

Chapter 2

Theory of Optical Precursors

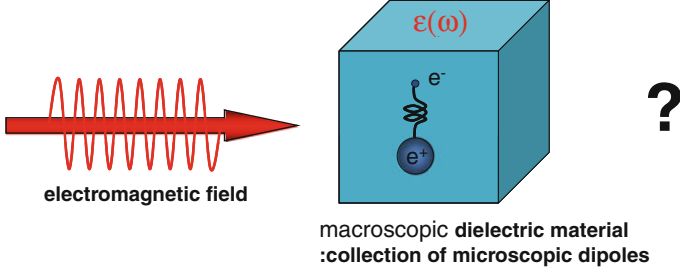
Abstract In this chapter, we discuss the theoretical framework of optical precursors based on the incident electromagnetic waves interacting with dielectric media. The simplest way to interpret the light-matter interaction is the medium optical response to the incident light characterized by dielectric constant $\varepsilon(\omega)$ as a function of incident light frequency ω . To build a theoretical model of optical precursors, first we derive macroscopic dielectric constant $\varepsilon(\omega)$ starting from the microscopic dipole moment $p(\omega)$. Based on Maxwell's equation and transfer function, the transmitted step-modulated electromagnetic field through dielectric media will be derived in general form of inverse Fourier transform of transmitted spectrum. The general expression of transmitted field can be solved numerically or analytically depending on the specific parameter regimes, such as Brillouin regime or resonant regime. The discussion extends from single Lorentz medium to electromagnetically-induced transparency medium, where the main signal transmits without loss.

2.1 Lorentz Medium and Transfer Function

Let's start with the conventional approach, in which we consider collection of dipole moments oscillating at the characteristic frequency ω_0 under externally shined electromagnetic field $E(x, t)$ in x direction as depicted by Fig. 2.1. This can be modeled as Lorentz oscillator driven by external force of E-field denoted as F ,

$$\begin{aligned}
 F &= m\ddot{x} \\
 \Leftrightarrow -eE(x, t) - 2m\gamma\dot{x} - m\omega_0^2x &= m\ddot{x} \\
 \Leftrightarrow -eE(x, t) &= m(\ddot{x} + 2\gamma\dot{x} + \omega_0^2x)
 \end{aligned} \tag{2.1}$$

where x denotes the displacement from the equilibrium position of the Lorentz oscillator. m denotes the mass of electron, and e denotes the electron charge. 2γ is



$$p(\omega) \Rightarrow P(\omega) \Rightarrow \chi(\omega) \Rightarrow \varepsilon(\omega)$$

Fig. 2.1 Light interaction with dielectric media

the damping constant, and ω_0 denotes resonant frequency. Equation (2.1) indicates the model follows damped harmonic oscillators governed by Hooke's law. Assume that the external electromagnetic field oscillates at ω . By Fourier transform, Eq. (2.1) is rewritten as displacement x .

$$\begin{aligned} -eE(x, \omega) &= m(-\omega^2 - 2i\omega\gamma + \omega_0^2)x \\ x &= \frac{eE(x, \omega)/m}{\omega^2 - \omega_0^2 + 2i\omega\gamma} \end{aligned} \quad (2.2)$$

Then microscopic dipole oscillator $p(\omega)$ and microscopic polarization $P(\omega)$ are obtained as a function of frequency,

$$\begin{aligned} p(\omega) &= -ex = \frac{-e^2/m}{\omega^2 - \omega_0^2 + 2i\omega\gamma} E(x, \omega) \\ P(\omega) &= Np(\omega) = \frac{-Ne^2/m}{\omega^2 - \omega_0^2 + 2i\omega\gamma} E(x, \omega) \end{aligned} \quad (2.3)$$

where N is the number of oscillators. Equation (2.3) implies how the macroscopic polarization is formed by the incident E-field. To relate $E(x, \omega)$ and $P(\omega)$, the linear susceptibility $\chi(\omega)$ can be introduced as

$$P(\omega) = \varepsilon_0 \chi(\omega) E(x, \omega) \quad (2.4)$$

where

$$\chi(\omega) = \frac{-Ne^2/m}{\omega^2 - \omega_0^2 + 2i\omega\gamma} \quad (2.5)$$

Now the medium response, macroscopic polarization $P(\omega)$, is added to the original E-field to form a total displacement field $E(x, \omega)$.

$$D(x, \omega) = \varepsilon(\omega)E(x, \omega) = \varepsilon_0 E(x, \omega) + P(\omega) = (1 + \chi(\omega))\varepsilon_0 E(x, \omega) \quad (2.6)$$

where $\varepsilon(\omega)$ is the dielectric function and the plasma frequency is ω_{pl} , and we have,

$$\varepsilon(\omega) = 1 + \chi(\omega) = 1 - \frac{\omega_{pl}^2}{\omega^2 - \omega_0^2 + 2i\omega\gamma} \quad (2.7)$$

where $\omega_{pl} = \sqrt{Ne^2/\varepsilon_0 m}$. Dielectric function $\varepsilon(\omega)$ represents the medium response to an incident light. In general, the first term of $\varepsilon(\omega)$ is the background dielectric constant ε_0 .

The dielectric function is the key to understand the physical interpretation of optical precursors. Plasma frequency ω_{pl} indicates the strength of absorption, one of the mechanisms in light-matter interaction. Full width half max 2γ implies the life time of the system, so that it affects the time scale of the transients or optical precursors. Finally, the atomic resonant frequency ω_0 with respect to the carrier frequency ω_p determines the field strength of the transients, i.e. optical precursors.

In the next section, the dielectric function of a Lorentz medium plays a role in the transfer function, $T(z, \omega) = e^{ik(\omega)z}$, as shown in Fig. 2.2, to evaluate emerging field $E(z, t)$ out of the dielectric material.

To deal with the optical field propagation, we first unify the notation of the optical field throughout the whole book. The real electric field in plane wave is expressed as,

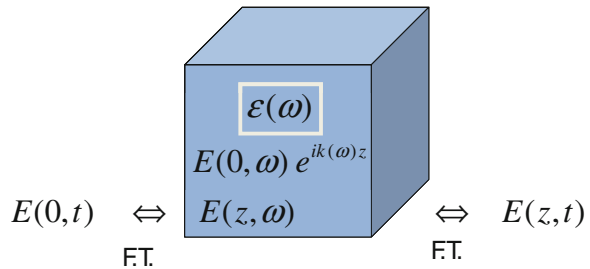
$$\vec{n} \cdot \mathbf{E}(z, t) = \frac{1}{2} \text{Re} \left\{ E_{evp}(z, t) \vec{n} e^{i(kz - \omega t)} \right\} \quad (2.8)$$

where \vec{n} is the polarization unit vector, and $E_{evp}(z, t)$ is the complex envelope. Fourier transform of the complex envelope gives the spectrum of the optical field,

$$\tilde{\mathbf{E}}(\omega, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathbf{E}(t, z) e^{i\omega t} dt \quad (2.9)$$

