Planetary Precessional Transmissions: Synthesis and Generation Technologies

Ion Bostan

Abstract Some problems of mechanical transmissions can be solved with special effects by developing new types of transmissions based on planetary precessional transmissions with multiple gear, that were developed by the author. Absolute multiplicity of precessional gear (up to 100 % pairs of teeth simultaneously involved in gearing, compared to 5–7 %—in classical gearings) provides increased lifting capacity and small mass and dimensions. The article presents the analysis of the main structures of precessional transmissions, the theoretical aspects of non-standard profiles generation.

Keywords Precessional transmission · Gear · Tooth profile

List of Symbols

i	Gear ratio
Ψ	Semi product angle of rotation
$\Delta\Psi_3$	Diagram error
δ	Pitch angle of the roller axis
θ	Nutation angle
β	Roller taper angle
z	Number of teeth
α	Gearing angle

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1 Introduction

A problem for engineering companies (especially in the metalworking industry, automotive, chemical and metallurgical industries) is to satisfy the ever-increasing requirements to the transmissions used in majority of industrial machinery and technological equipment related to bearing capacity, compactness, mass and dimensions, low cost of production, etc., and, in particular, to kinematical characteristics, structural compatibility with other aggregates of the equipment, etc. Gearings are considered the most sophisticated components of machines. Machine reliability depends very much on the gearing mechanical transmission operation, in general. The quality indices of traditional gears were increased largely by changing involute gearings, and by creating new gearings, such as Novikov-Hlebanija (Krasnoshhekov et al. 1976) Symark (Kaabushiki 1977), etc.

In the field of planetary transmission it was considered properly to follow the way of developing new types with increased performances. Scientific analysts consider that in the field of technical sciences worldwide an essentially new type of mechanical transmission is being invented every 20–25 years. Thus, the German engineer L. Braren developed the cycloid planetary transmission “CYCLO” in 1923 (A Unique Concept 2012). The Russian engineer A. Moskvitin invented the harmonic friction transmission in 1944 (Tzejtlin and Tzukerman 1969), and in 1959 the American engineer C. W. Musser developed the harmonic gear transmission (Tzejtlin and Tzukerman 1969).

In the late seventies a new type of mechanical transmission has been developed at the Polytechnic Institute of Chisinau (now the Technical University of Moldova). The new type of mechanical transmission entered into international terminology circuit as planetary precessional transmission (PPT). The first patent was issued under this name in 1983. Planetary precessional transmission differs from the classical one by the new principle of motion and load transformation and transmission, i.e. by using sphere-spatial motion of the satellite and variable convex—concave profile. Due to these innovative features gearing multiplicity in planetary precessional transmission reaches 100 % (in classical transmissions - 3–7 %) which provides increased bearing capacity, reduced dimensions and weight, extended kinematical range ±10 … ±3,599 (in harmonic transmissions 79 … 300), high kinematical accuracy, etc. The research team involved in research on precessional planetary transmissions published over 800 scientific articles, obtained about 170 patents, implemented about 20 practical achievements in the field of fine mechanics and specialized technological equipment, in robotic complexes for the exploration of ferro—manganese concretions from the World Ocean bottom (USSR concept), in spaceflight technique, etc.

Know-how in the elaboration of multicouple precessional gear, manufacturing technology and control methods, and a range of precessional transmission diagrams belong to the research team from the Technical University of Moldova.

- The specific character of sphere-spatial (precessional motions of the precessional transmissions pinion make impossible the utilisation of classical involute teeth profiles. This fact requires the elaboration of new profiles adequate to the sphere-spatial motion of pinion, which would ensure high performances to the precessional transmission. Carrying out on the principle of the transfer function continuity and gear [5] based on the principles of the transfer function continuity and gear multiplicity which aims to:

- the elaboration of the gear mathematics model with account of the peculiarities;
- the analytical description of teeth profiles by a system of parametric equations on spherical surface and normal teeth section for inner and plane gear;
- CAD determination of geometrical and cinematic parameters influence of the gear upon the teeth profiles shape and the justification of their rational limits of variation;
- the elaboration of the theoretical basis evaluation of teeth gear multiplicity in precessional transmissions;
- area definition of gear multiplicity existence by 100 % teeth couples.
- the production of non-standard teeth profiles requires a new manufacturing technology. In the complexity of problem "*gear-synthesis-profile study- manufacturing*" the elaboration of efficient methods of teeth manufacturing which ensures a maximum productivity and reduced cost while satisfying the requirements related to the gear with precessional motion plays an important role. To solve this problem the following has been done:
- we elaborated the mathematical model of teeth generation which shows the interaction of teeth in precessional gear;
- we investigated the kinematics of the mechanism of method realisation for teeth generation;
- we determined the tool path of motion and the family envelope of the generating surface by using the computer;
- we elaborated and manufactured from metal milling and tooth grinding tools, inclusively their longitudinal modification.

2 Synthesis of Planetary Precessional Transmissions

2.1 Kinematical Structure

Depending on the structural diagram, precessional transmissions fall into two main types—*K-H-V* and *2K-H* (according to Kudreavtzev's classification), from which a wide range of constructive solutions with wide kinematical and functional options that operate in multiplier regime come out.

Planetary precessional transmissions K-H-V. Kinematical structure of precessional transmission *K-H-V* (Fig. 1a) has four components: planet (wheel) carrier *H*, satellite gear *g*, central wheel *b* and the casing (framework). Satellite gear *g* and the central wheel *b* are under internal gearing, and their teeth generators cross in a

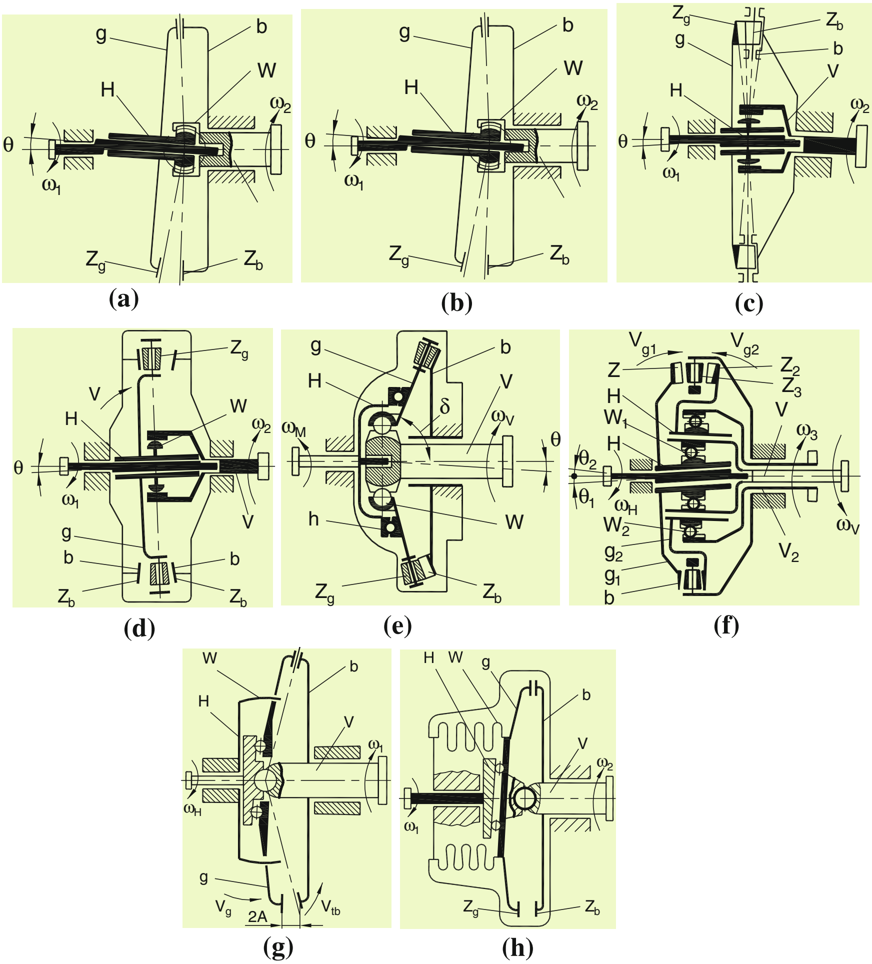


Fig. 1 Kinematical diagram of precessional transmission K-H-V

point called *the centre of precession*. Satellite gear g is mounted on the planet carrier H, designed as a reclined crank, which axis forms with the axis of the central wheel a certain angle θ. The reclined crank H, spinning, forwards a sphere-spatial motion to the satellite gear in relation to the ball joint installed in the centre of precession.

