Chapter 2
The Classical Background

Abstract The development of physical science over the last two millenia is traced from the summary of Lucretius, through the early Christian era, to the transformation into critical science after Copernicus. This revolution saw the birth of physics and chemistry to replace Aristotelian authority and alchemy, guided by the principles formulated by Isaac Newton and John Dalton. The new awareness blossomed into the formulation of a comprehensive theoretical mechanics and the recognition of seventy well-characterized chemical elements to replace the four elements of antiquity.

2.1 Introduction

Modern theories of the physical sciences have developed through several refinements from ancient philosophical models and, not surprisingly, many an outdated concept has remained hidden in modern expositions. Most persistent are those that appear self-evident to the non-critical or casual observer and therefore quietly tolerated without further analysis. Some of the most debilitating inconsistencies in theoretical science are of this type and often the hardest to gainsay.

A useful strategy to weed out hidden fallacies is by historical scrutiny of the theoretical progress of science, starting from the classical roots. Most of the authentic ancient sources have been lost, but a reliable account of physical theories in Roman times (∼55 BCE) has been preserved in poetic form, as compiled by Lucretius [1]. The poem develops around six primary propositions:

(i) Nothing is ever created out of nothing
(ii) Nothing is ever annihilated
(iii) Matter exists in the form of invisible particles (atoms)
(iv) Besides matter, the universe contains empty space (vacuity)
(v) The universe consists of matter (with its properties and accidents) and of vacuity and of nothing else
(vi) The atoms are indestructible

These are augmented by two further propositions:

(vii) The universe is boundless
(viii) The universe has no centre

J.C.A. Boeyens, *The Chemistry of Matter Waves*, DOI 10.1007/978-94-007-7578-7_2,

The motion and shape of atoms are described by two sets of secondary propositions.

On atomic movement:

(i) The atoms are always on the move, either falling or rebounding
(ii) They move faster than light
(iii) The atoms normally curve downwards
(iv) Occasionally they swerve slightly from the vertical
(v) They were never either more or less congested than now
(vi) The apparent mobility of matter is an optical illusion

On atomic shape:

(i) The various properties of objects are due to the varieties in the size and shape of atoms
(ii) The number of atomic shapes is large but finite
(iii) The number of atoms of any one shape is infinite
(iv) All visible objects are compounds of different kinds of atoms
(v) Only certain compounds can exist
(vi) The atoms themselves are devoid of colour, heat, sound, taste, and smell, and sentience

These propositions are supported by three general corollaries:

(i) The world is one of an infinite number
(ii) Nature is self-regulating, without interference from the gods
(iii) The world had a beginning and will soon have an end

A number of important conclusions, drawn from appropriate analysis of the basic propositions, deserve special mention:

(i) The universe is not bounded in any direction. It stretches away in all directions without limit
(ii) Solid matter results from a closer union between atoms by the entanglement of their own interlocking shapes
(iii) Through undisturbed vacuum all bodies must travel at equal speed though impelled by unequal weights. The heavier will never be able to fall on the lighter from above
(iv) There is no visible object that consists of atoms of one kind only

Many of these propositions have a surprising modern ring to them, although based on a totally outdated cosmology of a flat earth in infinite space. The disturbing reality is that precisely these objectionable features again underpin the "standard cosmology" of the West. Despite the evidence from general relativity, space is still considered in the expanding universe cosmology as Euclidean (*i.e.* flat) and of infinite extent in all directions. Despite intimate experience with the electromagnetic field, the expanding universe is modelled exclusively in terms of matter moving through vacuity, precisely as presumed by Lucretius.

It would seem that apart from trivial refinement of the atomic model, the philosophical paradigm has not changed in two millenia. Where Lucretius ascribed chemical interaction to the entanglement of interlocking atomic shapes the modern quantum chemist achieves the same in terms of entangled hybrid orbitals, and with the same conviction as Lucretius.

The first millenium after Lucretius saw little change in the understanding of the physical world, except for an infusion of theological dogma and the revival of creation myths that have survived into the present as big-bang cosmology. Instead of progressing, theoretical physics regressed to the Aristotelian model, leaving it to Galileo to rediscover the Lucretian proposition of falling bodies. As the progenitor of chemistry the art of alchemy descended into mysticism and astrology.

