Chapter 2
Finite Temperature Density Profile in SFDM

Victor H. Robles and T. Matos

Abstract Recent high-quality observations of low surface brightness (LSB) galaxies have shown that their dark matter (DM) halos prefer flat central density profiles. On the other hand the standard cold dark matter model simulations predict a more cuspy behavior. Feedback from star formation has been widely used to reconcile simulations with observations, this might be successful in field dwarf galaxies but its success in high mass LSB galaxies remains unclear. Additionally, including too much feedback in the simulations is a double-edged sword, in order to obtain a cored DM distribution from an initially cuspy one, feedback recipes require to remove a large quantity of baryons from the center of galaxies, however, other feedback recipes produce twice more satellite galaxies of a given luminosity and with much smaller mass to light ratios from those that are observed. Therefore, one DM profile that produces cores naturally and that does not require large amounts of feedback would be preferable. We find both requirements to be satisfied in the scalar field dark matter model. Here, we consider that the dark matter is an auto-interacting real scalar field in a thermal bath of temperature T with an initial Z_2 symmetric potential, as the universe expands the temperature drops so that the Z_2 symmetry is spontaneously broken and the field rolls down to a new minimum. We give an exact analytic solution to the Newtonian limit of this system and show both, that it satisfies the two desired requirements and that the rotation curve profile is not longer universal.

Subject headings: galaxies:formation–galaxies:halos–galaxies:individual (NGC 1003, NGC 1560, NGC 6946)–galaxies:fundamental parameters

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2.1 Introduction

One model that has received much attention is the scalar field dark matter (SFDM) model. It is our aim to show that in this model there is an scenario of galaxy formation (see Sect. 2.2) different from the standard model used in CDM simulations and that naturally produces core density profiles, reproduces rotation curves of large and small galaxies on equal footing as MOND and empirical dark matter models do, and that may not need of unrealistic feedback scenarios to agree with data. In previous works it has been verified that the SFDM model reproduces the same cosmological observations that CDM [3, 13, 18, 24, 26].

The idea was first considered by Guzmán and Matos [12]. In the SFDM model the main hypothesis is that the dark matter is an auto-interacting real scalar field that condensates forming Bose-Einstein Condensate (BEC) "drops" [18]. We interpret these BEC drops as the halos of galaxies, such that its wave properties and the Heisenberg uncertainty principle stop the DM phase-space density from growing indefinitely, and thus, it avoids cuspy halos and reduces the number of small satellites [15].

We extend this idea to the case in which the DM temperature and the exited states of the SF are consider together. We consider that the dark matter is a scalar field (SF) Φ, with a repulsive interaction embedded in a thermal bath of dark matter particles of temperature T, we also consider the finite temperature corrections up to one-loop in the perturbations. This is described by the potential [7, 17]

$$V(\Phi) = -\frac{1}{2}\frac{\hat{m}^2c^2}{\hbar^2}\Phi^2 + \frac{\hat{\lambda}}{4}\Phi^4 + \frac{\hat{\lambda}}{8}k_B^2T^2\Phi^2 - \frac{\pi^2k_B^2T^4}{90\hbar^2c^2}. \tag{2.1}$$

for the case when $k_B\mathrm{T} \gg \hat{m}c^2$. Here k_B is Boltzmann's constant, $\hat{\lambda} = \lambda/\left(\hbar^2c^2\right)$ is the parameter describing the interaction, $\hat{\mu}^2 := \hat{m}^2c^2/\hbar^2$ is a parameter, and T is the temperature of the thermal bath. The first term in $V(\Phi)$ relates to the mass term, the second to the repulsive self-interaction, the third to the interaction of the field with the thermal bath, and the last to the thermal bath only.