To analyze the kinematics of these transmissions, with an additional connection of the satellite g with the body: At a rotation of the drive shaft (planet carrier H), central wheel b will rotate with a certain angle $\Delta\psi_b$. To determine the position of the driven shaft V depending on the position of the drive shaft H, it is necessary to define the equation of the satellite gear motion. At a constant angular speed of the drive shaft, sphere-spatial motion of the satellite g is described by a system of equations, expressed by Euler angles $\psi = \omega_H t, \phi = \phi(t), \theta = const$, where ψ is the angle of precession (the rotation of axis $O'O'$ of the satellite wheel g with regard to axis OO of the central wheel b); φ—is the angle of the satellite rotation itself around its axis $O'O'$; θ—is the nutation angle (pitch of axis $O'O'$ of the satellite wheel to the central wheel axis OO). It should be noted that the equation of free rotation of the satellite wheel $\varphi = \varphi(t)$ is determined by type of kinematical connection between the satellite gear and the frame. For the transmission with connection mechanism in the form of a toothed coupling (Fig. 1b) gear ratio varies in the limits:

$$i^{g}_{HV} = -\frac{z_g \cos\theta - z_b}{z_b};\ i^{g}_{HV} = -\frac{z_g \cos\theta - z_b}{z_b \cos\theta}, \tag{1}$$

reaching extreme values 4 times for each rotation of the drive shaft H. If necessary, this drawback can be eliminated using as connection mechanism double universal joint, ball synchronous couplings, etc. Average gear ratio will be:

$$i^{g}_{HV_{med}} = -\frac{z_g - z_b}{z_b}. \tag{2}$$

for $z_g = z_b + 1, i^{g}_{HV} = -1/z_b$, i.e. the drive and driven shafts have opposite directions. For $z_g = z_b - 1, i^{g}_{HV} = 1/z_b$, i.e. the shafts rotate the same direction. Precessional transmissions *K-H-V* fall under two basic types:

- with satellite wheel fixed to the casing;
- with central wheel fixed to the casing.

Precessional transmissions diagrams, where the central wheel b is fixed to the casing (bed), and the satellite gear g is fixed to the driven shaft V, are shown in (Fig. 1a–d). In precessional transmissions *K-H-V* with the fixed central wheel an important component is the connection device W of the satellite gear with the driven shaft V.

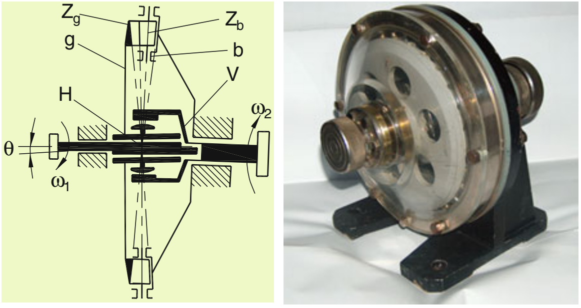
Device W performs transmission of motion between the shafts with parallel axes of the satellite-wheel and the central wheel that have the difference of teeth $z_g = z_b \pm 1$. Average gear ratio of these transmissions is determined from the relationship:

$$i^{b}_{HV} = -\frac{z_g}{z_b - z_g}; \tag{3}$$

$i^{b}_{HV} = -z_g$ for $z_b = z_g + 1$; $i^{b}_{HV} = z_g$ for $z_b = z_g - 1$.

A very important feature of precessional planetary transmissions K-H-V is the possibility of transmitting rotary motion in tight (hermetic) spaces. To this end, Fig. 1, g and h shows two kinematical structures of hermetic precessional transmission.

Based on the structural scheme (Fig. 1c), the demonstrational model of precessional reducer was developed, designed and manufactured (Fig. 2a, b). Manufacture of gear cover made of transparent material allows viewing the operating principle of precession gear.



(a) (b)

Fig. 2 The first (1983) demonstrational model of planetary precessional reducer K-H-V ($i = -29$)

In planetary precessional transmission (Fig. A, d) the satellite-wheel is placed between two fixed gear wheels, bearing the same number of teeth. This transmission has high bearing capacity and ensures reduced diametric dimensions

Advantages of planetary precessional transmissions K-H-V:

- high bearing capacity due to gearing multiplicity;
- wide range of speed reduction $i = 8 \ldots 60$, and in special structures $i = 60 \ldots 3{,}600$;
- operation in reducer and multiplier regimes;
- ensure the transmission of motion in tight spaces via membranes.

Planetary precessional transmissions 2K-H. Precessional transmissions *2K-H* can be developed following two basic schemes: one-side or bilateral location of the central wheels. In the case of bilateral arrangement (Fig. 3a, b, e, f), axial dimensions increase; and at unilateral location (Fig. 3c)—radial dimensions increase. The choice of structural diagram depends on the beneficiary needs. The diagram in Fig. 3c, b shows that the fixed wheel *b* and the movable wheel *a* are arranged on the same side of the satellite wheel—*g*. Pulleys Z_{g1} and Z_{g2} can be placed on separate or joint axes. In the case of location on the same axis, i.e. when $Z_{g1} = Z_{g2}$, the gear ratio is determined from the relation:

$$i^{b}_{HV} = -\frac{z_g}{z_b - z_a}. \tag{4}$$

if $Z_a = Z_{g2} - 1$ and $Z_b = Z_{G1} + 1$, then $i^{g}_{HV} = \frac{z_a}{2}$, and if $Z_a = Z_{g2} + 1$ and $Z_b = Z_{g1} - 1$, then $i^{g}_{HV} = -\frac{z_g}{2}$.

Precessional transmissions *2K-H* have higher performances, inclusive kinematical ones (Fig. 3a). Precessional transmissions *2K-H* comprise the satellite-wheel *g*, with two crown gears Z_{g1} and Z_{g2}, that are in gear with the fixed *b* and movable *a* central wheels. The gear ratio is determined from the relation:

$$i = -\frac{z_{g_1} z_a}{z_b z_{g_2} - z_{g_1} z_a}. \tag{5}$$

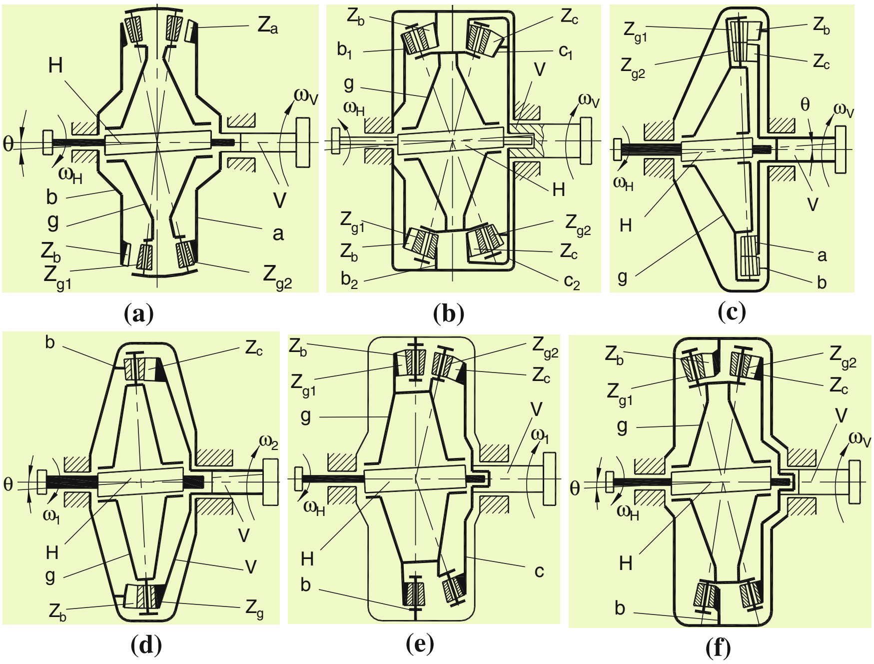


Fig. 3 Kinematical diagram of precessional transmissions 2K-H

Analysis of relation (5) shows that precessional transmissions *2K-H* provide achievement of a larger range of gear ratio. Maximum kinematical effect is reached for the teeth relations:

$$z_b = z_{g_2}; z_{g_1} = z_{g_2} + 1; z_a = z_{g_2} - 1;$$
$$i^b_{HVmax} = z^2_{g_1} \; pentru \; z_a = z_{g_1}, z_{g_2} = z_{g_1} + 1, z_b = z_{g_1} - 1.$$

