Preface

This book has evolved from a course in *Mathematical Writing* offered to second-year undergraduate students at Queen Mary, University of London.

Instructions on writing mathematics are normally given to postgraduate students, because they must write research papers and a thesis. However, there are compelling reasons for providing a similar training at undergraduate level, and, more generally, for raising the profile of writing in a mathematics degree.[1]

A researcher knows that writing an article, presenting a result in a seminar, or simply explaining ideas to a colleague are decisive tests of one's understanding of a topic. If a sketched argument has flaws, these flaws will surface as soon as one tries to convince someone else that the argument is correct. The act of exposition is inextricably linked to thinking, understanding, and self-evaluation.

For this reason, undergraduate students should be encouraged to elucidate their thinking in writing, and to assume greater responsibility for the quality of the exposition of their ideas. Their first-written submissions tend to be cryptic collections of symbols, which easily hide from view learning inadequacies and fragility of knowledge. It is quite possible to perform a correct calculation having only limited understanding of the subject matter, but it is not possible to write about it. A good writing assignment exposes bad studying habits (approaching formal concepts informally, or treating them as mere processes—see [1]), and provides a most effective tool for correcting them.

This course's declared objective is to teach the students how to develop and present mathematical arguments, in preparation for writing a thesis in their final year. For those who will not write a thesis, this course represents an indispensable minimal alternative, which is also more manageable in terms of teaching resources. The writing material is taken mostly from introductory courses in calculus and algebra. This suffices to challenge even the most capable students, who commented on the "unexpected depth" required in their thinking, once forced to offer verbal explanations. The students are asked to use words and symbols with the

[1] The poor quality of student writing in Higher Education has raised broad concern [44].

same clarity, precision, and conciseness found in books and lecture notes. This demanding exercise encourages logical accuracy, attention to structure, and economy of thought—the attributes of a mathematical mind. It also forces us to understand better the mathematics we are supposed to know.

The development of writing techniques proceeds from the particular to the general, from the small to the large: words, phrases, sentences, paragraphs, to end with short compositions. These may represent the introduction of a concept, the proof of a theorem, the summary of a section of a book, and the first few slides of a presentation.

The first chapter is a warm-up, listing do's and don'ts of writing mathematics. An essential dictionary on sets, functions, sequences, and equations is presented in Chaps. 2 and 3; these words are then used extensively in simple phrases and sentences. The analysis of mathematical sentences begins in Chap. 4, where we develop some constructs of elementary logic (predicates, quantifiers). This material underpins the expansion of the mathematical dictionary in Chap. 5, where basic attributes of real functions are introduced: ordering, symmetry, boundedness, and continuity. Mathematical arguments are studied in detail in the second part of the book. Chapters 7 and 8 are devoted to basic proof techniques, whereas Chap. 9 deals with existence statements and definitions. Some chapters are dedicated explicitly to writing: Chap. 1 gives basic guidelines; Chap. 6 is concerned with mathematical notation and quality of exposition; Chap. 10 is about writing a thesis. Solutions and hints to selected exercises are given in "Solutions to Exercises".

The symbol $[\not\in]$ appears often in exercises. It indicates that the written material should contain *no mathematical symbols*. (The allied symbol $[\not\in, n]$ specifies an approximate word length n of the assignment.) In an appropriate context, having to express mathematics without symbols is a very useful exercise. It brings about the discipline needed to use symbols effectively, and is invaluable for learning how to communicate to an audience of non-experts. Consider the following question: $[\not\in, 100]$.

> *I have a circle and a point outside it, and I must find the lines through this point which are tangent to the circle. What shall I do?*

The mathematics is elementary; yet answering the question requires a clear understanding of the structure of the problem, and a fair deal of organisation.

> *Write down the equation of a line passing through the point. This equation depends on one parameter, the line's slope, which is the quantity to be determined.*
>
> *Adjoin the equation of the line to that of the circle, and eliminate one of the unknowns. After a substitution, you'll end up with a quadratic equation in one unknown, whose coefficients still depend on the parameter.*
>
> *Equate the discriminant of the quadratic equation to zero, to obtain an equation—also quadratic—for the slope. Its two solutions are the desired slopes of the tangent lines. Any geometrical configuration involving vertical lines (infinite slope) will require some care.*

The most challenging exercise of this kind is the MICRO-ESSAY, where the synthesis of a mathematical topic has to be performed in a couple of paragraphs, without using any symbols at all. This exercise prepares the students for writing abstracts, a notoriously difficult task.

The available literature on mathematical writing is almost entirely targeted to postgraduate students and researchers. An exception is *How to think like a mathematician*, by K. Houston [20], written for students entering university, which devotes two early chapters to mathematical writing. The advanced texts include *Mathematical writing,* by D.E. Knuth, T.L. Larrabee, and P.M. Roberts [24], *Handbook of writing for the mathematical sciences* by N. Higham [19], and *A primer of mathematical writing* by S.G. Krantz [22]. Equally valuable is the concise classic text *Writing mathematics well,* by L. Gillman [13]. Unfortunately, this 50-page booklet is out of print, and used copies may command high prices.

The timeless, concise book *The elements of style,* by W. Strunk Jr. and E.B. White [37] is an ideal complement to the present textbook. Anyone interested in writing should study it carefully.

This book was inspired by the lecture notes of a course in logic given by Wilfrid Hodges at Queen Mary in 2005–2006. This course used writing as an essential tool, in an innovative way. Wilfrid has been an ideal companion during a decade-long effort to bring writing to centre stage in our mathematics degree, and I am much indebted to him.

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