Now let's consider an incident field $\vec{\mathbf{E}}(0, t)$ and consequent emerging field $\vec{\mathbf{E}}(z, t)$ out of dielectric medium. The medium is characterized via the transfer function $T(z, \omega) = e^{ik(\omega)z}$, which is derived from Maxwell's equation,

Fig. 2.2 Schematic diagram describing transfer function $\varepsilon(\omega)$



$$\vec{\nabla}^2 \vec{\mathbf{E}}(z, t) - \frac{1}{c^2} \frac{\partial^2 \vec{\mathbf{E}}(z, t)}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \vec{\mathbf{P}}(z, t)}{\partial t^2} \quad (2.10)$$

According to Eq. (2.10), the polarization $\vec{\mathbf{P}}(z, t)$ acts as a source of total electric field. Let's remind of the fact that the total field consists of the incident electric field $\vec{\mathbf{E}}(z, t)$ and the modified one by the interaction with the medium polarization. We assume that a polarized plane-wave field propagation vector along the z -direction and isotropic medium and for convenience we denote the electric field as $E(z, t)$. The propagation of light is described by the scalar 1-dimensional wave equation, which is given as follows:

$$\frac{\partial^2 E(z, t)}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E(z, t)}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 P(z, t)}{\partial t^2} \quad (2.11)$$

Here, by performing Fourier transform on Eq. (2.11), it is easy to obtain explicit form of the medium response as a function of frequency and the spectrum of the electric field $\tilde{E}(0, \omega)$.

$$\begin{aligned} \frac{\partial^2 E(z, \omega)}{\partial z^2} + \frac{\omega^2}{c^2} (1 + \chi(\omega)) E(z, \omega) &= 0 \\ \Rightarrow \frac{\partial^2 E(z, \omega)}{\partial z^2} + k^2(\omega) E(z, \omega) &= 0 \end{aligned} \quad (2.12)$$

where $k(\omega) \equiv \omega \sqrt{\mu \epsilon(\omega)}$. Then we express propagated field as a function of frequency.

$$E(z, \omega) = E(0, \omega) e^{ik(\omega)z} \equiv E(0, \omega) T(z, \omega) \quad (2.13)$$

By performing inverse Fourier transform, we could evaluate the transmitted field through medium as,

$$\begin{aligned} E(z, t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(z, \omega) e^{-i\omega t} d\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(0, \omega) e^{i(k(\omega)z - \omega t)} d\omega \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(0, \omega) e^{\phi(\omega, \theta)} d\omega \end{aligned} \quad (2.14)$$

where $\phi(\omega, \theta) = i\omega \frac{z}{c} (n(\omega) - \theta)$, $\theta \equiv \frac{ct}{z}$

$$n(\omega) = \sqrt{1 - \frac{\omega_{pl}^2}{\omega^2 - \omega_0^2 + 2i\omega\gamma}} \quad (2.15)$$

Generally, without approximations, analytic solution of Eq. (2.14) does not exist. To evaluate the integral and identify optical precursor components, Sommerfeld and Brillouin [1] introduced asymptotic theory associated with “saddle-points” methods, which are valid in the limit of $z \rightarrow \infty$. After several decades, Oughstun and Sherman developed modern asymptotic theory of optical precursors

[2]. The conventional asymptotic theory of optical precursors by SB and OS usually consider highly dissipative ($\gamma \sim 0.1 \omega_0$) and off-resonance condition ($\omega_p \neq \omega_0$), which result in precursor transmission with small intensity and a femtosecond time scale. The modern asymptotic analysis introduced by OS usually shows complicated expression of precursors field which can be numerically evaluated.

Recently, on the other hand, recent experimental works report optical precursors [3, 4] in “resonant regime” [5], indicating the characteristics of on-resonant condition as well as narrow linewidth. The existence of optical precursors has been verified in the resonant regime, where one can obtain analytic expression of optical precursors [6]. To prove the existence of optical precursors within the boundary of SB and OS, LeFev simplifies the OS’s modern asymptotic theory as we will discuss later [6].

2.2 Classical Theory of Optical Precursors: Asymptotic Method

In the conventional theory of optical precursors, the input field is taken as a step-modulated sinusoidal electric field of the form as illustrated in Fig. 2.3.

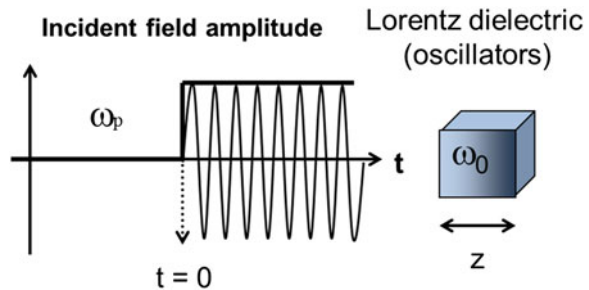
$$E(0, t) = E_0 \Theta(t) e^{-i\omega_p t} \quad (2.16)$$

where $\Theta(t)$ is the Heaviside unit step function. By performing Fourier Transform of the input pulse Eq. (2.16), the input spectrum $E(0, \omega)$ is

$$E(0, t) = E_0 \Theta(t) e^{-i\omega_p t} \Leftrightarrow \tilde{E}(0, \omega) = \frac{iE_0}{\sqrt{2\pi}(\omega - \omega_p)} \quad (2.17)$$

The step-modulated electric field is the starting point of the theories. Optical precursor theory for other input pulses, such as a hyperbolic tangent-modulated pulse [2], square pulse [7], Gaussian pulse [8], have been discussed. In this section, we only consider a step-modulated or square-modulated input pulse.

Fig. 2.3 Step modulated optical field interacts with a Lorentz dielectric medium



The modern asymptotic theory of OS is very complicated, so that it is difficult to obtain a physical meaning of the problem. The numerical evaluation over broad range of parameter spaces as well as understanding their characteristics will provide us insight into the propagation of transient pulses. Conventional method to handle the general type of integral Eq. (2.14) is asymptotic analysis based on the saddle-point method [9] which is valid in the limit when a distance into the medium $z \gg \alpha_0^{-1}$. The inverse of absorption coefficient α_0^{-1} indicates the distance over which the incident electromagnetic field intensity decreases by $1/e$ of its initial value.