The proposed gears allow obtaining maximum kinematical effect for other numbers of gear ratios, for example:

$$\begin{aligned} i_{max} &= z^2_{g_1} \; pentru \; z_a = z_{g_1}, z_{g_2} = z_{g_1} - 1, z_b = z_{g_1} + 1, \\ i_{max} &= z^2_{g_2} - 1 \; pentru \; z_b = z_{g_2}, z_{g_1} = z_{g_2} - 1, z_a = z_{g_2} + 1 \end{aligned} \tag{6}$$

While developing precessional transmissions 2K-H a problem comes out related to the selection of the optimal number of wheel teeth, which ensures the obtaining of certain gear ratio. To facilitate the rational selection of the number of teeth by relation (5), possible variants of their choice for the range of gear ratio $i = -13\ldots2{,}401$ were determined using computer. Calculations were performed for the ratio numbers of teeth: $z_{g_1} = z_b + 1; z_{g_2} = z_a + 1; z_{g_1} = z_{g_2} \pm 1, 2, 3\ldots$.

Based on the structural diagram *2K-H* (Fig. 3a) the demonstrational prototype of the precessional reducer *2K-H* was designed and manufactured. Housing made

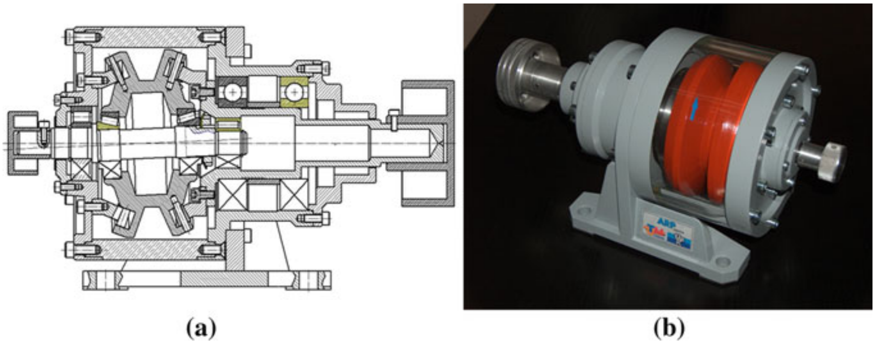


Fig. 4 The first (1988) demonstrational model of precessional reducer 2K-H

of transparent material allows viewing sphere-spatial specific motion (precessional) of the satellite block, the process of toothed wheel gearing with the toothed crowns of the satellite block (Fig. 4a, b).

Precessional transmission (Fig. 3d) shows that the central wheel b is linked to the casing, and the central wheel c—to the driven shaft V. Satellite gear g, mounted between the central gears a and b, gearing simultaneously with the teeth of the fixed b and movable c central wheels, conveys the driven shaft V a reduced motion of rotation. The number of teeth conducting the load simultaneously in the gearing is $z = (z_4 - 1)/2$ for the number of teeth z_4 with couples. The remained teeth contact between each other with passive flanks.

It is necessary to specify a number of features of precessional transmissions *2K-H*, which gives them higher performance compared to similar planetary transmission with cylindrical wheels:

- Precessional transmissions do not require compliance of equal distance between the axes, which fact broadens their optimum design field;
- Kinematics of precessional transmissions does not limit the choice of teeth modules of the gear torque or rollers location pitch, which increases the possibilities of generation of tooth numbers torque and gear ratio interval;
- Characteristics of the designed precessional gearings, unlike traditional involute gears, enlarge considerably kinematical possibilities and performances of precessional transmissions.

Advantages of planetary precessional transmission 2K-H:

- Increased bearing capacity due to the multiplicity of gearing;
- wide range of speed reduction $i = \pm 10 \ldots 3{,}600$ fully guaranteed by the involvement of a satellite with two central wheels;
- 2K-H gearing provides self-locking and one-way or opposite rotation of the drive and driven shafts;
- operates as reducer, multiplier (special constructions) and differential.

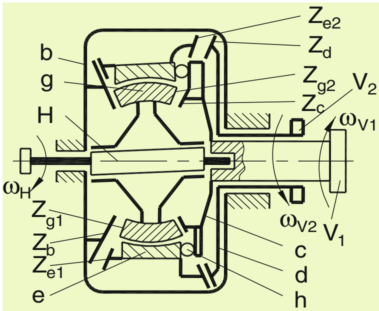


Fig. 5 Kinematical diagrams of complex precessional transmissions

Complex planetary precessional transmission. If specific areas, where you need a very high transmission ratio, a complex structural diagram is proposed, which is a planetary transmission precession 2K-H in two steps. Arrangement of the wheels in the bi-planetary combined transmission version *2K-H* (according to *V.N. Kudreavtzev's* classification) ensures maximum kinematical effect under reduced dimensions and low mass (Fig. 5a) (Bostan 2011). To this end, satellite g is installed on a reclined crank H, and the satellite e is installed on the outer spherical surface of the first satellite. Both satellites are equipped with two serrated crowns, which gear simultaneously with the fixed wheel b and with movable wheels c and d. Wheel carrier H rotates and conveys precessional motion to satellite g, and to central wheel C and the driven shaft V_1—reduced rotational motion. Rotational motion a of the wheel c turns into precessional motion of the satellite e through the rolling bodies, installed between the front inclined part of wheel c and the front part of satellite e gears with the teeth of fixed b and movable d wheels, sending to the last wheel and shaft V_2 rotational motion with the degree of reduction:

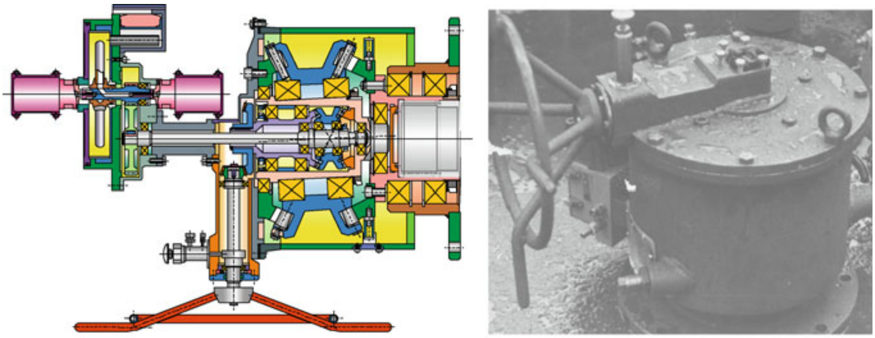
$$i = \frac{z_{g_1} z_c z_{e_1} z_d}{z_b z_{g_2}(z_b z_{e_2} - z_{e_1} z_d) - z_{g_1} z_c(z_b z_{e_2} - z_{e_1} z_d)} = 12,960,000. \qquad (7)$$

This transmission, with the number of teeth $z_b = 59$, $z_{e2} = z_{g2} = 61$, $z_{e1} = z_d = z_{g1}$ $z_c = 60$, for example, will allow obtaining the gear ratio $i = 12{,}960{,}000$.

On the basis of the structural diagram (Fig. 5) an industrial prototype of the driving device for gas pipelines fittings was elaborated, designed and manufactured (Fig. 6a, b). Drive mechanism involves a bi-planetary precessional transmission *2K-H*, in combination with a transmission with one-step cylindrical gear wheels, and fulfils gear ratio $i = 20{,}000$ and torque moment $T = 30{,}000$ Nm.

Advantages of complex planetary precessional transmissions 2K-H:

- high kinematical effect, estimated with motion gear ratio over $i = 1{,}000{,}000$;
- compactness, reduced dimensions and mass.



(a) (b)

Fig. 6 The first (1991) planetary precessional reducer 2K-H in two steps ($T = 30{,}000$ Nm, $i = 20{,}000$)

3 Generation Technologies

3.1 General Remarks

Development of mechanical transmission with gear, different from the classical one, requires complex research in various fields. This finding refers to planetary precessional transmission with multicouple gear, which is characterized by essential constructive-kinematical features. In solving complex problems related to "*gear synthesis—profile research—fabrication*" an important role belongs to developing efficient methods of teeth manufacturing, which would ensure maximum productivity, reduced cost and quality.

