AM (acquisition metaphor) and PM (participation metaphor) perspectives on learning.

Yet, both of these metaphorical stances toward learning in mathematics and other complex domains stood in sharp contrast to what was ongoing with regard to mathematics education within this historical timeframe. Specifically, this era saw a rise in basic skill assessments within public schools and calls for teacher accountability. On the other hand, what should count as "basics" for school mathematics curricula was being questioned by some, and there was a concomitant rise in the mathematics requirements for high-school graduation during this period. In addition, mathematics educators saw a need to go beyond computational skills to include estimation, problem solving, and the use of technology (Lambdin and Walcott 2007; Walmsley 2007).

To this end, NCTM published the 1989 *Curriculum and Evaluation Standards for School Mathematics*, with other documents to follow. This document emphasized the importance of teacher-facilitated investigations for children, designed to help foster problem solving, deep understanding, and ownership of mathematical ideas. As an example, the *investigative approach* attempted to blend skills, concepts, and mathematical inquiry by presenting children with worthwhile, challenging tasks or projects that encourage exploration. Within this approach, students are encouraged to share their ideas, and the teacher prompts and guides students when they are struggling (Baroody 2004). This move within the mathematics education community was supported by emerging research that illustrated young children are capable of more sophisticated mathematical thought than was proposed by Piaget (Baroody 2000). However, heated debates about the balance between computational skill and problem solving, known as the "math wars," characterized the second half of this era (Herrera and Owens 2001).

The expanding popularity and capabilities of hypermedia technology, which would become even more apparent in the years to follow, were also having an effect on what was taught and how it was taught within schools and classrooms throughout the industrialized world. From graphing calculators to personal computers and from online communities to the proliferation of media sites dedicated to young children and mathematics, it was becoming unnecessary and, perhaps, impossible to contain children's interactions to the classroom. The universe of actual and virtual "others" who could afford scaffolding to children engaged in mathematics-related activities was expanding by leaps and bounds—altering the face of mathematics teaching and learning for all time (Shaffer and Kaput 1999).

Another circumstance adding to the "messiness" of this era was the globalization of society and commerce and the ensuing international comparisons of student academic performance. Specifically, international mathematics and science studies that began in the 1990s highlighted the need for reform at all levels. Mathematics educators and policymakers in Western countries were particularly alarmed at how their students performed relative to many of the Asian countries. These results inspired several countries, including New Zealand, Australia, and Canada, to create initiatives that targeted the early years (Young-Loveridge 2008).

Personages

Because of the influence of social and cultural anthropologists during this particular era, we want to describe the particular contributions of two such individuals, Jean Lave and Barbara Rogoff. In decades past, anthropologists like Margaret Mead brought their theoretical interests and research methodologies to the study of particular social and cultural groups (e.g., Samoans); often quite distinct from their own. Over the course of this era, however, those trained as anthropologists found compelling evidence of mathematical thinking and capabilities within certain communities of practice such as apprentice tailors, milk deliverers, or dieters (Carraher et al. 1985). In her groundbreaking volume on *Cognition in Practice* (1988), Lave brought this fascinating work to the attention of the wider educational community and argued compellingly that such everyday cognition should be valued since it demonstrated evidence of mathematical thinking and learning within "authentic" contexts.

As with her contemporary Lave, Rogoff (1990) was invested in the sociocultural collective and was especially concerned with studying how children appropriate or master patterns of participation in group activities, including those activities that involved mathematical thinking and performing. For instance, in one of her classical studies, Rogoff and colleagues (Rogoff et al. 1995) investigated Girl Scouts engaged in the planning and tracking of orders and the delivery of cookie orders, offering evidence of important conceptual and procedural learning within this community valued activity.

There are several significant aspects to this sociocultural perspective for early mathematics teaching and learning. For one, there was a rejection of the Vygotskian notion of internalization because it signified separate psychological planes for the individual and the community (Sawyer 2004), when the "child and the social world are mutually involved" and, thus, cannot be "independently defineable" (Rogoff 1990, p. 28). As a consequence, determinations of young children's mathematical capabilities had to rely on an analysis of the collective actions and social interplay—not the assessments of any individual student. For another, classrooms and schools were not held as the conduits of formal, abstracted mathematical concepts or procedures but as sociocultural venues in which particular values, customs, and participatory structures are developed.

Views of Children and the Teaching of Mathematics

The shifting perspective toward more socially-nested forms of learning mathematics confronted differing orientations and agendas articulated by the educational and political establishments of this time. Specifically, the performance of young children in mathematics across the global community did not necessarily or consistently favor more participatory or social models of instruction as those advocated by socioculturalists. Rather the international profile was quite mixed; from the structured and more formal approach to early mathematics within certain countries (e.g., Singapore) to more informal and exploratory methods of others (e.g., New Zealand). However, international studies revealing these differences in teaching styles across various countries highlighted the idea that even *teaching is a cultural activity* (Stigler and Hiebert 1999). "Teaching, like other cultural activities, is learned through informal participation over long periods of time. It is something one learns to do by growing up in a culture rather than by formal study" (p. 2).

