

Contents

Preface	vii
Part I Morse Theory	
Introduction to Part I	3
1 Morse Functions	7
1.1 Definition of Morse Functions	7
1.2 Existence and Abundance of Morse Functions	8
1.3 The Morse Lemma, the Index of a Critical Point	12
1.4 Examples of Morse Functions	16
Exercises	18
2 Pseudo-Gradients	23
2.1 Gradients, Pseudo-Gradients and Morse Charts	23
2.2 The Smale Condition	36
2.3 Appendix: Classification of the Compact Manifolds of Dimension 1	48
Exercises	51
3 The Morse Complex	53
3.1 Definition of the Complex	53
3.2 The Space of Connections Between Two Critical Points, or of “Broken Trajectories”	57
3.3 Orientation, Complex over \mathbf{Z}	67
3.4 The Homology of the Complex Depends Neither on the Function Nor on the Vector Field	68
3.5 Cobordisms	75
Exercises	78

4	Morse Homology, Applications	79
4.1	Homology	79
4.2	The Künneth Formula	81
4.3	The “Poincaré” Duality	83
4.4	Euler Characteristic, Poincaré Polynomial	84
4.5	Homology and Connectedness	87
4.6	Functoriality of the Morse Homology	91
4.7	Long Exact Sequence	98
4.8	Applications	101
4.9	Appendix: The Morse Homology is the Cellular Homology ..	110
	Exercises	121

Part II The Arnold Conjecture, Floer Homology

Introduction to Part II	127
--------------------------------------	-----

5	What You Need to Know About Symplectic Geometry ..	129
5.1	Symplectic Vector Spaces	129
5.2	Symplectic Manifolds, Definition	130
5.3	Examples of Symplectic Manifolds	131
5.4	Hamiltonian Vector Fields, Hamiltonian Systems	134
5.5	Complex Structures	139
5.6	The Symplectic Group	144
6	The Arnold Conjecture and the Floer Equation	151
6.1	The Arnold Conjecture	151
6.2	Outline of the Proof, Floer Homology	154
6.3	The Action Functional	156
6.4	The Gradient, the Floer Equation	162
6.5	The Space of Solutions	164
6.6	Proof of the Compactness	175
6.7	Appendix: Functions, Closed Forms, Covers	183
6.8	Appendix: Structure of a Banach Manifold on $\mathcal{L}W$	186
7	The Geometry of the Symplectic Group, the Maslov	
	Index	189
7.1	Toward the Definition of the Index	189
7.2	The Maslov Index of a Path	196
7.3	Appendix: Construction and Properties of ρ	202
8	Linearization and Transversality	221
8.1	The Results	221
8.2	The Banach Manifold $\mathcal{P}^{1,p}(x, y)$	225
8.3	The Space of Perturbations of H	230
8.4	Linearization of the Floer Equation: Computation of the Differential of \mathcal{F}	234

8.5	The Transversality	242
8.6	The Solutions of the Floer Equation Are “Somewhere Injective”	255
8.7	The Fredholm Property	269
8.8	Computing the Index of L	285
8.9	The Exponential Decay	296
9	Spaces of Trajectories	305
9.1	The Spaces of Trajectories	305
9.2	Broken Trajectories, Gluing: Statements	311
9.3	Pre-gluing	313
9.4	Construction of $\widehat{\psi}$	315
9.5	Properties of $\widehat{\psi}$: $\widehat{\psi}$ Is an Immersion	333
9.6	Properties of $\widehat{\psi}$: Uniqueness of the Gluing	334
10	From Floer to Morse	359
10.1	The Results	359
10.2	The Linearization of the Flow of a Pseudo-Gradient Field, Proof of Theorem 10.1.3	362
10.3	Proof of Theorem 10.1.2 (Regularity)	371
10.4	The Morse and Floer Trajectories Coincide	376
11	Floer Homology: Invariance	383
11.1	The Morphism Φ^F	384
11.2	Proof of Theorem 11.1.16	397
11.3	Invariance of Φ^F : Proof of Proposition 11.2.8	413
11.4	Proof of Theorem 11.3.14	426
11.5	Conclusion of the Proof of the Invariance of the Floer Homology: Proof of Proposition 11.2.9	437
11.6	Conclusion	451
12	The Elliptic Regularity of the Floer Operator	453
12.1	Elliptic Regularity: Why and How?	453
12.2	Proof of Lemma 8.7.2	459
12.3	Proof of Theorem 12.1.2	461
12.4	(Nonlinear) Elliptic Regularity of the Floer Operator, Proofs	465
13	The Lemmas on the Second Derivative of the Floer Operator and Other Technicalities	477
13.1	Versions of the Floer Operator	477
13.2	The Two Lemmas on dF	478
13.3	The Operator \mathcal{F}_ρ	480
13.4	Proof of the Two Lemmas: The First One	484
13.5	Proof of the Two Lemmas: The Second One	491
13.6	Another Technical Lemma	497

13.7	Two Other Technical Lemmas	500
13.8	Variants with Parameter(s) of the Lemmas on the Second Derivative	507
14	Exercises for the Second Part	515
14.1	Exercises on Chapter 5	515
14.2	Exercises on Chapter 6	522
14.3	Exercises on Chapter 7	526
14.4	Exercises on Chapter 8	530
14.5	Exercises on Chapter 10	531
14.6	Exercises on Chapter 11	531
 Appendices: What You Need to Know to Read This Book		
A	A Bit of Differential Geometry	535
A.1	Manifolds and Submanifolds	535
A.2	Critical Points, Critical Values and Sard's Theorem	540
A.3	Transversality	542
A.4	Vector Fields as Differential Equations	547
A.5	Riemannian Metrics, Exponential Map	552
B	A Bit of Algebraic Topology	555
B.1	A Bit of Algebraic Homology	555
B.2	Chern Classes	559
C	A Bit of Analysis	561
C.1	The Arzelà–Ascoli Theorem	561
C.2	Fredholm Theory	562
C.3	Distribution Spaces, Weak Solutions	570
C.4	Sobolev Spaces on \mathbf{R}^n	573
C.5	The Cauchy–Riemann Equation	579
References		585
Index of Notation		589
Index		591

Morse Theory and Floer Homology

Audin, M.; Damian, M.

2014, XIV, 596 p. 114 illus., Softcover

ISBN: 978-1-4471-5495-2