Manufacture of precessional gear wheels with convex-concave and variable tooth profile cannot be achieved by existing generation technologies, but through fundamentally new technology. Generation technology of precessional wheel teeth must ensure continuity of motion transformation function with the following conditions: non-standard and variable tooth profile, and satellite carrying out sphere-spatial motion with a fixed point. To achieve the above, a new procedure for teeth processing is proposed by self-generating method with precessional tool against rotating blank.

To develop the theoretical basis for generating tooth profile by running the precessional tool it is necessary to determine the character of continuous contact of the tool cutting edge and profile of the processed wheel tooth for a complete "tool-blank" precessional cycle. In this connection a mathematical model of tooth self-generating method by running the precessional tool was elaborated, which fully reflects the actual interaction of teeth in precessional transmission. For this purpose the following was described:

- kinematical connection of the precessional tool with the blank that ensures continuity of motion transformation function in the linkage "tool-blank";

- path of motion of the tool centre in the fixed system of coordinates;
- path of motion of the tool centre in the movable system of coordinates, connected with the rotating blank;
- the generating contour of the tool in the movable system of coordinate and the system envelope of generating surfaces of the tool for a cycle of precession;
- projection of tool contour envelope in the plane system of coordinates.

3.2 Technological System for Teeth Generation by Sphere-Spatial Motion of Tool as Truncated Cone

3.2.1 Kinematics of Gear Generation System

To achieve teeth generation method a tool carrier device was developed, which diagram is shown in Fig. 7. In the designed device the node, which involves the tool in sphere-spatial motion, is stopped from rotating around the common axis of the main shaft-blank—a kinematic joint. Rotation of blank 3 and main shaft 1is coordinated by the division kinematic chain of machine tool. Kinematic joint of tool with the body must be built so as to ensure continuity of the transmission function of rotation motion, i.e. $\omega_1/\omega_3 = const$. Continuity of transmission function of rotation motion is determined by path of motion of point C, belonging to the movable system of coordinates. To research the kinematics of the device that involves the tool in precessional motion: Imaginary satellite gear (profile generating

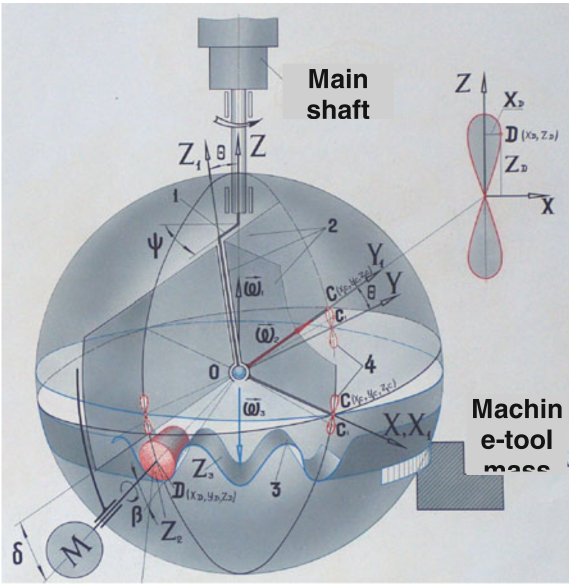


Fig. 7 Principled spatial diagram of teeth processing method by precessional tool running

tool) 2 with an imaginary number of teeth Z_2 (determined by the machine-tool kinematics) gears with the blank 3, fixed on machine-tool table, with the number of teeth $Z_2 = Z_2 \pm 1$. At a turning of the main shaft 1 the blank rotates at angle ψ_3, that corresponds to the angle between the difference of the wheel teeth:

$$\psi_3 = \frac{2\pi}{Z_3}(Z_2 - Z_3). \tag{8}$$

To define the position function of the given device $\psi_3 = f(\psi)$ it is necessary to determine beforehand the equations of the tool motion in the fixed *OXYZ* and movable $OX_1Y_1Z_1$ systems of coordinates. The link between the mentioned systems of coordinates is determined by the Euler angles. Sphere-spatial motion of tool (imaginary wheel) at uniform rotation of the main shaft 1 ω_1 is described by the system of equations

$$\psi = \omega_1 t, \ \theta = const., \ \varphi = \varphi(t), \tag{9}$$

Design of the working device for teeth generating technology should provide limitation of tool rotation around the main shaft of the tool-machine by a certain technical solution, for example by kinematical coupling "bolt-gutter".

In this case the coordinates of the bolt contact point *C* (Fig. 8) with the groove in the movable system of coordinates OX1Y1Z1 will be:

$$X_{1C} = 0, \ Y_{1C} = R_c, \ Z_{1C} = 0, \tag{10}$$

where R_c is the radius of point *C* location.

At sphere-spatial motion of tool 2, the motion of point *C* located in plane *OZX* is limited by the groove walls, i.e. the condition is realised for each value of ψ:

$$X_C = 0. \tag{11}$$

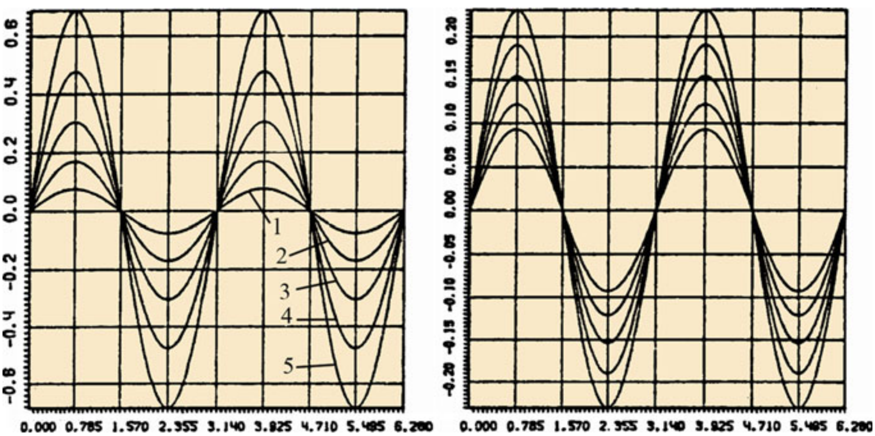


Fig. 8 Dependence of tool position error on the angle of rotation ψ of the main shaft for various angles of nutation θ

Using the transition matrix of the movable system of coordinates $OX_1Y_1Z_1$ connected with the tool and the bolt limiting its rotation around the shaft Z in the fixed system of coordinates, condition $X_c = 0$ o can be written in the form:

$$X_C = \begin{Vmatrix} X_{1c} \\ Y_{1c} \\ Z_{1c} \end{Vmatrix} = 0. \tag{12}$$

Or in extended form

$$X_c = a_{11}X_{1c} + a_{12}Y_{1c} + a_{13}Z_{1c} = 0$$

By replacing a_{11}, a_{12} *and* a_{13} in (11) we obtain:

$$X_c = X_{1c}(\cos\psi\sin\varphi - \sin\psi\sin\varphi\cos\theta) - Y_{1c}(\cos\psi\sin\varphi + \sin\psi\cos\varphi\cos\theta) \\ + Z_{1c}\sin\psi\sin\theta = 0 \tag{13}$$

For the contact point of the bolt with the groove coordinates (1.5) we have:

$$X_c = -R_c(\cos\psi\sin\phi + \sin\psi\cos\phi\cos\theta) = 0. \tag{14}$$

Thus,

$$(\cos\psi\sin\varphi + \sin\psi\cos\varphi\cos\theta) = 0. \tag{15}$$

By solving Eq. (15) we determine the linkage equation between the angle of tool self-rotation 2 and the angle of rotation of the main shaft:

$$\varphi = -arctg(\cos\theta tg\psi). \tag{16}$$

In such case the equations of tool 2 precessional motion take the form:

$$\psi = \omega_1 t,\ \theta = \text{const.}\ \varphi = \text{minus; arctg}(\cos\theta \cdot \text{tg}\psi). \tag{17}$$

To establish the dependence of the angle of rotation of blank ψ_3 on the angle of rotation of the main shaft ψ we describe the blank motion composed of the involved rotational motion with the crank of the main shaft ψ_{3e} and the relative motion of rotation with regard to the crank of the main shaft ψ_{3r}.