Despite the variable approaches to teaching mathematics, this era saw a rising interest in non-school factors that influence the learning of mathematics. For example, research with young children has illustrated that the degree to which students struggle with mathematics can be highly context dependent. In other words, knowledge learned in one situation does not necessarily transfer to other situations. For example, work by Carraher et al. (1985) showed that Brazilian children who demonstrated sophisticated thinking about arithmetic in contextualized settings often struggled with the same problems presented in numerical form (Sophian 1999). These findings support the earlier notions that children can and do gain much informal knowledge about mathematics outside of school, and that this knowledge should be a source for learning. Connecting this knowledge to teaching has been the focus of large-scale projects such as *Cognitively Guided Instruction* (Carpenter and Fennema 1991).

The home environment also was seen to play a role in the early learning of mathematics. For instance, in a study of four- to six-year olds, Blevins-Knabe and Musun-Miller (1996) found that increased number activities in the home generally predicted scores on standardized tests of mathematics achievement. However, this pattern did not hold across all ethnic groups or across groups of varying levels of education. More recently, Levine et al. (2010) further confirmed the importance of the home environment for later learning of mathematics. These researchers found that the amount of number talk from parents to their 14- to 30-month-olds predicted knowledge of cardinal number meanings at 46 months, even after controlling for socioeconomic status (SES).

At the same time, cross-cultural studies indicated that SES does play a role in early mathematical development. And while preschools do not inevitably close the SES gap, they have the potential to do so when they include high quality mathematics curricula (Starkey and Klein 2008). A contributing factor to growth in knowledge of mathematics during preschool is the degree to which teachers engage in math talk (Klibanoff et al. 2006).

Competing Views

Although the research on sociocultural influences on mathematical learning has been convincing, an opposing view has support as well. This view posits that humans are pre-disposed to acquiring skill with numbers. In a pioneering study, Starkey and Cooper (1980) were able to demonstrate that 16- to 30-week old infants are sensitive to changes in sets of up to four objects. Since that time, a significant amount of research has been conducted to substantiate the claim that humans have an innate sense of numerosity (Butterworth 1999, 2005; Dehaene 1997).

Further, those who retained a more cognitive view of early mathematics learning remained active during this period. Whether these examinations of young children's mathematical thinking and reasoning were conducted within research laboratories, classrooms, and in home environments, they focused on the mental processing and performance indicators that individual children demonstrated (e.g., Kerkman and Siegler 1997; Rittle-Johnson and Siegler 1998). Although such cognitive emphases were not commonplace among mathematics educators during this period, findings from such studies, combined with the rising interest in neuroscience and neurobiological served as omens to the now emergent era.

Emerging Era of Embodied Learning (2000–present)

There is always a tremendous risk involved in attempting to describe the current era. Some degree of distance is critical in making the appropriate determination. That being said, we will briefly consider what we see as signs or omens that distinguish this present phase in the history of early mathematics education. Historically, mathematics or the teaching of mathematics has been considered either not important or inappropriate for young children, but a significant body of research now suggests otherwise. Mounting evidence illustrates that young children have the interest and capacity to learn meaningful mathematics at early ages. That burgeoning research also suggests that adult guidance and support is needed to fully realize this potential and that the level of necessary support may vary across individuals (National

Research Council 2009). Moreover, there is significant evidence regarding the importance of early mathematics skills and their predictive power for later learning (Duncan et al. 2007).

In many ways, current beliefs about the learning of mathematics in early childhood represent an amalgamation of several perspectives highlighted during the past century. Psychological, social, and cultural perspectives each contribute to our current understandings of early mathematics education, and most recently, neurological perspectives have shed light on the nature of mathematical development (Butterworth 1999; Dehaene 1997). Specifically, the sociocultural orientations that arose in the prior era have remained evident within the educational research community, whereas the persistence of investment in basic skills development and the assessment of young children can still be identified within the K-12 experience.

We have chosen to label this final period as the Era of Embodied Learning because of the re-emergence of consideration of biological/neurological indicators of mathematics learning and development. Greater funding is being directed toward fMRI and ERP (event related potentials) studies of the young mathematical mind. Further, entire conferences and volumes are now dedicated toward the genetic and neurological foundations for mathematical learning and performance within very young populations (Butterworth 2005). For example, Blair et al. (2008) asserted that some children make arithmetic errors that reflect not only faulty procedural and conceptual knowledge but also a failure of executive functioning processes such as working memory and inhibitory control. Also considered to be an aspect of self-regulation, inhibitory control was found to be especially important in both mathematics and reading in the early years (Blair and Razza 2007). As a result, Blair and colleagues suggested that executive processes need explicit attention in the classroom.

Thus, the mind/body duality that had been characteristic of early decades has begun to give way to a realization that not only is mind nested in the sociocultural collective but also that the mind and the body of the child work as one. The more that is understood about the whole child—neurologically, biologically, and cognitive—individually *and* within the broader sociocultural context, then the better supportive and facilitative environments can be devised to support that child's mathematical learning and development. That is the bottom line of this emergent era.

Conclusions

Over the past century, we have seen the views of young learners and their capacity to understand and do mathematics shift as various theoretical perspectives have come to the forefront, while others fade into the background. Currently, the research community seems to have embraced multiple perspectives, simultaneously acknowledging individual, social, and cultural influences on mathematical thought. With these acknowledgments has come an increased awareness of the importance of mathematics for young children and a belief in their interest and capacity for learning it. *Mathematics for all* has been embraced by early childhood educators and researchers alike, and the result has been a strong movement toward the reconceptualization of mathematics learning for young children. As the ensuing chapters illustrate, great strides have already been made in this direction.

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