The galaxy size DM halos (fluctuations) can be described in the non-relativistic regime, where they can be seen as a Newtonian gas. When the SF has self-interaction, we need to add a quartic term to the SF potential and in the Newtonian limit the equation of state of the SF is that of a polytrope of index 1 [13, 26]. Some studies of the stability of these SF configurations have shown that stable large scale configuration are not preferred, Colpi et al. [6], Balakrishna et al. [1], Valdez et al. [27], though the critical mass for stability depends of which parameter values were used, all reach the same conclusion, very large configurations like cluster scales (masses of $\mathrm{M} \geq 10^{13}\ M_\odot$) are usually unstable, therefore, these structures were most likely form just as in the CDM model, by hierarchy [20, 26], i.e., by mergers of smaller halos. The idea of the SFDM model is the following, after inflation big structures start hierarchically to grow up like in the CDM model and its growth will be boosted by the SB mechanism. Inside of them, small structures, like galaxies, dwarf galaxies, etc. condense forming BECs. Thus, all predictions of the CDM model at big scales are reproduced by SFDM [5, 6, 10, 20], while smaller configurations will be formed by condensation.

To study the evolution of the SF perturbations we use the perturbed Klein-Gordon equation [26]

$$\delta\ddot{\Phi}+3H\delta\dot{\Phi}-\frac{1}{a^2}\nabla^2\delta\Phi+V_{,\Phi_0\Phi_0}\delta\Phi+2V_{,\Phi_0}\phi-4\Phi_0\dot{\phi}=0. \tag{2.2}$$

we take $c=1$ in this subsection. Equation (2.2) can be rewritten as:

$$\Box\delta\Phi+\left.\frac{d^2V}{d\Phi^2}\right|_{\Phi_0}\delta\Phi+2\left.\frac{dV}{d\Phi}\right|_{\Phi_0}\phi-4\dot{\Phi}_0\dot{\phi}=0, \tag{2.3}$$

where the D'Alambertian operator is defined as

$$\Box:=\frac{\partial^2}{\partial t^2}+3H\frac{\partial}{\partial t}-\frac{1}{a^2}\nabla^2. \tag{2.4}$$

With $\dot{}=\partial/\partial t$ and $H=(\ln a)\dot{}$ and a the scale factor. Essentially Eq. (2.2) represents a harmonic oscillator with a damping $3H\delta\Phi$ and an extra force $-2\phi V_{,\Phi_0}$. Here the potential is unstable and during the time when the scalar field remains in the maximum, the scalar field fluctuations grow until they reach a new stable point. If we use Eq. (2.1) the change from a local minimum to a local maximum happens when $T=T_C$, thus, we see why T_C determines the moment in which the DM fluctuations can start growing. This implies that the galactic scale halos could have formed within this period and with similar features.

We now suppose that the temperature is sufficiently small so that the interaction between the SF and the rest of matter has decoupled, after this moment the field stops interacting with the rest of the particles. We also assume that the symmetry break (SB) took place in the radiation dominated era in a flat universe. We mentioned that after the SB, the perturbations can grow until the reach their new minimum, thus, each perturbation has a temperature at which it formed and we will denote it by T_Φ. Under these assumptions the equation for an SF perturbation which is formed at T_Φ reads

$$\Box\delta\Phi+\frac{\hat{\lambda}}{4}\left[k_B^2\left(T_\Phi^2-T_C^2\right)+12\Phi_0^2\right]\delta\Phi-4\dot{\Phi}_0\dot{\phi}+\frac{\hat{\lambda}}{2}\left[k_B^2\left(T_\Phi^2-T_C^2\right)+4\Phi_0^2\right]\Phi_0\phi=0 \tag{2.5}$$

In the SFDM model the initial fluctuations come from inflation as in the standard CDM paradigm, later on the field decouples from the rest of the matter and goes through a SB which can increase the fluctuations amplitude forming the initial structures of the universe.