By the end of the first millenium CE there was total consensus in the Western World over the workings of the cosmos and the odd heretic, who dared to challenge the revealed truth, could readily be disposed of. Only two problems, destined to disturb this tranquility, remained: how to make gold and where to find the universal remedy for all disease. In searching for the philosopher's stone and an elixir to end the quest, the variety of unexpected secondary products that turned up could no longer be understood within the standard model of alchemy. The emerging scepticism soon spread to other aspects of natural philosophy and it became feasible to challenge metaphysics with real physics; alchemy with chemistry. However, the development of a new scientific paradigm had to await the emergence of a new cosmology, which was initiated by Copernicus.

2.1.1 The Copernican Revolution

Mediaeval science was liberated from its paralysis, imposed by the canonized Aristotelian and Ptolemaic systems, by the new cosmology, inaugurated by acceptance of the heliocentric model proposed by Copernicus. The awkward questions that tortured Lucretius, such as the whereabouts and status of the sun at night, the cause of seasonal changes and the support structure of the earth in space, disappeared almost miraculously, although the new system was resisted by the establishment for more than a century.

The real hero of the revolution was Johannes Kepler, who supposedly [2, p. 178], murdered his superior, Tycho Brahe, in order to gain access to the data that eventually substantiated heliocentric planetary motion without epicycles. Although Kepler's three laws of planetary motion on elliptic, rather than circular, orbits[1] were, for ideological reasons, received with scepticism, they were embraced by a new generation of scientists and eventually inspired the long-awaited new paradigm. The idea of planetary orbits, stabilized by gravity, culminated in Newton's memorable work and it finally also invalidated the notion of astrological interaction between heaven and earth and its stranglehold on alchemy. The way was cleared for the development of modern physics and chemistry.

[1] Only circles were assumed to reflect heavenly perfection.

2.2 Newtonian Physics

The tortuous route from Kepler's laws to Newton's reformulation in terms of the mechanical concepts of inertia and force was reviewed only recently [3] and the details of Newtonian mechanics have been confirmed in writing so many times as to make another repetition superfluous. It is considered more important to concentrate on the philosophical aspects of his work, how these impacted on later developments, and still have a decisive influence on current science.

The single most important concept introduced by Newton in order to rationalize Kepler's laws was the general inverse-square law of gravity, postulated to operate between any two mass points in the universe with a force,

$$F = G\frac{m_1 m_2}{r^2}.$$

Newton was the first to admit that [4, p. 314]:

> ...the Cause of Gravity is what I do not pretend to know...

and further elaborated:

> That Gravity should be innate, inherent and essential to Matter, so that one Body may act upon another at a distance thro' a Vacuum, without the Mediation of anything else, by and through which their Action and Force may be conveyed from one to the other, is to me so great an Absurdity, that I believe no Man who has in philosophical Matters a competent Faculty of thinking, can ever fall into.

The evident implication is that the mathematical formalism that correctly accounts, not only for planetary motion, but also for terrestrial gravitational effects, is by itself sufficient to uphold the general theory, without any physical understanding of the interaction.

Two and a half centuries had to elapse before a satisfactory physical explanation of gravity would emerge from the general theory of relativity. During this time constant positive scrutiny of Newton's gravitational model created the firm conviction that mathematical formalism suffices as the sole criterion to establish a physical theory. As a pertinent example, it is frequently repeated as a great virtue of quantum mechanics that it is adequately formulated on the hand of postulates with no more than mathematical meaning. It is probably fair to say that this has been the hallmark of twentieth-century theoretical physics. The delusional effects of such false security cannot be overemphasized and will be highlighted again in the course of this work. The problem does not lie with the mathematics *per se*, but with an inappropriate physical interpretation that may become established by default.

The particle concept that still dominates physical science is another spin-off of Newton's mathematical model. It arose from Newton's comment in *Principia* [4, p. 295]:

> After I had found that the force of gravity towards a whole planet did arise from and was compounded of the forces of gravity towards all its parts, and

> to every one part was in the inverse proportion of the squares of the distances from the part, I was yet in doubt whether that proportion as the square of the distance did actually hold... it might be that the proportion that was accurate enough at greater distances would be wide of the truth near the surface of the planet, where the distances of the particles are unequal, and their situations dissimilar.

The eventual mathematically inspired conclusion was that the force between two spheres

> ...increases or decreases in proportion to the distance between their centres according to the same square law as applied to the particles themselves. And this is a noteworthy fact.

As paraphrased by Hall [4]:

> Once more mathematics demonstrated a physical improbability as truth: the sphere acted on any body outside it, however close, from its own centre; and this was true of any law of attraction. But if the inverse-square law applied outside the sphere, then inside it the attraction to the centre was directly as the distance.