In the compound motion of blank $\psi_{3e} = \psi$, and ψ_{3r} represents a certain function $f(\varphi)$ of the angle of rotation of tool φ, that is:

$$\psi_3 = \psi + f(\phi). \tag{18}$$

For ideal precession of the drive mechanism of machine/tool function $f(\varphi)$ will take the form:

$$\psi_3 = \psi + \frac{Z_2}{Z_1}\phi. \tag{19}$$

By considering Eq. (16) we obtain the position function of the kinematical linkage mechanism of the device:

$$\psi_3 = \psi - \frac{Z_2}{Z_1} arctg(\cos\theta tg\psi). \tag{20}$$

Momentary gear ratio of the kinematical linkage mechanism of the device is obtained deriving (20) after ψ:

$$i_{31} = \frac{d\psi_3}{d\psi} = \frac{\omega_3}{\omega_1} = 1 - \frac{Z_2}{Z_3} \cdot \frac{\cos\theta}{\cos^2\psi + \cos^2\theta\sin^2\psi}. \tag{21}$$

Average gear ratio for a rotation of the main shaft will be

$$i_{31}^{med} = \frac{1}{2\pi}\int_0^{2\pi} i\psi d\psi = \frac{1}{2\pi}\left[\psi - \frac{z_2}{z_3} arctg(\cos\theta tg\psi)\right]\Big|_0^{2\pi} = -\frac{Z_2 - Z_3}{Z_3}. \tag{22}$$

Analysis of dependence (22) demonstrates that for the ratio of teeth $z_2 < z_3$ the direction of main shaft rotation of gear cutting machine and blank (imaginary wheel) coinsides, and for the ratio of teeth $z_2 > z_3$ is different. Division kinematical chain of machine tool must provide the following kinematical link: at full rotation of the main shaft the blank (imaginary wheel) should rotate under angle $\psi_3 = 2\pi(Z_2 - Z_3)/Z_3$. This kinematical link defines the average gear ratio of the manufactured gear. Given the fact that the kinematical link "tool—blank" is done by the machine—tool dividing chain under condition $\omega_1/\omega_2 = const.$, the angular velocity variation caused by the kinematic link mechanism (neasurică) of the tool with the frame will transpose on the tooth profile, therefore, it will introduce a diagram error $\Delta\psi_3$ in the tooth profile. The diagream error $\Delta\psi_3$ can be identified by angular positioning error of the blank ψ_3 relative to position ψ_3^{med} of the same blank, which conditionally would rotate uniformly with the gear ratio $i_{31}^{med} = -(Z_2 - Z_3)/Z_3$. in this case the diagram error will be:

$$\Delta\psi_3 = \psi_3 - i_{31}^{med} = \frac{Z_2}{Z_3}[\psi - arctg(\cos\theta tg\psi]. \tag{23}$$

So, the kinematical link of the tool with the frame introduces some diagram error in the tooth profile.

Figure 8 shows the graph of diagram error of tool position error ψ_3 at one rotation of the main shaft and motion of point D in OZY plan. If point C makes a motion in OZY plane the error is transmitted intact to the tool, and the last generates the tooth profile with the same error. To ensure continuity to motion processing function it is necessary to modify the tooth profile by diagram error value $\Delta\psi_3$ by communicating additional motion to the tool.

Correctness of additional motion of the tool was established using a computer calculation program. It was found that generation precision of the manufactured wheel teeth 3 depends on the continuity of its angular speed $\dot{\varphi}$ of the tool 2.

Function analysis (19) shows that for $\varphi = -\psi$ the instantaneous transmission ratio $i_{31} = const$. For condition $\varphi = -\psi$ from Eq. (19) we have:

$$\psi_3 = \psi - \frac{Z_2}{Z_3}\varphi = \frac{Z_2 - Z_3}{Z_3}\psi = \frac{Z_2 - Z_3}{Z_3}\omega_1 t.$$

From this analysis we find that any technical solution to eliminate the influence of diagram error of tooth profile precision generation with precessional tool would be 3D profiling of the contact surfaces of the groove of kinematical link mechanism, which supports the bolt (delimiter of rotation). The bolt contact with the shaped surfaces of groove transmits also the reaction torque from the node, on which the tool is installed to the frame. To achieve the proposed technical solution to exclude the error of 3D profiling of supporting surface of the link channel with the bolt it is necessary to describe the profile of contact surfaces with parametric equations. In this case we take an arbitrary point C on the tool axis with coordinates X_{1c}, Y_{1c}, Z_{1c} (Fig. 7), and identify the path of motion in the fixed system of coordinates $OXYZ$ to satisfy the condition $i_{31} = const$. Using the matrix form for the transition from the coordinate system $OX_1Y_1Z_1$ to the fixed system $OXYZ$ we get:

$$\left\| \begin{matrix} X_c \\ Y_c \\ Z_c \end{matrix} \right\| = A \left\| \begin{matrix} X_{1c} \\ Y_{1c} \\ Z_{1c} \end{matrix} \right\| \tag{24}$$

or by components:

$$\begin{aligned} X_c &= a_{11}X_{1c} + a_{12}Y_{1c} + a_{13}Z_{1c}; \\ Y_c &= a_{21}X_{1c} + a_{22}Y_{1c} + a_{23}Z_{1c}; \\ X_c &= a_{31}X_{1c} + a_{32}Y_{1c} + a_{33}Z_{1c}. \end{aligned} \tag{25}$$

where $a_{i,j}$, $i,j = 1\ldots3$ are cosines of angles between the axes of coordinates.

Considering that instantaneous gear ratio $i_{31} = const$. when $\varphi = -\psi$ then Eq. (25) are transcribed as:

$$\begin{aligned} X_C &= X_{1C}(\cos^2\psi + \cos\theta\sin^2\psi) + Y_{1C}(1-\cos\theta)\cos\psi\sin\psi + Z_{1C}\sin\theta\sin\psi; \\ Y_C &= Y_{1C}(1-\cos\theta)\cos\psi\sin\psi + Y_{1C}(\sin^2\psi + \cos\theta\cos^2\psi) - Z_{1C}\sin\theta\cos\psi; \\ Z_{1C} &= Z_{1C}\sin\theta\sin\psi + Y_{1C}\sin\theta\cos\psi + Z_{1C}\cos\theta. \end{aligned} \tag{26}$$

For the case when point “C” is placed on axis OY_1 its position is defined by coordinates $X_{1C} = 0, Y_{1C} = R_c$, $Z_{1C} = 0$, and Eq. (26) take the form:

$$\begin{aligned} X_C &= R_C(1-\cos\theta)\cos\psi\sin\psi; \\ Y_C &= R_C\left(\sin^2\psi + \cos\theta\cos^2\psi\right); \\ Z_C &= R_C\sin\theta\sin\psi. \end{aligned} \tag{27}$$

Equation (27) represent parametrical equations of groove lateral surfaces, by which the limiting bolt of tool rotational motion around the fixed axis OZ, form a kinematical coupling, and provides the condition $i_{31} = const$. Thus, the shape of groove lateral surfaces by which the bolt forms the kinematical coupling of tool with the casing, described by parametrical Eq. (27), excludes the influence of diagram errors on tooth profile generated with precessional tool.

