

# Preface

This textbook is intended for an introductory followed by an advanced course in linear algebra, with emphasis on its interactions with other topics in mathematics, such as calculus, geometry, and combinatorics. We took a straightforward path to the most important topic, linear maps between vector spaces, most of the time finite dimensional. However, since these concepts are fairly abstract and not necessarily natural at first sight, we included a few chapters with explicit examples of vector spaces such as the standard  $n$ -dimensional vector space over a field and spaces of matrices. We believe that it is fundamental for the student to be very familiar with these spaces before dealing with more abstract theory. In order to maximize the clarity of the concepts discussed, we included a rather lengthy chapter on  $2 \times 2$  matrices and their applications, including the theory of Pell's equations. This will help the student manipulate matrices and vectors in a concrete way before delving into the abstract and very powerful approach to linear algebra through the study of vector spaces and linear maps.

The first few chapters deal with elementary properties of vectors and matrices and the basic operations that one can perform on them. A special emphasis is placed on the Gaussian Reduction algorithm and its applications. This algorithm provides efficient ways of computing some of the objects that appear naturally in abstract linear algebra such as kernels and images of linear maps, dimensions of vector spaces, and solutions to linear systems of equation. A student mastering this algorithm and its applications will therefore have a much better chance of understanding many of the key notions and results introduced in subsequent chapters.

The bulk of the book contains a comprehensive study of vector spaces and linear maps between them. We introduce and develop the necessary tools along the way, by discussing the many examples and problems proposed to the student. We offer a thorough exposition of central concepts in linear algebra through a problem-based approach. This is more challenging for the students, since they have to spend time trying to solve the proposed problems after reading and digesting the theoretical

material. In order to assist with the comprehension of the material, we provided solutions to all problems posed in the theoretical part. On the other hand, at the end of each chapter, the student will find a rather long list of proposed problems, for which no solution is offered. This is because they are similar to the problems discussed in the theoretical part and thus should not cause difficulties to a reader who understood the theory.

We truly hope that you will have a wonderful experience in your linear algebra journey.

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