It is remarkable how this conclusion has gone unnoticed by scores of scientists who have agonized for many years over the presumed infinite self-energy of a charged object such as an electron. Without going too deeply into further philosophical implications, the proposition that a geometrical mass-point takes the place of a physical body, has had enormous ramifications. Not least because of Newton's lack of distinction between particles and atoms. He stated [4, p. 237]:

> ...it seems probable to me that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, movable Particles...
>
> For the matter of all things is one and the same, which is transmuted into countless forms by the operations of nature... and hence we conclude the least particles of all bodies to be also extended, and hard and impenetrable, and movable, and endowed with their proper inertia.

It is clearly implied that all fundamental particles are alike. This proposition has evolved into the modern notion that all elementary particles are zero-dimensional points. Nothing causes more confusion in quantum chemistry than the electronic point particle.

2.3 Daltonian Chemistry

The reason why the Copernian proposal of a heliocentric system was resisted for such a long time, is because it challenged the views of the mediaeval philosopher-saints, who incorporated Aristotelian physics and the Ptolemaic Almagest as a subset of revealed biblical truth. In essence, the result was a universe, centred on the static earth, in the form of eight concentric crystal spheres that move in harmony, directed by the prime mover (God) on the ninth immovable heavenly sphere.

As all motion is interconnected, all terrestrial events are inextricably correlated with their counterparts on the crystal spheres. This awareness developed into a firm guiding principle for the early alchemists whose prospects of success was interpreted to depend equally on a correct reading of the stars as on experimental design [5].

The Copernican revolution changed all that. The earth, no longer the focal centre of the cosmos, was no longer perceived as the microcosm that merely responds to the whims of the *primum mobile*. Events on earth could now happen and be controlled, independent of the stars. As the emphasis on astrology diminished, the relationship between starting materials and reaction products started to be appreciated and alchemy gradually evolved into chemistry.

Robert Boyle, a contemporary of Newton is generally identified as the first modern chemist, whereas Newton's chemical research is usually referred to as alchemy. In this case tradition has almost certainly got it wrong. Boyle is credited with writing the book *Sceptical Chymist* and, apart from formulating a seminal gas law, had contributed very little to indicate a new direction for chemistry. On the contrary, Newton's "alchemy" was pursued in the belief [4, p. 323]:

> ... that chemistry touches on some of the fundamental properties of matter. Through chemistry knowledge might be won of the forces that hold together, or rearrange, the particles of substances.

This is still the spirit in which modern chemistry is practiced. Although Newton therefore contributed very little of chemical significance, he certainly showed the way in which chemistry was to develop.

In less than a hundred years after Newton succeeded in providing a firm foundation for new physics, the new directions in chemistry culminated in the formulation of Dalton's atomic theory which did the same for chemical science.

The formulation of Newton's laws of motion, by refuting the four-element theory of Aristotle, was of equal importance for the development of chemistry. Associated with heavenly spheres, the elements, identified as earth, water, air and fire, each had its own natural comfort zone to which it invariably returned when displaced by unnatural action. The lowest sphere was occupied by earth, always seen to fall back when raised to higher levels, The heavier an object, the faster it falls. Next came water, that preferred to float on the solid earth in the sphere where it belongs. Air occupied the next sphere, with fire at the top. This behaviour soon found a more rational explanation in terms of Newton's law of gravity, rather than each element accelerating towards its own sphere.

The properties of composite bodies could be rationalized in terms of their elemental composition. Heavy objects obviously contained a high proportion of earth and inflammable substances were full of fire. In this sense the four elements represented characteristic principles, rather than indentifiable substances and were interpreted in this way by the alchemists. For laboratory practice the alchemists identified the secondary qualities of metallicity (spirit), brittleness (body), and combustibility (soul), typified by the real 'elements' mercury, salt and sulphur.

The seven metals, astrologically linked with seven planets

silver (Moon), mercury (Mercury), copper (Venus), gold (Sol), iron (Mars), tin (Jupiter) and lead (Saturn),

contained the three qualities in characteristic, subtly different, adjustable proportion, which enabled transmutation.

