

Preface by Kai Lai Chung

Paul Lévy's books were not easy of access in Kunming, China, during the war years at the Southwest Associated Universities, owing to Japanese bombing of the city.¹ I arrived at Princeton in December 1945 but it was later that I saw Captain Gil Hunt poring over *l'Addition*.² He told me that von Neumann said: reading Lévy, one soon realizes that the author thinks mathematics in a different way from *nous autres*.

I first saw Lévy at Berkeley, summer of 1950, when he was giving a course which I attended. One day I went to his office and asked him about a result for Markov chains. At once he indicated a counter-example. It must be then or soon after that I mentioned the conjecture regarding the positivity of a transition function $p_{ij}(t)$: either it vanishes for all t or it never does. In his epoch-making paper³ he began with this occasion. He calls a state i "stable" if $-p'_{ii}(0) < \infty$, "instantaneous" otherwise. A basic property of a stable state he states, is that the time-set in which a stable state occurs is (almost surely) a sequence of disjoint intervals of which the cardinality is finite up to any finite time. This must be obvious to him but missed by Monsieur Doob who published two important papers on denumerable Markov chains. The proof actually needs a little "measure-theoretic" fiddling which I presented at Feller's seminar later. The proof in Lévy's paper of the truth of the conjecture above is valid if all states are stable, as confirmed in the doctorate thesis of R.V. Chacon (at Syracuse University). It was the recently deceased Donald G. Austin who gave a beautiful complete proof of the conjecture by so-called "path-analysis."⁴ It was like Lévy that he announced his oversight in a Comptes Rendus note soon after he recognized it. He wrote me to say that he liked Austin's proof.

Turning the clock back to 1947-9, when I was at Cornell, the discovery of Lévy's long paper,⁵ not available during the war, caused a sensation with Feller and Kac.

¹See Sem. Stoch. Process. (1988).

²The book Lévy (1937).

³Ann. Sci. Ec. Norm. Super. 68:327–381 (1951).

⁴See Chung: *Markov chains*, Springer (1960 & 1967).

⁵Compos. Math. 7:283–339 (1939).

Among the numerous new results, the “arcsin” law together with its complement “uniform law” for Brownian Movement was an unheralded event. Feller and I wrote a note transferring both results to the classical setting of coin-tossing, heads or tails with fifty-fifty chance. The resulting statistical interpretation is so contrary to common-sense “intuition” that had Chevalier de Méré been at the gaming table he would certainly have rushed to consult Fermat or Pascal. Indeed the news was even reported in a popular science bulletin, of which Feller kidded me as being the informer—I was not. Lévy’s article contains such a wealth of ideas and methods which remain to be explored to this day.

Probably the best known of Lévy’s multifarious achievements is the family of “infinitely divisible laws” with numerous ramifications and “applications” including the “normal” and “stable” laws (another kind of stability, both named by the discoverer). The general formula for their Fourier transform (ch.f.) is sometimes cited as “Khinchine-Lévy,” in silly English alphabetical order.⁶ In a letter Lévy said K. completed it in the unsymmetric Cauchy case (only). In the book by Gnedenko and Kolmogorov, it was properly named. What is not well known is that in Lévy’s hands, it is just the analytic portion of his profound study of the paths of “Lévy process(es),” formerly called “additive,” “differential” etc. I must refer the really interested reader to my Review of Doob’s book *Stochastic Processes*⁷ for further comments. Let me add that if Lévy’s other renowned publications are not even mentioned here, it is because of my lack of personal knowledge of them, as well as the lack of SPACE and TIME.

The celebrated Theorem on the convolution of two Normal laws was conjectured by Lévy who actually assumed its truth in his early work on the infinitely divisible family. It was Harald Cramér who proved Lévy’s daring conjecture, using properties of entire function in complex analysis. His Cambridge tract, lent to me by Prof. Pao Lu Hsu in tattered condition, was a precious guide. At the Bicentennial convocation of Princeton University in 1946, Cramér was a visiting professor who gave two successive courses on modern probability. I was assigned to be his assistant, and he approved my dissertation (1947). Prof. Pao Lu Hsu, then in USA, was also an invited delegate, but he did not come (see my Preface to his *Collected Papers*, Springer-Verlag, 1982 with a photo of Hsu, Cramér and Bartlett in Chapel Hill). It was on the bicentennial occasion that I first met Feller, Doob, Abraham Wald among many other mathematicians known to me by name before.