To understand some of the concepts underlying the saddle-points method, let's start with presenting the “stationary phase” approximation in the following integral,

$$I = \int f(\omega) e^{iq(\omega)} d\omega \quad (2.18)$$

where $f(\omega)$ is a slowly varying function in terms of ω . The phase term $q(\omega)$ is large enough, causing rapid scillation of the integrand $s^{iq(\omega)}$. In Fig. 2.4a, integration of the fast-oscillating field over the entire range of frequency ω is averaged out and thus result in zero-value except for slowly varying $q(\omega)$ near the stationary point, ω_{sp} , as illustrated in Fig. 2.4b. Note that the subscript ‘sp’ denotes the “stationary point”, but will be used as “saddle point”. To obtain the stationary points, the first derivative of the phase $\partial_\omega q(\omega)|_{\omega_{sp}} = 0$ is required. Phase has an extreme value at these stationary points $q(\omega_{sp})$, so that it is called “stationary phase”. A non-zero contribution to the integral can be obtained by the integration along the stationary point (ω_{sp}). The “steepest-decent” method is a subset of the saddle point method for the case when the phase $q(\omega)$ is a real number. The stationary-phase approximation only considers the first leading-order term of the “steepest-decent” method.

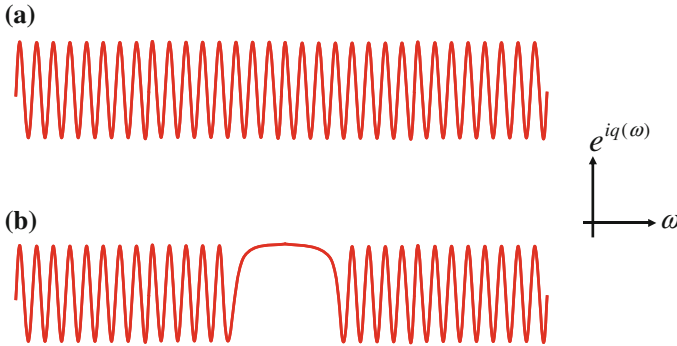


Fig. 2.4 Illustration of stationary phase. **a** Rapid oscillation without a stationary point, and **b** with a stationary point. The figure is from Jeong [9]

The above description provides us precise understanding of the concept of saddle-points in non-zero transmitted field components. Equation (2.14) is equivalent to the Eq. (2.18) when we let $f(\omega) \equiv \tilde{E}(0, \omega) = \frac{iE_0}{\sqrt{2\pi(\omega - \omega_p)}}$, and $q(\omega) \equiv z\varphi(\omega, \theta)/c$. Therefore, the saddle points, ω_{sp} , are obtained by the first derivative with respect to ω

$$\phi'(\omega_{sp}(z, t)) \equiv \partial_\omega \phi(\omega, \theta)|_{\omega_{sp}} = 0 \quad (2.19)$$

The integral has a non-zero value when it is evaluated near the extreme value of the phase.

With the saddle-points ω_{sp} obtained by Eq. (2.19), the corresponding phases $\varphi(\omega_{sp}, \theta)$, and their second derivatives $\varphi''(\omega_{sp}(z, t)) \equiv \partial_\omega^2 \varphi(\omega, \theta)|_{\omega_{sp}}$, the phase can be expressed as Taylor expansion

$$\phi(\omega, \theta) \approx \phi(\omega_{sp}, \theta) + \phi'(\omega_{sp}(z, t))(\omega - \omega_{sp}) + \frac{1}{2!} \phi''(\omega_{sp}(z, t))(\omega - \omega_{sp})^2 \quad (2.20)$$

and

$$\int_{-\infty}^{\infty} e^{\phi(\omega, \theta)} d\omega \approx \int_{-\infty}^{\infty} e^{\phi(\omega_{sp}, \theta) + \frac{1}{2} \phi''(\omega_{sp}(z, t))(\omega - \omega_{sp})^2} d\omega = e^{\phi(\omega_{sp}, \theta)} \frac{\sqrt{2\pi}}{\sqrt{-|\phi''(\omega_{sp}(z, t))|}} \quad (2.21)$$

by recalling $\int_{-\infty}^{\infty} e^{-\frac{\omega^2}{2\sigma^2}} d\omega = \sqrt{2\pi}\sigma$, where $\sigma \equiv 1/\sqrt{-|\phi''(\omega_{sp}(z, t))|}$. Therefore, Eq. (2.14) is given by

$$E_{\omega_{sp}}(z, t) = \frac{iE_0}{\sqrt{2\pi(\omega_{sp} - \omega_c)}} \frac{e^{\phi(\omega_{sp}, t) + i\psi}}{\sqrt{|\phi''(\omega_{sp}(z, t))|}} \quad (2.22)$$

where ψ is the angle of steepest descent.

For the case of a single-resonance Lorentz medium, there are two types of saddle points as we will see in this chapter. The two saddle points are related to the two transient parts of the emerging transmitted field. One of two saddle points has high-frequency components associated with Sommerfeld precursor $E_s(z, t)$, and the other is low frequency or DC component associated with Brillouin precursors $E_B(z, t)$.

As one might notice, pole contribution to the integral becomes dominant and non-zero value can be obtained at singular point $\omega = \omega_p$. This pole contribution is associated with the steady-state part of the emerging field, main signal $E_C(z, t)$ as we will see later. To evaluate transient transmitted field (SB precursors), Oughstun and Sherman [2] have developed Brillouin's asymptotic analysis by keeping a higher order term in the saddle-point equation. OS also have used modern mathematical methods "Olver-type path" [10] to search for a convenient path of integral.

Since the original work by OS [2] have demonstrated saddle points and each part of transmitted field with complicated mathematical expression, we would like to deal with the simplest expression of the phase and consequent derivatives most recently suggested by LeFew [11]. Let's again consider the phase of integrand in Eq. (2.14).

$$\varphi(\omega) = \frac{z}{c} i\omega[n(\omega) - \theta] = \frac{z}{c} i\omega \left[\sqrt{\frac{\omega^2 - (\omega_0^2 + \omega_{pl}^2) + 2i\omega\gamma}{\omega^2 - \omega_0^2 + 2i\omega\gamma}} - \theta \right] \quad (2.23)$$

Instead of using the conventional form, the complex frequency is set as $\eta \equiv i(\omega + i\gamma)$, which is shifted by $-\gamma$ from the imaginary axis of complex frequency ω and rotated by 90° [11]. Then, Eq. (2.23) can be simplified as

$$\varphi(\omega) = \frac{z}{c} (\eta + \gamma) \left[\frac{R_2}{R_1} - \theta \right] \quad (2.24)$$

where $R_1 \equiv \sqrt{(\eta^2 + \omega_0^2 - \gamma^2)/\omega_0^2}$, and $R_2 \equiv \sqrt{(\eta^2 + \omega_0^2 + \omega_{pl}^2 - \gamma^2)/\omega_0^2}$. The saddle-point equation is given by

$$\partial_\omega \phi(\omega, \theta)|_{\omega_{sp}} = R_1^3 R_2 \theta - R_1^2 R_2^2 + (R_2^2 - R_1^2) \eta (\eta + \gamma)|_{\eta_{sp}} = 0 \quad (2.25)$$