The period between Newton and Dalton saw the identification of a variety of different airs (gases) such as fixed air (carbon dioxide), acid air (HCl), alkaline air (ammonia), empyreal (dephlogisticated) air (oxygen), phlogisticated air (nitrogen), phlogiston (hydrogen), laughing gas (nitrous oxide), inflammable air (carbon monoxide) and inflammable air of marshes (methane) [6]. The confusing nomenclature was a remnant of the phlogiston theory of combustion, the overthrow of which by Lavoisier was a direct result of the discovery of new gases. The reformulation of known chemical processes in terms of mass-balanced reactions, such as

mercury calcinatus = mercury + oxygen
oxygen + hydrogen = water
oxygen + nitrogen = laughing gas
acid air + alkaline air = sal-ammonia
charcoal + oxygen = fixed air (carbonic gas)
wort of grapes = carbonic gas + alcohol

are all consistent with the conservation of matter, as conjectured by Lucretius.

The stage was set for the formulation of Dalton's atomic theory, in the form of four postulates [6]:

(i) The ultimate particles of a pure substance, whether simple or compound, are perfectly alike in size and weight.
(ii) The "simple atoms" of an elementary substance are indivisible, and can neither be created nor destroyed.
(iii) The "compound atoms" (or "molecules") of a compound are formed by the union of two or more elementary atoms.
(iv) Combination between atoms takes place in the simplest integral ratios, *e.g.* 1 atom of A with 1, 2 or 3 atoms of B.

This reads like a paraphrase of what Lucretius proposed almost two millenia earlier.

The difference is that Dalton had a responsive audience and a mass of data that could be rationalized in the form of a few simple laws of chemical combination, based on Dalton's proposals. These are the well-known laws of:

(i) Fixed Proportions
(ii) Multiple Proportions
(iii) Reciprocal Proportions

The first of these is almost self-evident, giving new meaning to the concept *element*, the second follows as a prediction from Dalton's postulates and the third implies *that each element has its own characteristic equivalent* or *combining weight*.

The third law enabled the specification of chemical formulae that reflect the relative number of equivalents of different elements in various compounds, but because of multiple proportions equivalent weights could not be assigned unambiguously. This ambiguity was finally resolved ten years later by Avogadro's Hypothesis, stating that,

> When the temperatures and pressures are the same, equal volumes of different gases contain equal numbers of molecules.

To make this generally applicable it was necessary to assume that molecules of an elementary gas might consist of groups of similar atoms, such as

$$N_2, O_2, O_3, H_2, P_4, S_8, \textit{etc}.$$

It is therefore possible to substitute molecules for volumes in any equation in which gases are involved. Thus, from the experimental result that

$$(1\text{ vol})\text{nitrogen} + (1\text{ vol})\text{oxygen} = (2\text{ vols})\text{nitric oxide}$$

it follows that

$$N_2 + O_2 = 2NO.$$

The next fifty years were devoted to clear up the confusion between atoms and molecules; between equivalent and atomic weights, finally achieved by Cannizaro, resulting in a consistent set of atomic weights for elements known at the time.

2.4 The Aftermath

Newton saw himself [7]

> ...to have been like a boy playing on the sea-shore, ...now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

The same is evidently true of John Dalton. In both cases their bold conjectures provided the foundation for continued research, aimed at the ultimate refinement of their seminal ideas, which were never in doubt.

2.4.1 Dalton's Legacy

The chemists were so single-minded in their dedication that the brilliant insight of William Prout, that made sense of their blind pursuits, was rejected with such hostility that he had to publish his theory anonymously. Noting the large number of atomic weights on the $(H = 1)$-scale improbably close to whole numbers Prout hypothesized [8]

> ...that *the atoms of all elements are formed by the condensation of atoms of hydrogen*, this element being the **primary matter** or **protyle**.

It is as if the mule-headed obstinacy that sustained alchemy, against all reason, for two millenia has been driving chemical research ever since. The same obstinacy upheld the phlogiston theory in the face of overwhelming counter-evidence. Even the discoverers of oxygen (Priestly) and hydrogen (Cavendish) refused to consider the experimentally proven alternative theory of combustion. At the present time one marvels at the pathological reluctance to abandon the discredited model of orbital hybridization and linear combination of atomic orbitals as an explanation of chemical interactions.

In defence of Prout's hypothesis it was pointed out repeatedly that non-integral atomic weights, such as 35.5 for chlorine, could result from a mixture of atoms with different mass, but the same chemical properties. The only reason why this proposition was rejected, was because it conflicts with Dalton's conjectures. The discovery of isotopes, almost a hundred years later, dovetailed with Prout's proposal, without lessening Dalton's contribution.