3.2.2 Analytical Description of the Precessional Tool Path of Motion

According to the principle of teeth generation by proposed method the tool should copy with certain accuracy the shape and path of motion of the pin tooth in the real gearing (involving the central wheel with teeth—satellite gear with pin teeth). In this connection it was necessary to research the tool path of motion with the angle of position to the blank $\delta \geq 0$. For this purpose, a point D was identified on the tool axis (Fig. 9) with coordinates X_{1D}, Y_{1D}, Z_{1D} in the movable system of coordinates $OX_1Y_1Z_1$ and parametrical equations of its motion in the movable system of coordinates were described. For i_{31}-*const*. after a number of transformations we obtain:

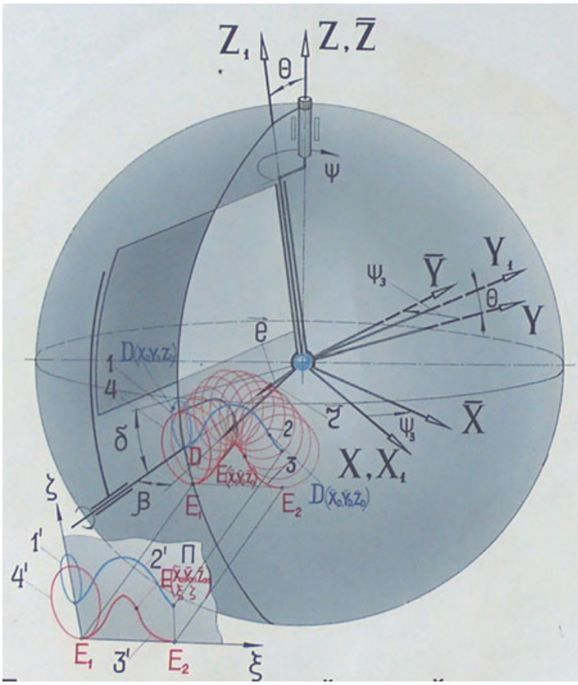


Fig. 9 Determination of the tool surface family envelope

$$\begin{aligned} X_D &= \alpha_{11}X_{1D} + \alpha_{12}Y_{1D} + \alpha_{13}Z_{1D}; \\ Z_D &= \alpha_{21}X_{1D} + \alpha_{22}Y_{1D} + \alpha_{23}Z_{1D}; \\ Z_D &= \alpha_{31}X_{1D} + \alpha_{23}Y_{1D} + \alpha_{33}Z_{1D}. \end{aligned} \tag{28}$$

With condition $\varphi = -\psi$ and constant instantaneous gear ratio i_{31}-*const* we have:

$$\begin{aligned} X_D &= X_{1D}(\cos^2\varphi + \cos\Theta\sin^2\varphi) + Y_{1D}(1-\cos\Theta)\cos\varphi\sin\varphi + Z_{1D}\sin\Theta\sin\varphi; \\ Y_D &= X_{1D}(1-\cos\Theta)\sin\varphi\cos\varphi + Y_{1D}(\sin^2\varphi + \cos\Theta\cos^2\varphi) - Z_{1D}\cos\varphi\sin\Theta; \\ Z_D &= X_{1D}\sin\Theta\sin\varphi + Y_{1D}\sin\Theta\cos\varphi + Z_{1D}\cos\Theta. \end{aligned} \tag{29}$$

For $\delta = 0$ tool coordinates will take the form:

$$X_{1D} = 0, Y_{1D} = -R_u, Z_{1D} = 0. \tag{30}$$

In this case the equations of tool motion depending on the angle of rotation ψ of the main shaft will be:

$$\begin{aligned} X_D &= -R_u(1-\cos\Theta)\cos\varphi\sin\varphi, \\ Y_D &= -R_u(\sin^2\varphi + \cos\Theta\cos\varphi), \\ Z_D &= -R_u\sin\Theta\cos\varphi. \end{aligned} \tag{31}$$

In the case of toothed wheels with angle $\delta > 0$ the tool should be located under the same angle. Then point D will have the following coordinates:

$$\begin{aligned} X_{1D} &= 0, Y_{1D} = -R\cos d, \\ Z_{1D} &= -Rsind, \end{aligned} \tag{32}$$

And the equations of the path of motion of tool in the fixed system of coordinates OXYZ have the form:

$$\begin{aligned} X_D &= -R_u\cos\delta(1-\cos\Theta)\cos\varphi\sin\varphi - R_u\sin\delta\sin\Theta\sin\varphi; \\ Y_D &= -R_u\cos\delta(\sin^2\varphi + \cos\Theta\cos\varphi) + R_u\sin\delta\sin\Theta\cos\varphi; \\ Z_D &= -R_u\cos\delta\sin\Theta\cos\varphi - R_u\sin\delta\cos\Theta. \end{aligned} \tag{33}$$

Exact performance of the tool path of motion according to Eq. (33) was taken into account in the process of elaboration of the tool-carrier device, shown in Fig. 7.

3.2.3 Determination of Family Envelope of Tool Generating Contour

Tooth profile of the processed wheel represents the family envelope of tool generating contour in its relative motion with the tooth. The envelope is determined from the equations of the working surface of the generating tool and parameters of relative motion at folding.

To simplify the process of envelope determination it is necessary to pass to the coordinates of tool centre D in the movable system of coordinates (Fig. 9), linked with the blank 3:

$$\begin{aligned} \overline{X}_D &= X_D \cos\psi_3 + Y_D \sin\psi_3; \\ \overline{Y}_D &= -X_D \sin\psi_3 + Y_D \cos\psi_3; \\ \overline{Z}_D &= Z_D. \end{aligned} \tag{34}$$

where $\overline{X_D}, \overline{Y_D}, \overline{Z_D}$ are the coordinates of the tool centre in the movable system of coordinates; $\psi_3 = \psi/i$ is the blank angle of rotation; i—gear ratio of the kinematical chain "main shaft—blank". Equation (34) define the path of motion of the tool centre, evolving on the sphere. Further, the envelope equations on the sphere were defined (curve 3, Fig. 9).

Further, we find the tool 1 conical working surface (with geometrical shape as frustum) in the movable system of coordinates, applying the condition known from the differential geometry:

$$\overline{re} = r\cos\beta, \text{ or } X \cdot \bar{X}_D + Y \cdot \bar{Y}_D + Z \cdot \bar{Z}_D = R \cdot r\cos\beta, \tag{35}$$

where $\bar{e}$ is the unit vector oriented to the cone axis; β—taper angle of tool.

Wrapping Eq. 5 on the sphere is obtained as result of solving jointly the equations, which describe the family wrapping of tool 1 working surfaces:

$$\begin{aligned} \Phi(X, Y, Z, \psi) = X\bar{X}_D + Y\bar{Y}_D + Z\bar{Z}_D - Rr\cos\beta = 0, \\ \frac{d\Phi}{d\psi} = (X, Y.Z, \psi) = 0 \end{aligned} \tag{36}$$

And the equation of spherical surface:

$$X^2 + Y^2 + Z^2 - R^2 = 0. \tag{37}$$

Therefore we find:

$$\begin{aligned} &\frac{d\Phi}{d\psi} = X\frac{\partial \bar{X}_D}{\partial \psi} + Y\frac{\partial \bar{Y}_D}{\partial \psi} + Z\frac{\partial \bar{Z}_D}{\partial \psi} = 0, \\ &\frac{\partial \bar{X}_D}{\partial \psi} = \frac{\partial X_D}{\partial \varphi}\cos\psi_3 - \frac{X_D}{u}\sin\psi_3 + \frac{\partial Y_D}{\partial \psi}\sin\psi_3 + \frac{Y_D}{u}\cos\psi_3, \\ &\frac{\partial Y_D}{\partial \psi} = -\frac{\partial X_D}{\partial \psi}\sin\psi_3 - \frac{X_D}{u}\cos\psi_3 + \frac{\partial Y_D}{\partial \psi}\cos\psi_3 - \frac{Y_D}{u}\sin\psi_3, \\ &\frac{\partial \bar{Z}_D}{\partial \psi} = \frac{\partial Z_D}{\partial \psi}. \end{aligned} \tag{38}$$

$$\begin{aligned}
\frac{\partial X_D}{\partial \psi} &= -R\cos\delta(1-\cos\Theta)\cos^2\psi - R\sin\delta\sin\Theta\cos\psi \\
\frac{\partial Y_D}{\partial \varphi} &= -R\cos\delta(1-\cos\Theta)\sin^2\psi - R\sin\delta\sin\Theta\sin\psi, \\
\frac{\partial Z_D}{\partial \psi} &= -R\cos\delta\sin\Theta\sin\psi.
\end{aligned} \tag{39}$$

After introducing (38), (39) into (36) and (37) we obtain:

$$\begin{aligned}
X_o &= \frac{-(ab+de) \pm \sqrt{(ab+de)^2 + (1+a^2+d^2)(R^2-b^2-l^2)}}{1+a^2+d^2}; \\
Y_o &= aX_o + b; \\
Z_o &= dX_o + e,
\end{aligned} \tag{40}$$

where:

$$\begin{aligned}
a &= \frac{X_c \frac{\bar{\partial} Z_D}{\partial \psi} - Z_C \frac{\bar{\partial} X_D}{\partial \psi}}{Z_c \frac{\bar{\partial} Y_D}{\partial \psi} - Z_C \frac{\bar{\partial} Z_D}{\partial \psi}}, \quad a = \frac{R^2 \cos\beta \frac{\bar{\partial} Z_D}{\partial \psi}}{Z_D \frac{\bar{\partial} Y_D}{\partial \psi} - Y_C \frac{\bar{\partial} Z_D}{\partial \psi}}, \\
d &= -\frac{(X_D - aY_D)}{\bar{Z}_D}, \quad e = \frac{R^2\cos\beta - bY_D}{\bar{Z}_D}.
\end{aligned} \tag{41}$$