With four saddle-points, η_{sp}^\pm (η_s^\pm and η_B^\pm) or ω_{sp}^\pm (ω_s^\pm and ω_B^\pm), obtained from Eq. (2.25), one can evaluate $E_S(z, t)$ and $E_B(z, t)$ numerically from Eq. (2.22) as

$$E_S(z, t) \approx \sum_{\omega_s^\pm} \frac{\mp E_0 e^{\phi(\omega_s^\pm(z, t), t) - \frac{i}{2} \text{Arg}[\phi''(\omega_s^\pm(z, t))]} }{\sqrt{2\pi}(\omega_s^\pm - \omega_p) \sqrt{|\phi''(\omega_s^\pm(z, t))|}} \quad (2.26)$$

$$E_B(z, t) \approx \sum_{\omega_B^\pm} \frac{\mp E_0 e^{\phi(\omega_B^\pm(z, t), t) - \frac{i}{2} \text{Arg}[\phi''(\omega_B^\pm(z, t))]} }{\sqrt{2\pi}(\omega_B^\pm - \omega_p) \sqrt{|\phi''(\omega_B^\pm(z, t))|}} \quad (2.27)$$

Besides the saddle-points, another non-zero contribution to the integral Eq. (2.14) arises from $1/(\omega - \omega_p)$ at singular point $\omega = \omega_p$. The pole contribution to the integral is related to the steady-state response of the medium to the incident field, which is known as the main signal.

$$E_C(z, t) = 2\pi i \text{Res}[\omega = \omega_p] \quad (2.28)$$

With the numerical frame work of asymptotic theory: Eqs. (2.26–2.28), the total transmitted field intensity can be obtained as in Fig. 2.5. The left column shows the results of asymptotic theory in Eqs. (2.26–2.28), which are compared to analytic results based on (2.48–2.49) [right column]. Although the recent asymptotic analysis [11] has significantly reduced the numerical errors for resonant regime, one cannot avoid intrinsic difference between the theory and the experimental data right at the front edge [12].

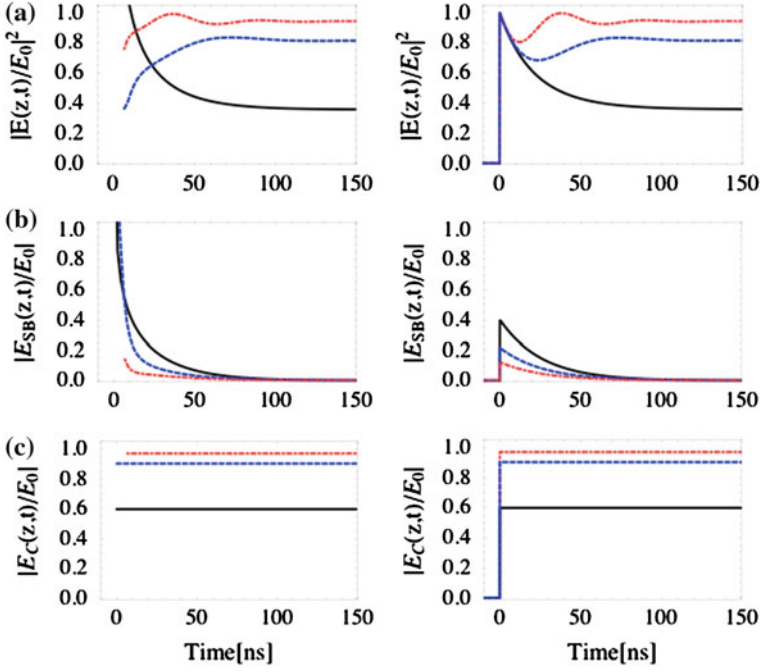


Fig. 2.5 The absolute value of total transient field envelope for the asymptotic theory [left column, Eqs. (2.26–2.28)] and for analytic expression [right column, Eqs. (2.48–2.49)]. **a** The total transient transmission. **b** The amplitude of precursors, and **c** main signal for $\Delta \sim 4\gamma$ (denoted by red dash-dot line), $\Delta \sim 2\gamma$ (denoted by blue dashed line), $\Delta \sim 0$ (denoted by black solid line)

2.3 Optical Precursor Theory for Resonant Medium

The main assumptions of the resonant regime are small plasma frequency ($\omega_{pl} \ll \sqrt{8\omega_0\gamma}$), narrow material resonance ($\gamma \ll \omega_0$), nearly resonance with material oscillators ($\omega_p \sim \omega_0$), and slowly varying approximation (SVA) [13–15]. Under these assumptions, it is possible to evaluate analytic solution describing the propagation of the step-modulated field through the single –Lorentz dielectrics [14–16]. The total emerging field is given by

$$E(z, t) = E_{SB}(z, t) + E_C(z, t) \quad (2.29)$$

where the total transient response is $E_{SB}(z, t)$, which should be equivalent to the sum of two precursors $E_S(z, t) + E_B(z, t)$.

2.3.1 Analytic Expression for a Single-Resonance Lorentz Dielectrics: Two-Level System

In this section, let's consider weakly dispersive resonant medium for which analytic solution of Eq. (2.14) is achievable. The first assumption we take is small plasma frequency condition, $\omega_{pl} \ll \sqrt{8\omega_0\gamma}$, to eliminate square root by the Taylor expansion,

$$n(\omega) = \sqrt{1 - \frac{\omega_{pl}^2}{\omega^2 - \omega_0^2 + 2i\omega\gamma}} \approx 1 - \frac{1}{2} \frac{\omega_{pl}^2}{\omega^2 - \omega_0^2 + 2i\omega\gamma} \quad (2.30)$$

By the condition of medium's resonant frequency, we further simplify the denominator of Eq. (2.30) as $\omega^2 - \omega_0^2 + 2i\omega\gamma \approx 2\omega(\omega - \omega_0 + i\gamma)$, and hence

$$n(\omega) \approx 1 - \frac{\omega_{pl}^2}{4\omega(\omega - \omega_0 + i\gamma)} \quad (2.31)$$

Therefore, the simplified phase is given as

$$\phi(\omega) = i\omega \left[\frac{z}{c} n(\omega) - t \right] \approx -i\omega\tau - \frac{ip}{\Delta_0 + i\gamma} \quad (2.32)$$

where $p \equiv \alpha_0 z \gamma / 2$, retarded time $\tau \equiv t - z/c$, detuning from medium resonance $\Delta_0 \equiv \omega - \omega_0$, and absorption coefficient $\alpha_0 \equiv \omega_{pl}^2 / 2\gamma c$. Based on the simplified phase Eq. (2.32), there are two ways to obtain analytic expression. One is contour integral which is used when we deal with off-resonant expression, but lose information about the saddle points. The other is method of steepest decent [17] and saddle-point method for the on-resonance case.