In the meantime sustained efforts to either confirm or discredit Prout's hypothesis resulted in a consensual set of accurate atomic weights. However, the revolutionary postulate of elemental gaseous molecules, consisting of aggregates of identical atoms remained a major stumbling block for many years. Such a union is explicitly forbidden in the Lucretian system and was seriously frowned upon by Dalton. The vague notion of chemical affinity, which presupposed an attraction between some type of opposites, offered no explanation for the formation of elemental molecules. It caused the same agony as experienced before by Newton and his contemporaries looking for a physical explanation of gravity. In the same way it culminated in a passive acceptance of some mysterious attraction that leads to the formation of chemical *bonds*. As a palliative it was argued that the exceptionally close approach, which can be achieved between vanishingly small atoms, would imply strong inverse-square gravitational interaction.

The discovery of electrochemical effects later in the 19th century provided a partial explanation of chemical affinity in terms of polarization effects, but also largely confined to heteroatomic interactions. Commenting on chemical affinity Alfred Stewart [9] stated in 1926, when wave mechanics was already in the air:

> In the 'nineties the whole problem lacked a generalization. On the one hand lay the metallic salts with their capacity for ionization, which suggested an electrical conception of chemical affinity; but on the other side stood the enormous host of the carbon derivatives which showed no electrical character and which could be so easily symbolized by means of graphic formulæ. Attempts to force the carbon compounds into line with the inorganic compounds proved unsatisfactory; while the converse effort to reduce the inorganic series to a graphic formulation was equally ineffective: and towards the end of the century, the whole problem seemed beyond the power of chemists to solve upon a simple basis.
>
> It cannot be asserted that, even at the present day, the chemical affinity problem is solved; but at least some change in outlook is obvious. It is generally agreed that chemical affinity is electrical in nature; and that polar and non-polar compounds differ from each other in degree rather than in kind.

> The theory of G.N. Lewis is widely accepted as a means of formulating the machinery of chemical combination and dissociation, though at present it has hardly progressed beyond the stage of being a representation of the facts rather than an explanation. Still, even with its limitations, it furnishes the best mechanical model available to depict the union of one atom with another; and it has the decided advantage of bringing a unifying conception to bear upon the whole question.

The classical concepts of chemical affinity developed from the rapid advances in organic chemistry following the introduction by Berzelius of a simplified scheme of atomic symbols and molecular formulae to replace the awkward graphical symbolism of Dalton. Many of the rules were formalized by Kekulé [10] by assigning valencies of 1, 2, 3 and 4 to H, O, N and C and their congeners, later associated with affinity centres in order to rationalize the assumed three-dimensional structure of molecules. The well-known conjecture of Jacobus van't Hoff which associates optical activity with three-dimensional molecular chirality is perhaps the most important chemical concept that can be traced back directly to Dalton's atomic theory.

2.4.2 Classical Mechanics

The heirs of Newton were equally diligent in repeatedly solving the equation

$$F = ma$$

in all possible situations, but decidedly more open-minded in accepting equivalent alternative formulations, particularly those developed by Lagrange, Hamilton and others.

An important issue left undecided by Newton and his contemporaries was the mode in which light propagates. The bland statement, repeated in many textbooks, that Newton defended the corpuscular nature of light against the wave model of Huygens, has no validity. In his mathematical analysis of *Newton's rings* [11, p. 368], Newton introduced a 'fit' parameter, not physically defined, but in exact agreement with modern values of wavelength. Hence [4, p. 271]:

> Evidently Newton's theory of light was very far from being a simple corpuscular or emission theory. The wave-concept was always essential to it—not as a hypothesis, but as a feature of a mathematical theory from which verifiable predictions could be drawn.

A brief review of classical dynamics [12] shows that the Hamiltonian refinement of Newton's laws resulted in a similar formulation that links particle mechanics to wave motion.

For a system of n mass points the Newtonian equations of force along x, as a function of kinetic energy

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) = m_i \ddot{x}_i = X_i$$

is further modified by defining the force as a function of potential energy,

$$X_i = -\frac{\partial V}{\partial x_i}$$

to yield

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) + \frac{\partial V}{\partial x_i} = 0$$

with similar expressions for y_i and z_i.