Equation (40) determine the envelope on the sphere (curvature 3, Fig. 9). To define the envelope of teeth profile in cross section it is necessary to project it in plane Π, perpendicular on two generators, which cross two minimum successive points of profile on the sphere, i.e. points E_1 and E_2 and the centre of precession "O". Coordinates of points E_1 and E_2 are determined from the relations:

$$\begin{aligned}
X_{E_1} &= X_1 = X_o|_{\psi=0} = 0, \\
Y_{E_1} &= Y_1 = Y_o|_{\psi=0} = -R\cos(\delta+\Theta+\beta), \\
Z_{E_1} &= Z_1 = Z_o|_{\psi=0} = -R\sin(\alpha+\Theta+\delta), \\
X_{E_2} &= X_2 = X_o|_{\psi=\frac{2\pi z_2}{z_1}}, Y_{E_2} = Y_2 = Y_o|_{\psi=\frac{2\pi z_2}{z_1}}, Z_{E_2} = Y_2 = Y_o|_{\psi=\frac{2\pi z_2}{z_1}}.
\end{aligned} \tag{42}$$

Via points E_1 and E_2 is drawn a plane, perpendicular on generators OE_1 *and* OE_2. The equation of this plane is determined from the condition:

$$\left[\overline{E_1E_2} \cdot \overline{E_1E}\right][OE_1 \cdot OE_2] = 0 \tag{43}$$

where E is an arbitrary point on plane. Equation (43) is represented as:

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ X_2 - X_1 & Y_2 - Y_1 & Z_2 - Z_1 \\ X - X_1 & Y - Y_1 & Z - Z_1 \end{vmatrix} \begin{vmatrix} i & j & k \\ X_1 & Y_1 & Z_1 \\ X_2 & Z_2 & Z_2 \end{vmatrix} = 0, \tag{44}$$

or

$$A_1X + B_1Y + C_1Z + D = 0, \tag{45}$$

where:

$$\begin{aligned}
A_1 &= (Z_2 - Z_1)(X_2Z_1 - X_1Z_2) - (Y_2 - Y_1)(X_1Y_2 - X_2Y_1);\\
B_1 &= (X_2 - X_1)(X_1Y_2 - X_2Y_1) - (Z_2 - Z_1)(Y_1Z_2 - Y_2Z_1);\\
C_1 &= (Y_2 - Y_1)(Y_1Z_2 - Y_2Z_1) - (X_2 - X_1)(Z_1X_2 - X_1Z_2);\\
D_1 &= -A_1X_1 - B_1Y_1 - C_1Z_1.
\end{aligned} \tag{46}$$

Envelope of teeth profile in cross section was determined by designing the envelope from the sphere on a plane perpendicular on two generators that cross via two minimum successive points of the profile on the sphere. In this case envelope equations of tooth profile in plane will be:

$$X_{0_P} = \frac{D_1X_0}{A_1X_0 + B_1Y_0 + C_1Z_0};\; Y_{0_P} = X\frac{Y_0}{X_0};\; Z_{0_P} = X\frac{Z_0}{X_0}\ , \tag{47}$$

where X_0, Y_0, Z_0; X_1, Y_1, Z_1; X_2, Y_2, Z_2 are the coordinates of the centre of precession O and minimum points on the tooth profile.

Envelope equations of the described coordinates X, Y, Z in the system of coordinates $OXYZ$ are transcribed as equations with two coordinates ξ and ζ in the system of coordinates $E_1\xi\zeta$ (Fig. 9) linked to the described plane with equations

$$\begin{aligned}
E_1E_2 &= \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2};\\
E_1E &= \sqrt{(X - X_1)^2 + (Y - Y_1)^2 + (Z - Z_1)^2} = \sqrt{\xi^2 + \zeta^2};\\
E_2E &= \sqrt{(X - X_2)^2 + (Y - Y_2)^2 + (Z - Z_2)^2} = \sqrt{(E_1E_2 - \xi)^2 + \zeta^2}.
\end{aligned} \tag{48}$$

We obtain the envelope equations in two coordinates ξ and ζ in the system of coordinates $E_1\xi\zeta$, which represent teeth profile generated by precessional tool from Eq. (48):

$$\begin{aligned}
\xi &= \frac{(E_1E_2) + (X - X_1)^2 + (Y - Y_1)^2 + (Z - Z_1)^2 - (X - X_2)^2 + (Y - Y_2)^2 + (Z - Z_2)^2}{2E_1E_2};\\
\zeta &= \sqrt{(X - X_1)^2 + (Y - Y_1)^2 + (Z - Z_1)^2 - \xi^2}.
\end{aligned} \tag{49}$$

Figure 10 shows the profilograms of teeth profile generation with precessional tool performed in CAD/CAM/CAE/CATIA V5R7. On the profilograms curve 1 (Fig. 10*a*) describes the path of motion of the tool centre in the fixed $OXYZ$ system of coordinates, and curve 2—the path of motion of the tool centre in the movable system of coordinates $O\bar{X}\bar{Y}\bar{Z}$, curve 3—family wrapping of precessional tool surfaces (tooth profile), curve 4—generating tool contour.