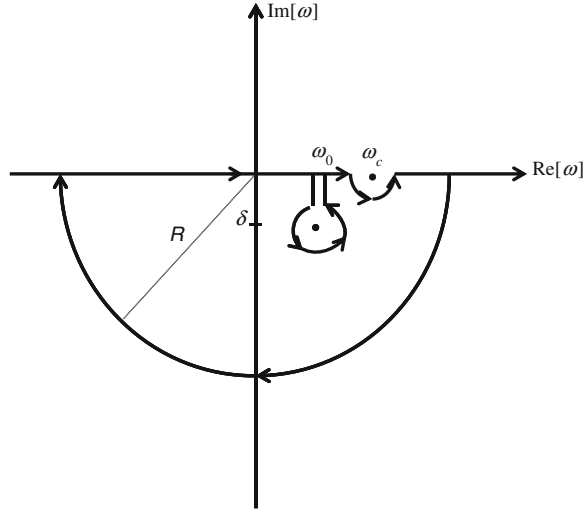
Method 1: Contour integral by Cauchy integral formula [16].

The first approach to solve Eq. (2.14) associated with simplified phase Eq. (2.32) is contour integral based on Cauchy integral formula [16] as illustrated in Fig. 2.6.

The original integral in Eq. (2.14) is performed along the real axis of the complex frequency plane in Fig. 2.6. By Cauchy theorem [17] the integral is given as

$$\begin{aligned} E(z, t) &= \frac{E_0}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{-i\omega\tau}}{\omega - \omega_p} e^{\frac{-iz_0\gamma/2}{\omega - \omega_0 + i\gamma}} d\omega \\ &= \frac{E_0}{2\pi i} \oint_{\omega_p} G d\omega + \frac{E_0}{2\pi i} \oint_{\omega_0 - i\gamma} G d\omega \\ &= E_c(z, t) + E_{SB}(z, t) \end{aligned} \quad (2.33)$$

Fig. 2.6 Schematics of contour integral



where the integrand is

$$G \equiv \frac{e^{i\omega\tau}}{\omega - \omega_p} e^{\frac{-iz_0 z \gamma / 2}{\omega - \omega_0 + i\gamma}} \quad (2.34)$$

The contour integral is divided into two associated with two singular points. First part is singular point at carrier frequency and it contribute to first term of the integral

$$E_C(z, t) = \frac{E_0}{2\pi i} \oint_{\omega_p} \frac{e^{-i\omega\tau}}{\omega - \omega_p} e^{\frac{-iz_0 z \gamma / 2}{\omega - \omega_0 + i\gamma}} d\omega \quad (2.35)$$

By setting the complex variable $v \equiv \omega - \omega_p$,

$$E_C(z, t) = \frac{E_0}{2\pi i} \oint_{v=0} \frac{dv}{v} e^{-i(v+\omega_p)\tau} e^{\frac{-iz_0 z \gamma / 2}{v+i\gamma}} \quad (2.36)$$

A solution to above expression can be evaluated by the residue theorem,

$$\oint_C f(v) dv = 2\pi i \sum (\text{enclosed residues}) \quad (2.37)$$

Thus, the solution has an analytic form indicating exponential decay of the envelope of the transmitted main signal.

$$E_C(z, t) = E_0 \Theta(\tau) e^{-i\omega_p \tau} e^{\frac{z_0 z \gamma / 2}{i\omega_0 - \gamma}} \quad (2.38)$$

There is different way to express Eq. (2.38) using “generating Bessel function”,

$$e^{\frac{i}{2}(u-\frac{1}{u})} = \sum_{m=-\infty}^{\infty} u^m J_m(x) \quad (2.39)$$

By letting $u = iy\sqrt{\tau/p}$, and $x = 2\sqrt{p\tau}$, we get different expression of Eq. (2.39) as

$$e^{i(\tau y - \frac{p}{y})} = \sum_{m=-\infty}^{\infty} i^m \left(\frac{y}{p}\right)^{\frac{m}{2}} J_m(2\sqrt{p\tau}) \quad (2.40)$$

where $y \equiv -\Delta_0 - i\gamma$. Therefore, the alternate form of Eq. (2.40) is given as

$$\begin{aligned} E_C(z, t) &= E_0 \Theta(\tau) e^{\frac{\alpha_0 z \gamma / 2}{i\Delta_0 - \gamma}} e^{-i\omega_p \tau} \\ &= E_0 \Theta(\tau) e^{-i\omega_p \tau + (i\Delta_0 - \gamma)\tau} \sum_{m=-\infty}^{\infty} i^m \left(\frac{\tau}{p}\right)^{\frac{m}{2}} (-\Delta_0 - i\gamma)^m J_m(2\sqrt{p\tau}) \\ &= E_0 \Theta(\tau) e^{-i\omega_p \tau + (i\Delta_0 - \gamma)\tau} \sum_{n=-\infty}^{\infty} \left(\frac{p}{i\Delta_0 - \gamma}\right)^n (p\tau)^{-n/2} J_n(2\sqrt{p\tau}) \end{aligned} \quad (2.41)$$

Now, let's look at the second term of Eq. (2.33), which is the contour integral around singular point of the exponent, $\omega_0 - i\gamma$, by setting $z \equiv \omega - \omega_p + i\gamma$.

$$E_{SB}(z, t) = \frac{E_0}{2\pi i} e^{-i(\omega_0 - i\gamma)\tau} \oint_{z=0} dz \frac{e^{-iz\tau - i\alpha_0 z \gamma / 2z}}{z + \omega_0 - \omega_p - i\gamma} \quad (2.42)$$

By rewriting the denominator as

$$\frac{1}{z - a} = -\frac{1}{a} \sum_{n=0}^{\infty} \left(\frac{z}{a}\right)^n = -\sum_{n=0}^{\infty} \frac{z^n}{a^{n+1}} \quad (2.43)$$

where $a \equiv \omega_p - \omega_0 + i\gamma$, and

$$e^{-iz\tau - ip/z} = \sum_{m=-\infty}^{\infty} (-iz)^m \left(\frac{\tau}{p}\right)^{m/2} J_m(2\sqrt{p\tau}) \quad (2.44)$$

where $p \equiv \alpha_0 z \delta / 2$, Then we rewrite Eq. (2.33) as

$$E_{SB}(z, t) = -\frac{E_0}{2\pi i} e^{(-i\omega_0 - \gamma)\tau} \oint_{z=0} dz \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{z^{m+n} i^m (\tau/p)^{m/2} (-1)^m J_m(2\sqrt{p\tau})}{(\omega_p - \omega_0 - i\gamma)^{n+1}} \quad (2.45)$$

By considering the residue theorem

$$\oint_{z=0} dz z^{m+n} = 2\pi i \delta_{m, (n+1)} \quad (2.46)$$

and $J_{-n}(x) = (-1)^n J_n(x)$,

$$E_{SB}(z, t) = -E_0 \Theta(\tau) e^{-i\omega_p \tau + (i\Delta - \gamma)\tau} \sum_{n=1}^{\infty} \left(\frac{p}{i\Delta_0 - \gamma} \right) (p\tau)^{-n/2} J_n(2\sqrt{p\tau}) \quad (2.47)$$

for $\sqrt{p/(\tau(\Delta_0^2 + \gamma^2))} < 1$. Exponential decay $e^{-\gamma\tau}$ and the Bessel function $J_{-n}(2\sqrt{p\tau})$ in Eq. (2.47) affect the transient time scale, and it indicates that the second term of the contour integral is the transient part of total transmission.