As one of the main interest of this work is finding an exact analytical solution to Eq. (2.5), we will not pursue here the task of solving it numerically. However, the numerical work done in Magaña et al. [19] has confirm that the behavior of the SF perturbation just after the SB is what we had expected from our analysis of Eq. (2.3). They have analysed with some detail the evolution of a perturbation with wavelength 2Mpc and density contrast $\delta = 1 \times 10^{-7}$ after the SB, they took as initial condition $a = 10^{-6}$ and evolve it until $a = 10^{-3}$. They also analysed the case $T \sim T_C$ and show that as the temperature decreases and goes below T_C, Φ_0 falls rapidly to a new minimum where it will remain oscillating. In a similar way, the SF fluctuation grows quickly as Φ_0 approaches the new minimum, and this takes place before recombination.

Therefore, one of the main differences from CDM lies in the initial formation of the DM halos, they are formed very rapidly and almost at the same time, from here we expect that they possess similar features. This difference between the SFDM and CDM models can be tested by observing well formed high-redshift galaxies and also by comparing characteristic parameters of several DM dominated systems, for instance, by observing that indeed, dwarf or LSB galaxies possess cores even at high-redshift, especially since CDM simulations of dwarf galaxies by Governato et al. [11] suggest that their DM density profiles were initially cuspy but later on turn into core profiles due to feedback processes.

We find that the ansatz

$$\delta\Phi = \delta\Phi_0 \frac{\sin(kr)}{kr}\cos(\omega t) \tag{2.6}$$

is an exact solution to Eq. (2.5) provided

$$\omega^2 = k^2c^2 + \frac{\lambda k_B^2}{2\hbar^2}\left(T_C^2 - T_\Phi^2\right). \tag{2.7}$$

Here $\delta\Phi_0$ is the amplitude of the fluctuation. From Eq. (2.7) we notice that now $k = k(T_\Phi)$. For an easier comparison with observations we use the standard definition of number density $n(x, t) = \kappa(\delta\Phi)^2$, where κ is a constant that gives us the necessary units so that we can interpret $n(x, t)$ as the number density of DM particles, as Φ has energy units. With this in mind, we can define an effective mass density of the SF fluctuation by $\rho = mn$ and a central density by $\rho_0 = m\kappa(\delta\Phi_0)^2$. It is important to note that, while $\delta\Phi_0$ is not be obtained directly from observations, the value of ρ_0 is a direct consequence of the RC fit, for this reason we will work with ρ_0 instead of $\delta\Phi_0$.

Combining Eq. (2.6) and the definition of n we obtain a finite temperature static density profile

$$\rho(r) = \rho_0 \frac{\sin^2(kr)}{(kr)^2}, \tag{2.8}$$

provided

$$k_B^2 T_\Phi^2 = k_B^2 T_C^2 - 4\Phi_0^2, \tag{2.9}$$

$$\Phi_0^2 = \Phi_{\min}^2 \tag{2.10}$$

Here $k(T_\Phi)$ and $\rho_0 = \rho_0(T_\Phi)$ are fitting parameters while λ, T_C, κ are free parameters to be constrained by observations.

For galaxies the Newtonian approximation gives a good description, therefore, from Eq. (2.8) we obtain the mass and rotation curve velocity profiles given by

$$M(r) = \frac{4\pi G\rho_0}{k^2}\frac{r}{2}\left(1 - \frac{\sin(2kr)}{2kr}\right), \tag{2.11a}$$

$$V^2(r) = \frac{4\pi G\rho_0}{2k^2}\left(1 - \frac{\sin(2kr)}{2kr}\right). \tag{2.11b}$$

respectively.

2.2 Discussion and Conclusions

In the SFDM model the big DM halos (density fluctuations) form after the SB and grow only after the SF rolls into the minimum of the potential, same which varies with the temperature. Nevertheless, during this time the halos are not in thermal equilibrium, locally the temperature is different from place to place. Therefore, the initial size of the condensation depends on the local halo temperature. From Eq. (2.7) we see that the size of the DM configuration, specified by R, is now temperature-dependent, therefore, as in Harko and Madarassy [14], we also solve the problem of having a unique scale length for all halos, but now in a new way, by using the SB mechanism. Therefore, different formation temperatures of galactic halos may result in different DM halo sizes.