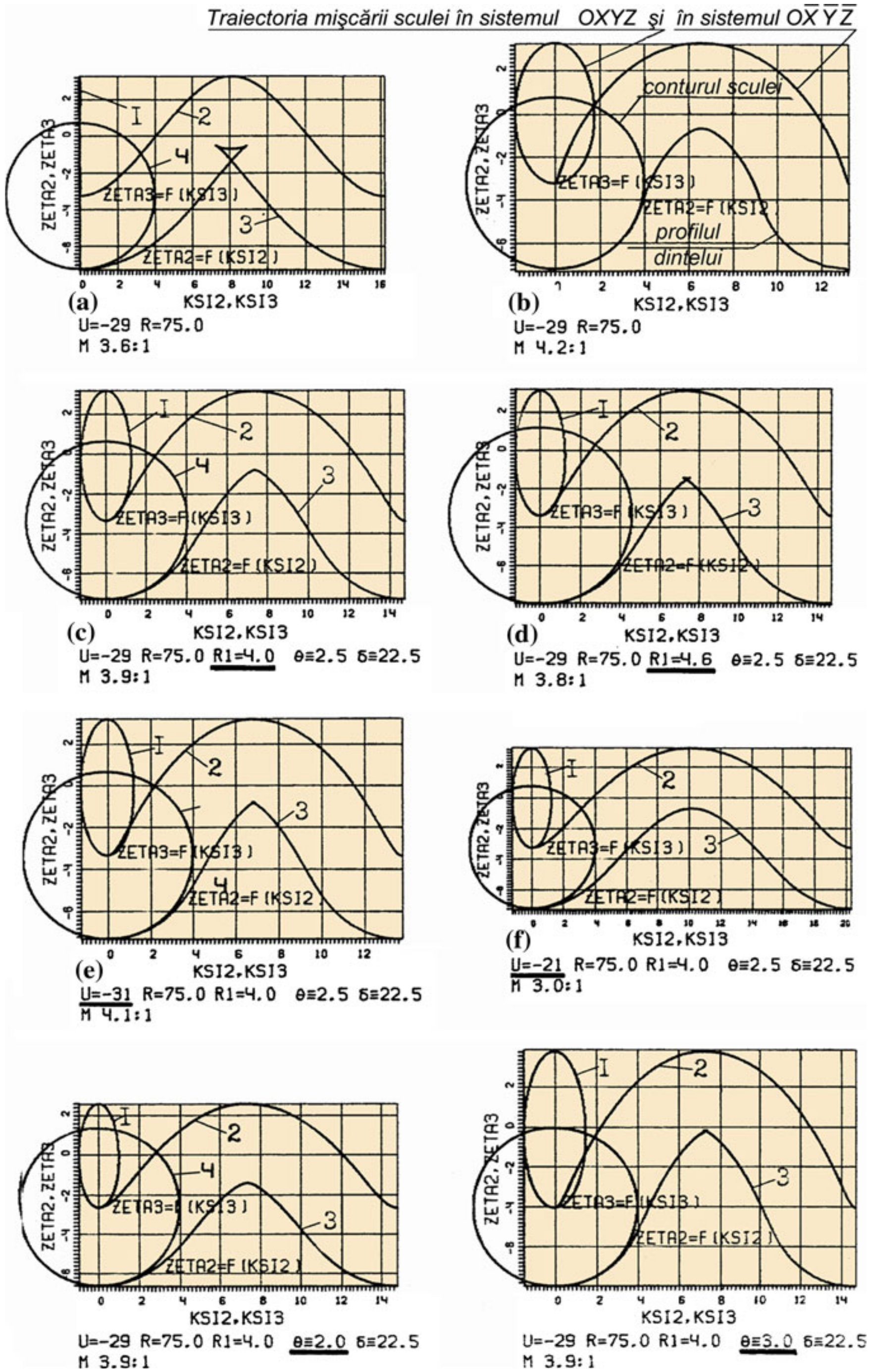


Fig. 10 Profilograms of teeth profile generation with cone-shape precessional tool: *1*, *2* path of motion of tool *centre* in the fixed OXYZ and movable OX'Y'Z' systems of coordinates; *3* teeth profile; *4* generating tool contour

Profilogram analysis (Fig. 10) demonstrates the degree and direction of influence on generated tooth profile of the position angle of tool δ (conical axoid angle) with regard to the axis of rotation of the blank, tool radius R and gear ratio i of the kinematical linkage "main shaft—blank".

3.2.4 Technological Equipment for Generating Teeth with Sfero-Space Motion of Truncated Cone Shaped Tool

The profile of central wheel tooth of precessional gear is variable depending on the values of conical axoid angle δ, taper angle of the rollers β, the nutation angle θ, the number of teeth of gears Z_1, Z_2 and the correlation between them. Fabrication of these profiles using traditional methods is practically impossible, because for each correlation value of all parameters $\delta, \beta, \ \theta$ and Z tooth profile changes shape, which requires the design and manufacture of the tool with the respective profile.

Therefore a new generating technology was proposed, which carries out a set of profiles of the teeth, using a tool with the same geometrical parameters. The method consists of the following: a series of motions coordinated between them against the rotating blank is communicated to the tool (milling cutter or grinding wheel with truncated cone-shaped geometry). The kinematic link of the blank with the tool provides rotation of one-toothed blank in a closed cycle of the motion communicated to the tool. The tool is given such a shape and motion that allows the processing of any possible profile of the set, including longitudinal and profile modification. The described surface on the peripheral side of the tool against the rotating blank reproduces a certain conceivable body, called the *imaginary wheel* (generators).

Using the kinematic chain of gear cutting machine-tool running, gear blank and the tool are brought in a coordinated motion—running motion, which reproduces the imaginary wheel gearing with the blank. Part of metal is removed at each elementary change of tool position in space in relation to the blank. Therefore the working surface of the wheel teeth processed is obtained as envelope of a consecutive series of positions of rotating tool profile generator contour against the blank.

A tool holder device was developed to realize the demanded motions of the tool (Fig. 11a), which can be adjusted to gear cutting machines models *5K32P53*, *5330P*, *53A50*, *5A60*, *5342*, with accuracy class *GOST 6-77*.

To compensate error diagram of the satellite at its sphere-spatial rotation, a kinematical joint connecting the cross-rail with the body is introduced into the teeth grinder, ensuring continuity of the transformation function of rotational motion $\omega_1/\omega_3 = const.$ in the kinematic chain "main shaft—tool—blank". In other words, at teeth processing by proposed method, their profile is corrected by an amount equal to the kinematical diagram error introduced by the sphere-spatial motion of tool with regard to the casing (bed).

It was defined that in real precessional transmissions *2K-H* the link of precessional satellite with the body introduces an error in the driven shaft position.

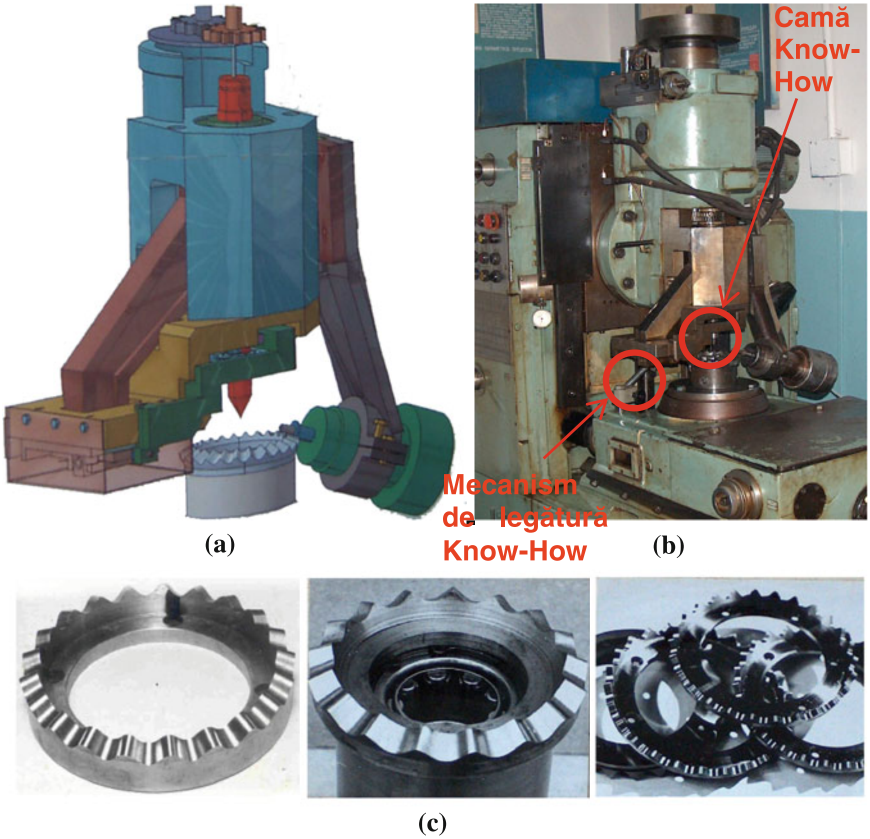


Fig. 11 Generation device for gears with non-standard profile (**a**), machine-tool with fabricated device (**b**) and samples of fabricated gears (**c**)

This fact provokes non-uniformity of its rotation at uniform turning of the drive shaft. Drawback is eliminated by transposition of driven shaft position error on the processed tooth profile. Diagram error elimination is achieved through the construction of cross-rail connection joint to the body, which through a cam installed on the crank shaft communicate auxiliary motion to the tool. The joint ensures continuity to the transformation function of rotational motion along the linkage *shaft-crank-tool-blank*. At tooth processing by proposed method their profile is correlated to value of the shift angle of driven shaft introduced by precessional satellite link in the real transmission.

In the developed tool-carrier device the point of intersection of the fixed axis OZ with the movable axis OZ_1 of the crank (centre of precession) is on the axis of rotation of the gear cutting machine table. To research the features of interaction between the tool and the wheel processed tooth ($\delta > 0$), which axis coincides with axis OZ of the device crank-shaft:

Figure 11, *a* shows the 3D computer model of the processing device for gear wheels with non-standard profile, designed in *AutoDeskInventor* and simulated in *MotionInventor*. Figure 11, *b* shows the picture of gear cutting machine-tool endowed with the device for profile generation by precessional tool. Figure 11, *c* presents samples of gear wheels with non/standard profile, worked out on this machine-tool.

4 Conclusions

Among the characteristics of the estimated results of research in the field of new and efficient drive development we can enumerate the following:

- the elaborated precessional gears ensure: high bearing capacity; high kinematical efficiency; high kinematical accuracy; low noise level and vibrations;
- generation procedure for variable convex-concave teeth profiles provide high efficiency and processing accuracy.

Structural optimization of the precessional transmissions will allow synthesis of new diagrams of precessional transmissions with constant and variable transmission ratio and elaboration of new diagrams of precessional transmissions for specific running conditions.

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