Preface

Our purpose in writing a calculus text has been to help students learn at first hand that mathematics is the language in which scientific ideas can be precisely formulated, that science is a source of mathematical ideas that profoundly shape the development of mathematics, and that mathematics can furnish brilliant answers to important scientific problems. This book is a thorough revision of the text *Calculus with Applications and Computing* by Lax, Burstein, and Lax. The original text was predicated on a number of innovative ideas, and it included some new and nontraditional material. This revision is written in the same spirit. It is fair to ask what new subject matter or new ideas could possibly be introduced into so old a topic as calculus. The answer is that science and mathematics are growing by leaps and bounds on the research frontier, so what we teach in high school, college, and graduate school must not be allowed to fall too far behind. As mathematicians and educators, our goal must be to simplify the teaching of old topics to make room for new ones.

To achieve that goal, we present the language of mathematics as natural and comprehensible, a language students can learn to use. Throughout the text we offer proofs of all the important theorems to help students understand their meaning; our aim is to foster understanding, not "rigor." We have greatly increased the number of worked examples and homework problems. We have made some significant changes in the organization of the material; the familiar transcendental functions are introduced before the derivative and the integral. The word "computing" was dropped from the title because today, in contrast to 1976, it is generally agreed that computing is an integral part of calculus and that it poses interesting challenges. These are illustrated in this text in Sects. 4.4, 5.3, and 10.4, and by all of Chap. 8. But the mathematics that enables us to discuss issues that arise in computing when we round off inputs or approximate a function by a sequence of functions, i.e., uniform continuity and uniform convergence, remains. We have worked hard in this revision to show that uniform convergence and continuity are more natural and useful than pointwise convergence and continuity. The initial feedback from students who have used the text is that they "get it."

This text is intended for a two-semester course in the calculus of a single variable. Only knowledge of high-school precalculus is expected.

Chapter 1 discusses numbers, approximating numbers, and limits of sequences of numbers. Chapter 2 presents the basic facts about continuous functions and describes the classical functions: polynomials, trigonometric functions, exponentials, and logarithms. It introduces limits of sequences of functions, in particular power series.

In Chapter 3, the derivative is defined and the basic rules of differentiation are presented. The derivatives of polynomials, the exponential function, the logarithm, and trigonometric functions are calculated. Chapter 4 describes the basic theory of differentiation, higher derivatives, Taylor polynomials and Taylor's theorem, and approximating derivatives by difference quotients. Chapter 5 describes how the derivative enters the laws of science, mainly physics, and how calculus is used to deduce consequences of these laws.

Chapter 6 introduces, through examples of distance, mass, and area, the notion of the integral, and the approximate integrals leading to its definition. The relation between differentiation and integration is proved and illustrated. In Chapter 7, integration by parts and change of variable in integrals are presented, and the integral of the uniform limit of a sequence of functions is shown to be the limit of the integrals of the sequence of functions. Chapter 8 is about the approximation of integrals; Simpson's rule is derived and compared with other numerical approximations of integrals.

Chapter 9 shows how many of the concepts of calculus can be extended to complex-valued functions of a real variable. It also introduces the exponential of complex numbers. Chapter 10 applies calculus to the differential equations governing vibrating strings, changing populations, and chemical reactions. It also includes a very brief introduction to Euler's method. Chapter 11 is about the theory of probability, formulated in the language of calculus.

The material in this book has been used successfully at Cornell in a one-semester calculus II course for students interested in majoring in mathematics or science. The students typically have credit for one semester of calculus from high school. Chapters 1, 2, and 4 have been used to present sequences and series of numbers, power series, Taylor polynomials, and Taylor's theorem. Chapters 6–8 have been used to present the definite integral, application of integration to volumes and accumulation problems, methods of integration, and approximation of integrals. There has been adequate time left in the term then to present Chapter 9, on complex numbers and functions, and to see how complex functions and calculus are used to model vibrations in the first section of Chapter 10.

We are grateful to the many colleagues and students in the mathematical community who have supported our efforts to write this book. The first edition of this book was written in collaboration with Samuel Burstein. We thank him for allowing us to draw on his work. We wish to thank John Guckenheimer for his encouragement and advice on this project. We thank Matt Guay, John Meluso, and Wyatt Deviau, who while they were undergraduates at Cornell, carefully read early drafts of the manuscript, and whose perceptive comments helped us keep our student audience in mind. We also wish to thank Patricia McGrath, a teacher at Maloney High School in Meriden, Connecticut, for her thoughtful review and suggestions, and Thomas

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New York, NY Peter Lax
Ithaca, NY Maria Terrell

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