Therefore, for $\sqrt{p/(\tau(\Delta_0^2 + \gamma^2))} < 1$, the total transmitted field is evaluated from the sum of Eqs. (2.41) and (2.47)

$$E_{SB}(z, t) = E_0 \Theta(\tau) e^{-i\omega_p \tau} \left(e^{\frac{p}{i\Delta_0 - \gamma}} - e^{(i\Delta - \gamma)\tau} \sum_{n=1}^{\infty} \left(\frac{p}{i\Delta_0 - \gamma} \right) (p\tau)^{-n/2} J_n(2\sqrt{p\tau}) \right) \quad (2.48)$$

The alternate form of the total transmission is derived by considering

$$\sum_{n=1}^{\infty} S_n = \sum_{n=-\infty}^{\infty} S_n - \sum_{n=-\infty}^0 S_n, \quad \text{and} \quad \sum_{n=-\infty}^{\infty} S_n = \sum_{n=0}^{\infty} S_{-n} \quad \text{where} \\ S_n \equiv \left(\frac{p}{i\Delta_0 - \gamma} \right)^n (p\tau)^{-n/2} J_n(2\sqrt{p\tau}). \quad \text{So in the other region where} \\ \sqrt{p/(\tau(\Delta_0^2 + \gamma^2))} > 1, \text{ we have,}$$

$$E_{SB}(z, t) = E_0 \Theta(\tau) e^{-i\omega_p \tau + (i\Delta_0 - \gamma)\tau} \left(- \sum_{n=-\infty}^{\infty} S_n + \sum_{n=-\infty}^{\infty} S_n - \sum_{n=-\infty}^0 S_n \right) \quad (2.49) \\ = E_0 \Theta(\tau) e^{-i\omega_p \tau + (i\Delta_0 - \gamma)\tau} \sum_{n=0}^{\infty} \left(\frac{-i\Delta_0 + \gamma}{p} \right)^n (p\tau)^{n/2} J_n(2\sqrt{p\tau})$$

The above expression for total transmitted field is useful if one would like to see the effect of detuning within near resonance regime, i.e., $\Delta_0 \approx \gamma$. However, $E_{SB}(z, t)$ cannot be separated to $E_S(z, t)$ and $E_B(z, t)$. To obtain analytic form of each precursor part, one requires restricted on-resonant condition of $\omega_p = \omega_0$ as discussed in method 2.

Method 2: Saddle-point approximation.

Let's redefine the phase $\phi(\omega, t)$ as $E(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(0, \omega) e^{i\phi(\omega, t)} d\omega$, and the first and second derivatives of the phase are

$$\phi'(\omega_{sp}^{\pm}, t) \approx -\tau + \frac{p}{(\Delta_{sp} + i\gamma)^2} \\ \phi''(\omega_{sp}^{\pm}, t) \approx -\frac{2p}{(\Delta_{sp} + i\gamma)^3} \quad (2.50)$$

From above equation, saddle-point can easily obtained from $\partial_{\omega} \phi(\omega, t)|_{\omega_{sp}} = 0$

$$\Delta_{sp}^{\pm} = \pm \sqrt{\frac{\alpha_0 z \gamma}{2\tau}} - i\gamma \quad (2.51)$$

where, $\Delta_{sp} \equiv \omega_{sp} - \omega_0$. Here, let's define $\xi(t) \equiv \sqrt{\frac{\alpha_0 z \gamma}{2\tau}} = \sqrt{\frac{\alpha_0 z \gamma}{2(t-L/c)}}$, then the de-tuned saddle-points is

$$\Delta_{sp}^{\pm} = \pm \xi(t) - i\gamma \quad (2.52)$$

and the second derivatives of the phase are written as

$$\phi''(\omega_{sp}^{\pm}) = -\frac{\alpha_0 z \gamma}{(\pm \xi(t))^3} \quad (2.53)$$

Now let's consider the general form of the transmitted field. The transmitted field is mainly attributed to the saddle point ω_{sp} contribution.

$$\begin{aligned} E(z, t) &\approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(0, \omega_{sp}) e^{i[\phi(\omega_{sp}) + \frac{1}{2}\phi''(\omega_{sp})(\omega - \omega_{sp})^2]} d\omega \\ &= \frac{E(0, \omega_{sp})}{\sqrt{2\pi}} e^{i\phi(\omega_{sp}, t)} \int_{-\infty}^{\infty} \exp\left(\frac{1}{2}\phi''(\omega_{sp}, \theta)(\omega - \omega_{sp})^2\right) d\omega \\ &= \frac{E(0, \omega_{sp})}{\sqrt{2\pi}} e^{i\phi(\omega_{sp}, t)} \sqrt{\frac{2\pi}{-i\phi''(\omega_{sp})}} \end{aligned} \quad (2.54)$$

From the spectrum of the input pulse, the two transient fields at two types of saddle-points are given as

$$\begin{aligned} E_S(z, t) &= \frac{iE_0}{2\pi(\omega_{sp}^+ - \omega_p)} e^{i\phi(\omega_{sp}^+, t)} \sqrt{\frac{2\pi}{-i\phi''(\omega_{sp}^+, t)}} \\ &\approx \frac{iE_0}{\sqrt{2\pi}\Delta_{sp}^+} \frac{e^{i\phi(\omega_{sp}^+, t) - i\pi/4}}{\sqrt{\alpha_0 z \gamma / \xi(t)^3}} \approx \frac{iE_0}{\sqrt{2\pi}} \frac{e^{i(\sqrt{2\alpha_0 z \gamma \tau} - \pi/4)}}{(2\alpha_0 z \gamma \tau)^{1/4}} \end{aligned} \quad (2.55)$$

$$\begin{aligned} E_B(z, t) &= \frac{iE_0}{2\pi(\omega_{sp}^- - \omega_p)} e^{i\phi(\omega_{sp}^-, t)} \sqrt{\frac{2\pi}{-i\phi''(\omega_{sp}^-, t)}} \\ &\approx \frac{iE_0}{\sqrt{2\pi}\Delta_{sp}^-} \frac{e^{i\phi(\omega_{sp}^-, t) + i\pi/4}}{\sqrt{\alpha_0 z \gamma / \xi(t)^3}} \approx \frac{iE_0}{\sqrt{2\pi}} \frac{e^{-i(\sqrt{2\alpha_0 z \gamma \tau} - \pi/4)}}{(2\alpha_0 z \gamma \tau)^{1/4}} \end{aligned} \quad (2.56)$$

Here, the first approximation can be made because of the on-resonance condition $\omega_p \approx \omega_0$, so that $\omega_{sp}^{\pm} - \omega_p \approx \omega_{sp}^{\pm} - \omega_0 = \Delta_{sp}^{\pm}$.