On May 19, 1964 Lévy wrote me a letter part of which announcing his election to *l’Académie* (in much detail), the other part about Mrs Doebelin’s passing. As response I asked him soon after to present a Note to the *Comptes Rendus* (my final one there). It is about a “pregnant integral equation” obtained by considering a last exit

⁶By the way, it would be interesting to show the following result without using Fourier transform: for any stable non-normal law with distribution function L , we have $1 - L(x) \sim Cx^{-\alpha}$, for a constant $0 < \alpha < 2$ as $x \rightarrow +\infty$. The problem has made some progress. The most interesting result is due to Letta and Pratelli who obtained $P(X > x) \leq cx^{-\alpha}$ for any symmetric stable law.

⁷Bull. Am. Math. Soc. (March 1954). Numerous other references to Lévy are mentioned, including his precursor Martingale.

time from a boundary for Markov chain, which I had spoken of in Budapest, Prague and elsewhere. An almost everywhere solution is easy but I was unable to show it everywhere. He rewrote it after telephoning Jacques Neveu for “information” and split it in two Notes which appeared in May, 1965. The open problem was first solved by Lennart Carleson with his fine analysis, of which I wrote a reproduction in a Chinese journal published in Dalian. Lévy used to write to me while summering in Menton, the border from Ventimiglia, where my wife and I sojourned once.

During our stay in Strasbourg in 1967, where our five-year old son went to French school and learned to speak very well, the Lévy's kindly invited us to dinner in their top-floor apartment on Ave. Théophile Gautier, just above the residence of the novelist Mauriac, we found out later. The guests included the Chevalleys and the Schwartzes (I first met Laurent at Prof. Chevalley's office in Princeton University). Monsieur Lévy offered me a little cup of Port-Salut, warning me of its *parfum*. Those were marvellous days.

After the Lévy Memorial Colloque at Palaiseau in 1987, we were *chez eux* at the Schwartzes, and Laurent told some stories of his *beau-père's* peculiar memory of what he had or had not demonstrated the day before. Mme Schwartz was most kind in giving me a photo of her father and one of Emile Borel, both reproduced in my *Elementary Probability* book with six other luminaries. Lévy had passed away in 1971. That happened to be the first passage time for my return to my homeland and to see my mother in Hangchow, Marco Polo's Qinsai. “TIME MUST HAVE A STOP”

In 1976 I was able to dedicate a paper to Lévy's memory.⁸ I was giving a course on Brownian Motion (sic) with a few faculty auditors (incl. S.T. Yau now in Harvard). A basic theorem in Lévy's *étude profonde*, proved by him using a “vertical window,” baffled me as too hard to do in class, but I was happy when I found a “horizontal window” to replace it, by use of a last-exit time (Theorem 1 of the mentioned paper). The lingo was due to my friend an auditor from Operation Research occurred.

The Brownian excursions being the groundwork for Lévy's fantastic invention of “local time,” I wrote with A.A. Balkema another paper entitled “Lévy's WAY to his local time” in our *Seminar on Stochastic Processes*, 1990 (Birkhäuser). The capitalized WAY is taken from Laotze and Chuangtze, pronounced like “TAO.”

My acquaintance with Maurice Fréchet is brief: I first saw him at a group walk in a Japanese garden in Tokyo, in 1960. He looked very handsome and was most cordial. Unfortunately somehow on the crowd we did not have a chance to talk. However our first contact occurred at the time when Paris fell to the Nazis. Suddenly I received a whole bunch of reprints sent by him (I wish I had kept the large envelope, but his reprints are here.) This must be his kind response to a Note on an inequality by E.J. Gumbel I sent to Borel for possible presentation to the *Académie*. Fréchet also sent a postcard when my professor Pao Lu Hsu and I sent a Note on recurrence (1946). Later I got a galley copy of Fréchet's first volume on probabilities of a finite number of events in the *Actualités* series. I had published a number of

⁸ Ark. Mat. 14 (1976).

papers on the same subject in the Annals of Mathematical Statistics (1941–1945). Owing to the lack of communication during the war, Fréchet's two volumes and my papers did not seem to *rencontre*. Gumbel invited me to his home in Bronx, New-York City, soon after my arrival in USA near the end of November (1945). We met again in Tokyo, and we had a last supper together at the inn when all other guests had left. Indeed, he was the first mathematician/statistician I met in America. He was most kind to visit me in Princeton where I was a graduate student. Much later he shared an office with a doctorate student of mine at Columbia University. This proves that there is TIME recurrence in this mundane world. I almost forgot to say above that Fréchet's *Généralités* in the Borel collection was one of my first introduction to probability after Borel's treatise. Prof. Hsu had a copy of Fréchet's second volume on Markoff chains which I read. Only much later when I taught a course on general topology I learned Fréchet's pioneer work on that, and other kinds of mathematics (incl. economics, I learned recently from Profs Bru and Barbut). This world is connected in SPACE too.

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Paul Lévy and Maurice Fréchet

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