Preface

Floer homology is today an essential technique in symplectic topology. Inspired by ideas of Witten and Gromov in the 1980s, it has made possible the resolution of many difficult problems, and continues to do so.

This book is devoted to the solution of one of these problems, a famous conjecture due to Arnold, which proposes to give a lower bound for the number of periodic trajectories of a Hamiltonian system using an invariant that depends only on the topology of the symplectic manifold on which this system evolves. This lower bound greatly resembles the famous Morse inequalities, which give a lower bound for the number of critical points of a function. The similarity is not accidental: Floer homology is an (infinite-dimensional) analogue of the homology of the manifold as computed by the Morse complex "à la Witten": in both cases the main role is played by the moduli spaces of trajectories connecting the critical points (of a function for the Morse homology, of a functional for the Floer homology).

In 2004–2005, we gave a course, or rather two courses, on these notions for first- and second-year graduate students. We of course began with Morse theory. We like Milnor's book very much, in which we had both learned about the existence, the abundance and especially the usefulness of Morse functions, so we started writing notes for the students, it was quite easy...

And then it became more difficult—there was no book giving the more modern point of view on Morse homology, with the construction and the invariance properties of the Morse complex defined using spaces of trajectories, which would allow us to move on to the construction of the Floer complex. We could no longer copy; we now needed to use a bit of imagination.

Having completed the first part of the course to the satisfaction of the students, we tackled Floer homology. The objects and techniques of Morse homology that we, topologists and geometers, use every day, were transformed into objects and techniques of Floer homology. The charm, or one of the charms, and the strength, of this theory, lie in the fact that in addition to geometry and topology, it uses much analysis, Fredholm operators and Sobolev spaces. Explaining this to genuine students is not an easy task. This is why we decided to continue writing lecture notes.

Even though many works of research have used and still use these techniques, and many students need them at present, no reasonably self-contained book existed on this subject.

Five years have passed, in which we have honed, corrected, lengthened, made more precise, added upper bounds, lower bounds, equalities, comparisons, stated and proved seventy-three theorems, one hundred and twenty-one propositions and one hundred and three lemmas, drawn ninety-eight figures[1] (and set out a certain number of exercises that, contrary to custom, do not contain the proof of any important result "left as an exercise for the readers")...

Five years have passed, in which other students have read these notes and have made remarks that convinced us that our notes satisfied a need and that it would be stupid not to improve them even further in order to turn them into a book.

Here is the book, devoted to the power and glory of homological methods "à la" Morse–Floer.

What You Need to Know...

It was difficult pretending to write a self-contained book. Nevertheless, it results from a course given to genuine graduate students—for whom we first needed to "recall" what a manifold is. Keeping them and their fellow students in mind, we therefore gathered at the end of the book, in three appendices, "what you need to know": the basic results of differential geometry, algebraic topology and analysis that we use. Sometimes we include complete definitions and/or proofs, other times we only give indications. The index should help readers find their way.

Thanks

To all students who have undergone this course, in particular to Emily Burgunder, Olivier Dodane, Shanna Li, Alexandre Mouton, Emmanuel Rey, Nelson Souza.

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[1] See footnote 2.

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To Claude Sabbah, for his technical advice, for having welcomed this book into his collection and especially for having patiently waited for it.

Preface to the English Edition

The first edition (in French) appeared in 2010. For this new version:

- Following the advice of the Springer referees, we added a few pages on the relation Morse/cellular homology (this is Section 4.9).
- Following the advice of some of our first readers, we corrected hundreds (and there is no exaggeration here) of misprints and mistakes (among which, we must confess, quite a few mathematical errors).[2]

We are very grateful to all these people and we thank them.

We are especially grateful to Felix Schlenk for his friendly suggestions and questions. A warm thank you again to Dusa McDuff for her kind advice and remarks.

We have also updated the list of references (see footnote 3 page 451), adding especially recent books and papers of some of the leading experts in the field, the very clear paper on the definition of the Conley–Zehnder index [43], as well as the new results obtained by one of the authors [23, 24] and a few words on the history of the Arnold conjecture by the other [6].

Note that the symbol $\square$ means either the end of the proof or the absence of proof. $\square$

Last, but not least, we thank Reinie Erné, who managed to make, very kindly and patiently, a great translation work of a quite technical text.

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[2] This led us to add a few items to those mentioned above, so that this edition contains seventy-four theorems, one hundred and twenty-four propositions, one hundred and eleven lemmas and one hundred and two figures.

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