

# Contents

<b>1</b>	<b>Numbers and Limits</b> .....	1
1.1	Inequalities .....	1
1.1a	Rules for Inequalities .....	3
1.1b	The Triangle Inequality .....	4
1.1c	The Arithmetic–Geometric Mean Inequality .....	5
1.2	Numbers and the Least Upper Bound Theorem .....	11
1.2a	Numbers as Infinite Decimals .....	11
1.2b	The Least Upper Bound Theorem .....	13
1.2c	Rounding .....	16
1.3	Sequences and Their Limits .....	19
1.3a	Approximation of $\sqrt{2}$ .....	23
1.3b	Sequences and Series .....	24
1.3c	Nested Intervals .....	36
1.3d	Cauchy Sequences .....	37
1.4	The Number $e$ .....	44
<b>2</b>	<b>Functions and Continuity</b> .....	51
2.1	The Notion of a Function .....	51
2.1a	Bounded Functions .....	54
2.1b	Arithmetic of Functions .....	55
2.2	Continuity .....	59
2.2a	Continuity at a Point Using Limits .....	61
2.2b	Continuity on an Interval .....	64
2.2c	Extreme and Intermediate Value Theorems .....	66
2.3	Composition and Inverses of Functions .....	71
2.3a	Composition .....	71
2.3b	Inverse Functions .....	74
2.4	Sine and Cosine .....	81
2.5	Exponential Function .....	86
2.5a	Radioactive Decay .....	86
2.5b	Bacterial Growth .....	87

2.5c	Algebraic Definition	87
2.5d	Exponential Growth	89
2.5e	Logarithm	91
2.6	Sequences of Functions and Their Limits	96
2.6a	Sequences of Functions	96
2.6b	Series of Functions	103
2.6c	Approximating the Functions $\sqrt{x}$ and $e^x$	107
<b>3</b>	<b>The Derivative and Differentiation</b>	<b>117</b>
3.1	The Concept of Derivative	117
3.1a	Graphical Interpretation	120
3.1b	Differentiability and Continuity	123
3.1c	Some Uses for the Derivative	125
3.2	Differentiation Rules	133
3.2a	Sums, Products, and Quotients	133
3.2b	Derivative of Compositions of Functions	138
3.2c	Higher Derivatives and Notation	141
3.3	Derivative of $e^x$ and $\log x$	146
3.3a	Derivative of $e^x$	146
3.3b	Derivative of $\log x$	148
3.3c	Power Rule	149
3.3d	The Differential Equation $y' = ky$	150
3.4	Derivatives of the Trigonometric Functions	154
3.4a	Sine and Cosine	154
3.4b	The Differential Equation $y'' + y = 0$	156
3.4c	Derivatives of Inverse Trigonometric Functions	159
3.4d	The Differential Equation $y'' - y = 0$	161
3.5	Derivatives of Power Series	166
<b>4</b>	<b>The Theory of Differentiable Functions</b>	<b>171</b>
4.1	The Mean Value Theorem	171
4.1a	Using the First Derivative for Optimization	174
4.1b	Using Calculus to Prove Inequalities	179
4.1c	A Generalized Mean Value Theorem	181
4.2	Higher Derivatives	186
4.2a	Second Derivative Test	191
4.2b	Convex Functions	192
4.3	Taylor's Theorem	197
4.3a	Examples of Taylor Series	202
4.4	Approximating Derivatives	209
<b>5</b>	<b>Applications of the Derivative</b>	<b>217</b>
5.1	Atmospheric Pressure	217
5.2	Laws of Motion	220
5.3	Newton's Method for Finding the Zeros of a Function	225
5.3a	Approximation of Square Roots	226

5.3b	Approximation of Roots of Polynomials	227
5.3c	The Convergence of Newton’s Method	229
5.4	Reflection and Refraction of Light	234
5.5	Mathematics and Economics	240
<b>6</b>	<b>Integration</b>	<b>245</b>
6.1	Examples of Integrals	245
6.1a	Determining Mileage from a Speedometer	245
6.1b	Mass of a Rod	247
6.1c	Area Below a Positive Graph	249
6.1d	Negative Functions and Net Amount	252
6.2	The Integral	254
6.2a	The Approximation of Integrals	257
6.2b	Existence of the Integral	261
6.2c	Further Properties of the Integral	265
6.3	The Fundamental Theorem of Calculus	271
6.4	Applications of the Integral	281
6.4a	Volume	281
6.4b	Accumulation	284
6.4c	Arc Length	284
6.4d	Work	287
<b>7</b>	<b>Methods for Integration</b>	<b>291</b>
7.1	Integration by Parts	291
7.1a	Taylor’s Formula, Integral Form of Remainder	295
7.1b	Improving Numerical Approximations	297
7.1c	Application to a Differential Equation	299
7.1d	Wallis Product Formula for $\pi$	299
7.2	Change of Variables in an Integral	302
7.3	Improper Integrals	310
7.4	Further Properties of Integrals	326
7.4a	Integrating a Sequence of Functions	326
7.4b	Integrals Depending on a Parameter	329
<b>8</b>	<b>Approximation of Integrals</b>	<b>333</b>
8.1	Approximating Integrals	333
8.1a	The Midpoint Rule	335
8.1b	The Trapezoidal Rule	336
8.2	Simpson’s Rule	339
8.2a	An Alternative to Simpson’s Rule	343
<b>9</b>	<b>Complex Numbers</b>	<b>347</b>
9.1	Complex Numbers	347
9.1a	Arithmetic of Complex Numbers	348
9.1b	Geometry of Complex Numbers	352

9.2	Complex-Valued Functions	361
9.2a	Continuity	362
9.2b	Derivative	362
9.2c	Integral of Complex-Valued Functions	364
9.2d	Functions of a Complex Variable	365
9.2e	The Exponential Function of a Complex Variable	368
<b>10</b>	<b>Differential Equations</b>	<b>375</b>
10.1	Using Calculus to Model Vibrations	375
10.1a	Vibrations of a Mechanical System	375
10.1b	Dissipation and Conservation of Energy	379
10.1c	Vibration Without Friction	381
10.1d	Linear Vibrations Without Friction	385
10.1e	Linear Vibrations with Friction	387
10.1f	Linear Systems Driven by an External Force	391
10.2	Population Dynamics	398
10.2a	The Differential Equation $\frac{dN}{dt} = R(N)$	399
10.2b	Growth and Fluctuation of Population	405
10.2c	Two Species	409
10.3	Chemical Reactions	420
10.4	Numerical Solution of Differential Equations	428
<b>11</b>	<b>Probability</b>	<b>435</b>
11.1	Discrete Probability	436
11.2	Information Theory: How Interesting Is Interesting?	446
11.3	Continuous Probability	452
11.4	The Law of Errors	463
	<b>Answers to Selected Problems</b>	<b>475</b>
	<b>Index</b>	<b>501</b>



<http://www.springer.com/978-1-4614-7945-1>

Calculus With Applications

Lax, P.D.; Terrell, M.S.

2014, XII, 503 p. 220 illus., Hardcover

ISBN: 978-1-4614-7945-1