In Fig. 2.1 we show the RC fits of two LSB galaxies and one HSB galaxy using the minimum disk hypothesis (neglecting the baryonic component) taken from a high-precision subsample of McGaugh [21] combined with Broeils [4] for NGC 1560.

We compare Eq. (2.11b) (solid line) with the Einasto profile (dashed line) and notice that these galaxies present two features, long flat tails in the outer region and wiggles. The flat outermost region of Eq. (2.11b) is a direct consequence of using exited states, the same behavior was present in previous works [2, 25] which used $T = 0$. However, the main difference now is that $T \neq 0$ gives exited states in halos which could be stable due to thermal and repulsive self interactions.

The wiggles (small oscillations) are perfectly reproduced by the SFDM model by using combinations of exited states, the value of j that appears in the panels of Fig. 2.1

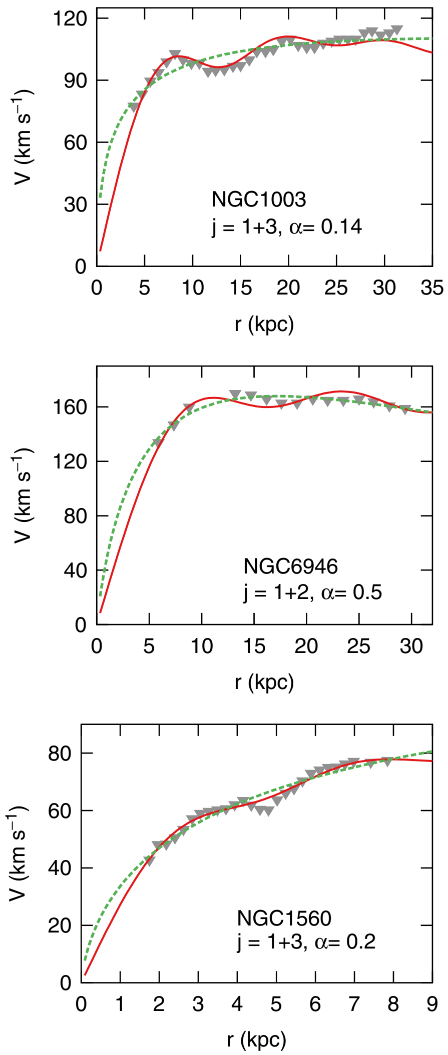

Fig. 2.1 Rotation curve fits to three galaxies. *Top panel*: NGC 1003, *middle panel*: NGC 6946, *bottom panel*: NGC 1560. *Solid lines* (*red* in the online version) are the fits using the SFDM model, *dashed line* (*green* in the online version) represents Einasto's fits, and *triangles* are the observational data. In NGC 1560 we see that the dip at $r \approx 5$kpc is reproduced more accurately in the SFDM profile. Einasto fits show different values of α in each galaxy, suggesting a non-universality in the DM halos, the same is concluded in the SFDM model

specifies the required combination of states for the fit shown. This combination of states in our RC fits suggests that there is not a universal DM profile, some reasons could be that (1) the subsequent evolution determines the final profile, as happens with CDM halos, (2) a collision of two halos with different states formed a halo with the combination of states that we observe today, and (3) the halo formed with the currently observed states and has remained unaltered for a long time. Further research is necessary to determine the most likely explanation. In Einasto's fits wiggles cannot be reproduced with DM only. In fact, if we want to reproduced the oscillations seen in high-resolution RCs with a non-oscillatory DM profile

(NFW, Einasto's, Burkert's etc.), we must include the gas and stars dynamics in the simulations [8], it would be interesting to show the stability of these oscillations after including baryons, as this might be a challenging task in LSBs galaxies due to their low gas surface densities.

For the Einasto's fits we notice that the parameter α changes for each galaxy. As noted in previous works [9, 16, 22, 23], the change in α implies that halos do not possess a universal profile, i.e., we should expect to see a non-universality in the halos of galaxies, this is exactly the same result we have obtained directly from the SFDM model but without assuming a priori a DM density profile.

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