The second approximation is due to the narrow-resonance condition ($\gamma \rightarrow 0$), so that $\Delta_{sp}^{\pm}(t) \approx \xi(t)$.

$$\begin{aligned}
E_{SB}(z, t) &= E_S(z, t) + E_B(z, t) \\
&= \sqrt{\frac{2}{\pi\sqrt{2\alpha_0 z\gamma\tau}}} E_0 e^{-i\omega_p\tau} \cos\left(\sqrt{2\alpha_0 z\gamma\tau} - \frac{\pi}{4}\right)
\end{aligned} \tag{2.57}$$

By assuming highly absorptive media, the total transmitted field only consists of Sommerfeld and Brillouin precursors, and main signal is zero due to absorption by two-level atoms. In the next section, we will discuss EIT media where the delayed main signal can be transmitted without absorption.

From the Bessel function approximation $J_0(x) \approx \sqrt{\frac{2}{\pi x}} \cos(x - \pi/4)$, we finally obtain the total transient field as,

$$E_{SB}(z, t) = E_0 J_0(\sqrt{2\alpha_0 z\gamma\tau}) \Theta(\tau) e^{-\gamma\tau} e^{i(k_0 z - \omega_0 \tau)} \tag{2.58}$$

The above expression is equivalent to the zeros order of Bessel term in Eq. (2.48) for on-resonance condition. One can prove that for a step-off pulse $E_0(z = 0, t) = E_0 \Theta(-t)$ with falling edge, the SB precursor is the same as equation (2.58) but with a minus sign. Therefore, for step input pulse $E_0(z = 0, t) = E_0 \Theta(\pm t)$, we obtain

$$E_{SB}(z, t) = \pm E_0 J_0(\sqrt{2\alpha_0 z\gamma\tau}) \Theta(\tau) e^{-\gamma\tau} e^{i(k_0 z - \omega_0 \tau)} \tag{2.59}$$

The interesting thing is that the identical expression derived in two approaches has been discussed over and over in the past with various impulse responses, such as 0π -pulse [13]. By solving Maxwell-Bloch equation, the analytic expression of 0π -pulse turned out to be exactly the same as Eq. (2.59). Although quite a few could notice that 0π -pulse is identical to resonant precursors [14], majority of optical community, even Crisp, had denied the existence of optical precursors in small area (0π)-pulse analysis. However, due to the experimental demonstration of resonant precursors, the analogy between optical precursors and 0π -pulse can be accepted and both phenomena are interpreted as coherent transients [15].

2.3.2 Main Signal Propagation in Electromagnetic Induced Transparency Medium

Conventional theory of optical precursors deals with a single Lorentz oscillator, as we have discussed so far. In a single Lorentz medium, the main signal corresponding to the pole $\omega \rightarrow \omega_0$ is absorbed heavily. What happens if we consider EIT medium? How does the change affect total transmitted field? We would like to answer such questions in this last section based on the Ref. [18].

For a three-level EIT system, the model of simple Lorentz oscillators does not apply. We can instead take a semi-classical approach to obtain the dielectric function. As depicted in Fig. 2.7a, the three-state system is coupled with a strong

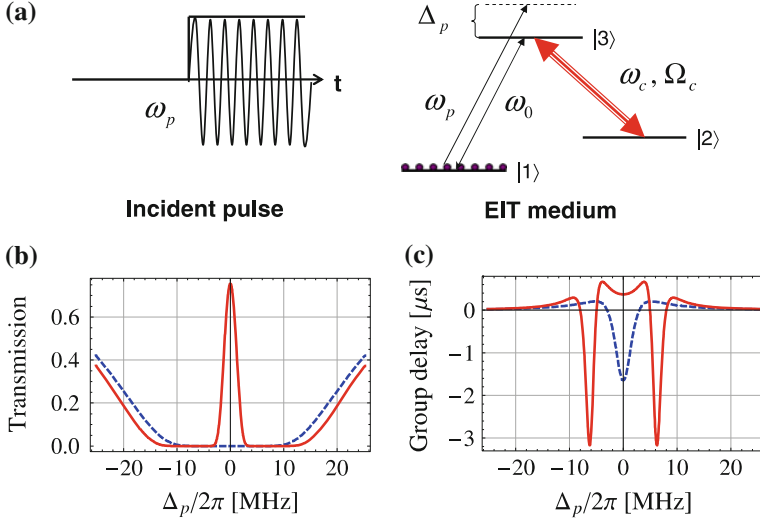


Fig. 2.7 Optical pulse propagation through an EIT. The figure was published in Jeong H and Du S (2009) Phys. Rev. A(R) 79: 011802

coupling field. Assume that the coupling field is on-resonance with the transition $|2\rangle \rightarrow |3\rangle$. The Hamiltonian in the rotating-wave wave is shown below, including the relaxation mechanism with decay rate of $|2\rangle$ ($|3\rangle$) as $\Gamma_2 = 2\gamma_{12}$ ($\Gamma_3 = 2\gamma_{13}$):

$$H_{eff} \doteq \hbar \begin{bmatrix} 0 & 0 & -\frac{1}{2}\Omega_p^* \\ 0 & -\Delta_p - \frac{i\Gamma_2}{2} & -\frac{1}{2}\Omega_c^* \\ -\frac{1}{2}\Omega_p & -\frac{1}{2}\Omega_c & -\Delta_p - \frac{i\Gamma_3}{2} \end{bmatrix} \quad (2.60)$$

where $\Delta_p = \omega_p - \omega_0$. From $|\varphi\rangle_R = a_1(t)|1\rangle + a_2(t)|2\rangle + a_3(t)|3\rangle$, the coupled differential equations:

$$i\hbar \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \\ \dot{a}_3 \end{bmatrix} = \hbar \begin{bmatrix} 0 & 0 & -\frac{1}{2}\Omega_p^* \\ 0 & -\Delta_p - \frac{i\Gamma_2}{2} & -\frac{1}{2}\Omega_c^* \\ -\frac{1}{2}\Omega_p & -\frac{1}{2}\Omega_c & -\Delta_p - \frac{i\Gamma_3}{2} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad (2.61)$$