Hamilton's principle of least action requires that the Lagrangian function, $L(q, \dot{q}) = T - V$, defined as the difference between the kinetic and potential energies of a system, in terms of generalized coordinates and velocities, be an extremum, *i.e.*

$$\delta \int_{t_0}^{t_1} \{T(q, \dot{q}) - V(q)\} dt = 0.$$

After integration by parts the variation condition yields the set of equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0, \quad j = 1, 2, 3, \ldots, 3n.$$

The Lagrangian equations are further modified by definition of the Hamiltonian function

$$H = \sum_{k=1}^{3n} p_k \dot{q}_k - L(q_k, \dot{q}_k)$$

where

$$p_k = m_k \dot{q}_k = \frac{\partial T}{\partial \dot{q}_k}, \quad k = 1, 2, 3, \ldots, 3n, \qquad \dot{p}_k = \frac{\partial L}{\partial q_k},$$

$$i.e. \quad \frac{\partial H}{\partial p_k} = \dot{q}_k, \qquad \frac{\partial H}{\partial q_k} = -\frac{\partial L}{\partial q_k} = -\dot{p}_k.$$

These are the Hamiltonian or canonical form of the equations of motion, with the advantage that they contain $6n$ partial differential equations of the first order rather than $3n$ of the second order. Noting that kinetic energy is a homogeneous quadratic function of velocities it is shown that $H = T + V = E$, the total energy, and $T = p^2/2m$.

In the case of a time-dependent Hamiltonian it is next demonstrated that

$$\frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2 + V + \frac{\partial S}{\partial t} = 0,$$

the so-called Hamilton–Jacobi equation, where $H = -\partial S/\partial t$, $dS/dt = L$ and the action S is known as Hamilton's Principal Function.

The Hamiltonian in integrated form is given by $S = -Et + W$, where $W(p,q)$ is known as Hamilton's Characteristic Function. It now becomes apparent why S has been called the action of the system. It represents the energy transferred to the system over a period of time. Substitution of $-\partial S/\partial t = H$ into the HJ equation shows that

$$H = T + V = \frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2 + V \tag{2.1}$$

to give another form of the HJ equation:

$$\left(\frac{\partial S}{\partial q}\right)^2 = 2m(E - V).$$

At any given instant the equation $S(q,t) =$ constant defines a surface in Euclidean space. As t varies the surface traces out a volume. At each point of the moving surface the gradient, ∇S is orthogonal to the surface. In the case of an external scalar potential the particle trajectories associated with S are given by the equation $m\dot{q} = \nabla S$. It follows that the mechanical path of a moving point is perpendicular to the surface of constant S for all q and t. A family of trajectories is therefore obtained by constructing the normals to a set of surfaces, each orbit being distinguished by its starting point q_0. For a single starting point the moving surface describes a spherical wavefront. This intimate connection between wave and particle formalisms is of special interest and has been used to forge a link between geometrical and wave optics. By analogy to this it was also used by Schrödinger in his original formulation of a wave equation for quantum matter [13].

The alternative approach is to describe the motion in wave formalism. In this case the wave equation

$$\nabla^2 U = \frac{1}{v^2}\frac{d^2 U}{dt^2}$$

is assumed to have a plane-wave solution of the form

$$U = Ae^{i(\omega t - k\phi)}$$

with variable amplitude A, phase $k\phi$, frequency ω, and variable velocity v. It satisfies the wave equation providing

$$\nabla^2 A - k^2 A(\nabla\phi)^2 + A\frac{\omega^2}{v^2} = 0.$$

Assuming the velocity in vacuum, $v_0 = \omega/k$ one has

$$\frac{\nabla^2 A}{A} - k^2\left[(\nabla\phi)^2 - \left(\frac{v_0}{v}\right)^2\right] = 0$$

where v is the velocity in another medium with index of refraction $n = v_0/v$. Since $\nabla^2 A = 0$ for the plane wave and $k^2 \neq 0$, it is implied that

$$(\nabla\phi)^2 = n^2.$$

The striking mathematical similarity between this so-called eikonal equation of geometrical optics and the HJ equation

$$(\nabla S)^2 = 2m(E - V)$$

is unmistakable. Setting $n^2 = 2m(E - V)/p^2$, where p has the dimensions of a momentum and assuming the proportionality $S = p\phi$, the equations become identical.

This result suggests the possibility that wave propagation can be approximated in terms of particle motion or to describe the motion of a particle in wave formalism, the basic assumption of wave mechanics. However, it is important to note that this eikonal analysis is only valid in the short wavelength region of geometrical optics and under no circumstances does it imply the popular notion of wave-particle duality.

The mechanical equivalents of the dimensionless optical parameters ϕ and n have the dimensions of momentum appropriate for wave-mechanical matter waves. These wave properties become increasingly less in evidence with increasing mass and becomes effectively unobservable for macroscopic massive objects.

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Springer

http://www.springer.com/978-94-007-7577-0

The Chemistry of Matter Waves
Boeyens, J.C.A.
2013, XIV, 243 p., Hardcover
ISBN: 978-94-007-7577-0