With ground state approximation $a_1 \approx 1$, the steady state for the system considering relaxation mechanism can be obtained from the following calculation:

$$\begin{aligned} 0 &= \left(-\Delta_p - \frac{i\Gamma_2}{2}\right)a_2 - \frac{1}{2}\Omega_c^*a_3 \\ 0 &= -\frac{1}{2}\Omega_p - \frac{1}{2}\Omega_c a_2 - \left[\Delta_p + \frac{i\Gamma_3}{2}\right]a_3 \end{aligned} \quad (2.62)$$

Therefore, the probability amplitude of state $|3\rangle$ is:

$$a_3 = \frac{2\Omega_p(\Delta_p + \frac{i\Gamma_2}{2})}{|\Omega_c|^2 - 4(\Delta_p + \frac{i\Gamma_3}{2})(\delta + \frac{i\Gamma_2}{2})} \quad (2.63)$$

The induced polarization density is:

$$\begin{aligned} P &= \langle \varphi | \hat{P} | \varphi \rangle \\ &= N(a_1^* a_3 \mu_{13} e^{-i\omega_p t} + a_2^* a_3 \mu_{23} e^{-i\omega_c t} + a_1 a_3^* \mu_{13}^* e^{i\omega_p t} + a_2 a_3^* \mu_{23}^* e^{i\omega_c t}) \end{aligned} \quad (2.64)$$

If only the polarization induced by the probe laser beam is considered, Eq. (2.62) becomes:

$$P = N(a_1^* a_3 \mu_{13} e^{-i\omega_p t} + a_1 a_3^* \mu_{13}^* e^{i\omega_p t}) \quad (2.65)$$

Therefore, we obtain the linear susceptibility χ of this three-level system:

$$\begin{aligned} \chi(\omega) &= \frac{2N\mu_{13}a_3}{\varepsilon_0 E_p} = \frac{N|\mu_{13}|^2}{\varepsilon_0 \hbar} \cdot \frac{4(\Delta_p + \frac{i\Gamma_2}{2})}{|\Omega_c|^2 - 4(\Delta_p + \frac{i\Gamma_3}{2})(\Delta_p + \frac{i\Gamma_2}{2})} \\ &= \frac{\alpha_0 z}{k_p z} \cdot \frac{2\Gamma_3(\Delta_p + \frac{i\Gamma_2}{2})}{|\Omega_c|^2 - 4(\Delta_p + \frac{i\Gamma_3}{2})(\Delta_p + \frac{i\Gamma_2}{2})} \end{aligned} \quad (2.66)$$

For EIT medium, with high optical depth condition $\alpha_0 z \gg 1$, the saddle points are far-detuned from the resonance, we can treat the SB precursor field the same as a two-level system. The analytic expression of the SB precursors is same as Eqs. (2.58–2.59). The big difference comes from the main signal part which is not absorbed within the EIT window. To describe the main signal, the above stationary phase approximation (2.50) is not appropriate because $\phi''(\omega_c) = 0$. To obtain the main field expression, we could deal with the impulse response (Green's function) of the EIT window $[(\omega_0 - \Delta_e, \omega_0 + \Delta_e)]$,

$$G_{EIT}(z, t) = \frac{1}{2\pi} \int_{\omega_0 - \Delta_e}^{\omega_0 + \Delta_e} e^{i[k(\omega) - \omega t]} d\omega \quad (2.67)$$

where $\Delta_e \equiv \Omega_c/2$. As we convolute the input signal $E(0, t)$ with Green's function $G_{EIT}(z, t)$, the main signal $E_C(z, t)$ can be expressed as

$$\begin{aligned} E_C(z, t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} G_{EIT}(z, t - \tau) E_0(0, \tau) d\omega \\ &= \int_0^{\infty} G_{EIT}(z, t - \tau) d\omega \end{aligned} \quad (2.68)$$

The interesting point is that Eq. (2.68) implies active control of main signal by varying the parameters of the medium and control beam.

With reasonable approximation, the analytical expression of the main field is obtained as follows [19]. The ground-state dephasing is negligible, i.e., $\gamma_{12} \approx 0$ or

$|\Omega_c|^2 \gg 4\gamma_{12}\gamma_{13}$. The carrier frequency of the probe beam is ω_p , while we now denote ω as the frequency deviation from ω_p due to the spectral constituents of the finite probe pulse. Since the EIT transparency window is very narrow, the EIT linear susceptibility can be expanded to the third order near $\omega = 0$:

$$\begin{aligned} \chi(\omega) &\approx \chi(0) + \chi'(0)\omega + \frac{1}{2}\chi''(0)\omega^2 + \frac{1}{6}\chi'''(0)\omega^3 \\ &= \frac{4i\alpha_0\gamma_{12}\gamma_{13}}{k_{p0}\Omega_c^2} + \frac{4\alpha_0\gamma_{13}}{k_{p0}\Omega_c^2}\omega + \frac{\omega^2}{2} \frac{32i\alpha_0\gamma_{13}^2}{k_{p0}\Omega_c^4} + \frac{\omega^3}{6} \frac{12\alpha_0\gamma_{13}(-\frac{32\gamma_{13}^2}{\Omega_c^6} + \frac{8}{\Omega_c^4})}{k_{p0}} \end{aligned} \quad (2.69)$$

The transfer function can then be approximated as:

$$T(\omega) = e^{-\gamma_{12}\tau_g} e^{-\omega^2/(2a^2)} e^{i\omega\tau_g} e^{i\omega^3/(3b^3)} \quad (2.70)$$

where $\tau_g = 2\alpha_0 z \gamma_{13} / |\Omega_c|^2$ is the group delay, $a = \sqrt{\alpha_0 z} / (2\tau_g)$ determines the EIT bandwidth, $b = |\Omega_c|^2 [24\alpha_0 z \gamma_{13} (|\Omega_c|^2 - 4\gamma_{13}^2)]^{-1/3}$. The impulse response function can be expressed as a convolution:

$$h(t) = \frac{ab}{\sqrt{2\pi}} e^{-\gamma_{12}\tau_g} e^{-\frac{1}{2}a^2(t-\tau_g)^2} * Ai(-bt) \quad (2.71)$$

$Ai(bt)$ is an Airy function, which comes from the third-order term of the linear susceptibility. The main field is the convolution of the input step pulse with the impulse response function:

$$\begin{aligned} E_{M\pm}(t) &= E_0 \Theta(\pm t) * h(t) \\ &= \frac{E_0}{2} e^{-\gamma_{12}\tau_g} (1 \pm \text{erf}[\frac{a(t-\tau_g)}{\sqrt{2}}]) * bAi(-bt) \end{aligned} \quad (2.72)$$

where erf denotes the error function, which indicates that the main field is delayed by τ_g . The Airy function adds a small modulation on top of the main field and leads to the “postcursor” introduced in Mache and Segard’s work in 2009. But in most cases, the Airy function effect is small and can be ignored.

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